Don Ethridge

DEMAND AND PRICE ANALYSIS

Some examples from agriculture

ECONOMIC AND STATISTICAL ANALYSIS DIVISION / ECONOMIC RESEARCH SERVICE

U.S. DEPARTMENT OF AGRICULTURE

Technical Bulletin No. 1316

"The seal of the true science is the confirmation of its forecasts; its value is measured by the control it enables us to exercise over ourselves and our environment."

HENRY L. MOORE

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by Frederick V. Waugh, Research Adviser,
Economic Research Service

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Preface

I hope this bulletin will be helpful to many groups of persons, including undergraduate and graduate students, young researchers who are beginning to get practical experience in demand and price analysis, and agricultural outlook workers, both in the U.S. Department of Agriculture and in the State extension services. This last group is especially important. The State outlook worker is not doing an adequate job when he simply "carries back the word from Washington" concerning the outlook for demand and prices in agriculture. He must understand the analysis in back of the outlook. He must take the outlook and apply it to the particular situation in his State; then he must be able to explain it to the farmer in simple terms. Similarly, research men must have a good basic understanding of demand analysis if they are to help legislators and administrators to improve farm programs.

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I sincerely thank the many people who helped me make this study, without implying that they agree with all the opinions, views, or judgments I have expressed.

Martin E. Abel and Forrest Walters of Economic Research Service worked closely with me on the entire manuscript. The following economists outside the Government were consulted and made valuable contributions to various parts of this report: George E. Brandow, Richard J. Foote, Karl A. Fox, George G. Judge, Geoffrey S. Shepherd, O. C. Stine, and Holbrook Working. Also, Ragnar Frisch and Leif Johansen of the University of Oslo made substantial contributions to Chapter 7 on marginal utility and indifference. In addition, Kenneth W. Meinken, and Sidney Hoos contributed to Chapters 9 and 10 on distribution among markets.

The following colleagues in the U.S. Department of Agriculture should be acknowledged for their contributions to all or most of the manuscript: James P. Cavin, Willard W. Cochrane, Rex F. Daly, Lee M. Day, Nathan M. Koffsky, Anthony S. Rojko, W. Neil Schaller, Forrest Scott, William H. Waldorf, and Hyman Weingarten.

I believe most economists and statisticians agree on what I consider to be the basic principles of demand analysis. But there are some differences of opinion about the importance of some current fashions in econometric literature. In such cases, I have expressed my personal views—even where they differ from the judgments of some respected economists and statisticians.

FREDERICK V. WAUGH.

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Highlights

● Why study demand and prices of farm products? Who uses the results and how? What value are such studies to the general public? The farmer, the cooperative association, and the food distributor must have accurate forecasts of consumption and prices for making intelligent adjustments in production and marketing. The administrator and the legislator must have sound theories and accurate measurements of demand as a guide to programs, policies, and legislation. For our democracy to work toward the general welfare, every citizen should know the basic facts of demand. Our economic performance can be no better than our understanding of economic theory and our ability to forecast the quantitative effects of proposed actions.

● The demand for food is quite inelastic, both with respect to price and income. This statistical fact lies at the heart of the farm problem. A small surplus in agriculture depresses prices severely. And farmers usually get only slight benefits from increases in consumer income. Moreover, demand at the farm level is more inelastic than at the retail level. In other words, farm prices are more flexible than retail prices. This is because price spreads between the farm and the retail store are not generally percentages. They are more nearly constant amounts in dollars and cents.

● Since World War II, there apparently have been substantial shifts in the demand for meats. The demand curve for beef has gone up. The curves for pork, lamb, and veal have gone down. Consumption of chickens has gone up sharply, as a result of lower prices; but the demand curve itself apparently has remained about fixed. Of course, there is competition among these foods. Because we need to understand a lot more about such competition, this bulletin analyzes the interrelationships of demand for beef, pork, and chickens.

● The marginal utility of money is an important economic concept with significant practical applications. The percentage change in the marginal utility of money resulting from a 1-percent increase in income is termed money flexibility. It is estimated in this bulletin by an analysis of food prices.

● The long-run demand for any commodity is likely to be more elastic (or less inelastic) than the short-run demand. Cotton is currently of special interest. The short-run domestic demand for cotton is known to be very inelastic. But it is also widely recognized that relatively high cotton prices may, in the long run, reduce cotton consumption substantially. This bulletin attempts to measure the long-run domestic demand for cotton, using a form of distributed lag. Elasticity of the long-run demand is estimated at about −1.8. This indicates that a 1-percent increase in cotton prices would eventually result in a drop of 1.8 percent in domestic cotton consumption.

● The income (or "returns") from a crop often is affected greatly by the crop's distribution among different places, times, forms, and groups of consumers. Most theoretical discussions have been limited to special cases of independent markets. This bulletin discusses general principles of distribution—whether or not the markets are independent. It then shows how these principles apply to the diversion of surplus wheat to exports and the diversion of surplus lemons to processed products.

DEMAND AND PRICE ANALYSIS:

Some Examples From Agriculture

By FREDERICK V. WAUGH

Research Adviser
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Aims of Research in Demand and Prices

Research in demand and prices has followed two different paths, searching two different goals. Most of the early studies were devoted to pure, abstract theory. Since Henry L. Moore's work, beginning in 1914, many statistical studies have tried to measure the quantitative relationships between prices and consumption.

Both these aims are good, desirable, and necessary. They need not conflict with one another. Clear theoretical concepts and correct theory are basic to progress in any field. A correct theory is "practical." It explains in principle how prices are actually made, and how consumers actually respond to changes in prices and in their incomes. It should guide sound statistical research on demand and prices. Also, good statistical research helps to sharpen up theoretical concepts and to amplify theory in the fields of demand and prices—as it has done in physics, for example.

Let us consider in more detail the two main aims of research in demand and prices.

To Develop a Theory of Demand

A theory is simply an explanation. Empiricism is not enough. Science is not a collection of miscellaneous facts; it is an orderly classification of facts and an explanation of their interrelationships.

The so-called pure theory of demand is based upon logical reasoning as to how men would act under certain stated conditions: For example, universal competition, full information, and the maximization of individual satisfactions. Such pure abstract reasoning can be an end in itself, just as it was in the geometry of the ancient Greeks. Pareto was quite explicit that his work was not aimed at numerical estimation of prices.[1] Rather, he was interested in pure logic.

There is still plenty of room for pure logic and for mathematical economic theory in the field of demand and prices. Such theories often are fascinating. But logic alone is not enough. It needs to be checked by actual observation of how prices are made in the market—just as Aristotle's reasoning about falling bodies had to be checked a

[1] Pareto, Vilfredo. MANUEL D'ÉCONOMIE POLITIQUE. Giard et Briere, Paris, 1909. P. 233: "Remarquons, d'ailleurs, que cette determination n'a nullement pour but d'arriver a un calcul numérique des prix."

thousand years later by Galileo, who was interested in how bodies actually did fall. The test of good theory—whether in physics or in economics—is whether it explains what actually happens in the real world.

There need be no conflict between theory and statistical analysis in the field of demand and prices. They are both essential. And they need to march forward hand in hand.

To Estimate or Forecast

While some are interested mainly in economic theory, others want to forecast expected prices—or, at least, to estimate the prices that could be expected under certain assumed conditions. For this purpose, economic theory must be concrete, quantitative, statistical.

Such work calls for applied economic theory. But sometimes the main emphasis has been upon measurement, and theory has been kept in the background. For example, Cassels and Black wrote in 1933, "The statistical price research which has become so important within the past 15 years has been directed mainly to practical ends, and not to the confirmation, amplification, or correction of any theoretical explanation of value."[2]

But, as good theory is essential to good forecasting, good statistical measurement and statistical analysis are essential to developing economic theories that adequately describe how prices are actually made in real markets.

In most practical cases, estimating and forecasting are not aims in themselves. Rather, we need estimates and forecasts in order to accomplish other important purposes. The farmer, processor, distributor, or speculator needs a forecast of probable future prices to decide intelligently when to buy or sell.

Moreover, economic forecasts and projections are not fatalistic prophecies like those made by the ancient astrologers, numerologists, and soothsayers. The U.S. Department of Agriculture is making economic projections 5, 10, or more years into the future. These projections are estimates of what the economic situation would be in agriculture several years from now under each of several alternative farm programs. Such projections should be of great value in helping farmers and other citizens to decide what kind of agricultural programs they want.

1. Lessons From History

> . . . It may be doubted whether Jevon's hope of constructing demand curves by statistics is capable of realization.—F. Y. Edgeworth in *Demand Curves*. From Palgrave's Dictionary of Political Economy.

Even though practically all of the statistical work in demand has been done during the past 50 years, it now covers hundreds of books and articles. I shall not attempt to catalog them here. Rather, I shall point out a few of the important trends.

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[2] Social Science Research Council. RESEARCH IN PRICES OF FARM PRODUCTS. New York, 1933.

The Concept of a Demand Curve

The two most important concepts of demand were first clearly stated by Frenchmen in the middle of the 19th century. Cournot wrote:

> Let us admit therefore that the sales or the annual demand D is, for each article, a particular function F (p) of the price p of such article . . . Observation must therefore be depended on for furnishing the means of drawing up between proper limits a table of the corresponding values of D and p; after which, by the well-known methods of interpolation or by graphic processes, an empiric formula can be made to represent the function in question; and the solution of problems can be pushed as far as numerical applications.[1]

A similar concept was developed independently by another Frenchman, Dupuit.[2] Marshall extended and popularized these concepts.[3]

Another Frenchman, Walras, developed a different sort of concept, or at least a substantial elaboration of the Cournot concept.[4] He visualized the demand for any commodity as a function of all other commodities and services, together with consumer income. Each consumer, in trying to maximize his own satisfactions, would spend his money in such a way that the marginal utility of a dollar's worth of each commodity was equal to that of each other commodity or service. Thus, a two-dimensional relation between price of a commodity and the quantity of that commodity purchased would be determined only upon the assumption that all other prices and consumer income were held constant.

Walras' concept was elaborated by Pareto.[5] A modern version will be found in Hicks, especially in his mathematical appendix.[6]

The Walras-Pareto-Hicks concept provides an elaborate and intriguing model for pure economic theory. It is not adapted to the measurement of demand by statistical analysis. Most statistical work on demand has used concepts similar to those proposed by Cournot, Dupuit, and Marshall. Moore and Schultz discussed the Walras-Pareto concept in detail, but they based their statistical analysis on Marshall.

Early Statistical Studies

The year 1914 will be remembered as the beginning of World War I. Many statistical economists will also remember the year as marking the publication of two of the first serious attempts to measure demand statistically.

One of these was a 5-page paper by Lehfeldt.[7] He attempted to measure the elasticity of the true demand curve for wheat, considering the whole world as a single market. For this purpose he used a curve similar to what was later called an "orthogonal regression." He first

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[1] Cournot, Augustin. RESEARCHES INTO THE MATHEMATICAL PRINCIPLES OF THE THEORY OF WEALTH. 1838. Translation by Nathaniel T. Bacon. P. 47. Macmillan, New York. 1929.

[2] Jules Dupuit's writings (1844 through 1853) were republished under the title, DE L'UTILÍTÉ' ET DE SA MESURE. Torino, Italy. 1933.

[3] Marshall, Alfred. PRINCIPLES OF ECONOMICS. 8th ed. Macmillan.

[4] Walras, Léon. ÉLEMENTS D'ÉCONOMIE POLITIQUE PURE. Paris, 1900.

[5] Pareto, Vilfredo. MANUALE DI ECONOMIA POLITICA. Societa Editrice Libraria, Milano, Italy, 1907.

[6] Hicks, J. R. VALUE AND CAPITAL. Clarendon Press, Oxford, 1939 and later editions.

[7] Lehfeldt, R. A. THE ELASTICITY OF DEMAND FOR WHEAT. Econ. Jour., XXIV. Pp. 212–217. June 1914.

found the logarithms of the world average price and total quantity of wheat for each year, took the year-to-year changes in these logarithms, and used the ratio of the standard deviations of these changes as an estimate of demand elasticity. His justification was:

> . . . it seemed to me that the best way to deal with the deviations of p and q was to take the ratio of their standard deviations . . . In support of this view I am glad to be able to quote the opinion of Dr. G. D. Maynard, with whom I have discussed the problem.

Professor Lehfeldt evidently had no interest in estimating the expected price of wheat associated with given quantities, nor in estimating the expected consumption associated with given prices. His curve would have been entirely unsuitable for either of these purposes.

The year 1914 also marked the publication of the first of a very remarkable series of books by Henry L. Moore.[8] I do not think Stigler[9] overstated the case when he wrote:

> If one seeks distinctive traits of modern economics, traits which are not shared to any important degree with Marshallian or earlier periods, he will find only one: the development of statistical estimation of economic relationships. Mathematical analysis became increasingly more common after Walras's first edition; statistical descriptions of economic phenomena were expanding throughout the nineteenth century; bold pronouncements on public policy are as old as economics. But statistical economics, the name given by Henry Moore, is the one important modern development . . . Henry Moore was its founder.

Moore's writings have become classics in statistical demand analysis. They pointed the way to most of the statistical research that has been done in this area in the past 50 years.

In retrospect, Professor Moore seems to have tried to do two things at once: first, develop curves that could be used to forecast expected prices; and, second, measure the true demand curves of economic theory. His general method was least squares regressions. They were applied in a wide variety of ways: to the original data, to year-to-year changes, and to percentages of trends. He used various mathematical functions; for example, those providing for constant and for changing elasticity at different parts of the curve.

In general, Moore's curves are still sound when they are used for his first purpose; that is, for estimating expected prices or expected consumption. But few statistical analyses will do both things at once; that is, provide the closest estimates of expected prices and of the true demand curve of economic theory. Still, when most of the shifts in demand can be explained, and thus, when the residual errors are small, the best estimating curve and the true curve of theory are nearly identical. Often, when searching for the best estimating equation, we are likely to get a very good estimate of the true theoretical curve.[10]

One of the early mileposts in this history was Holbrook Working's study of the demand for potatoes in 1922.[11] Working has since made a number of other statistical studies. He has also written many informative articles about economic theory and research methods.

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[8] Moore, Henry L. ECONOMIC CYCLES: THEIR LAWS AND CAUSE (1914); FORECASTING THE YIELD AND PRICE OF COTTON (1917); GENERATING ECONOMIC CYCLES (1923); SYNTHETIC ECONOMICS (1929). Macmillan, New York.

[9] Stigler, George J. HENRY L. MOORE AND STATISTICAL ECONOMICS. Econometrica, Vol. 30, No. 1, Jan. 1962.

[10] This point was made in a letter of Oct. 1, 1963, from Holbrook Working to Frederick V. Waugh.

[11] Working, Holbrook. FACTORS DETERMINING THE PRICE OF POTATOES IN ST. PAUL AND MINNEAPOLIS. Univ. Minn. Agr. Expt. Sta. Bul. 10, 1922.

Statistical studies of demand were in their infancy when the Bureau of Agricultural Economics was set up in the early 1920's. Such studies were greatly stimulated by H. C. Taylor, O. C. Stine, H. R. Tolley, and other leaders in the BAE.

Two of the early examples of such studies were Killough on oat prices, and Haas and Ezekiel on hog prices.[12 13] Scores of reports of statistical demand studies have been written in the BAE and successor agencies since the 1920's. They are still forthcoming. Also, many reports have been published, and are being published, by land-grant colleges and by the Journal of Farm Economics.

These studies had a very practical purpose. Often the central purpose was to estimate or forecast prices. Sometimes it was to estimate or forecast consumption or trade. Such forecasts were intended to give the farmer and the food trades basic information they needed for making profitable adjustments in production and marketing. Out of this effort and similiar work in the land-grant colleges grew the agricultural outlook service.

This service of the U.S. Department of Agriculture does not stop with publishing technical appraisals and interpretations of the economic situation and outlook for a large number of commodities. It also is being used more and more to estimate the probable effects of alternative agricultural programs, such as those to adjust production or support prices. The purpose is essentially to project expected prices, farm income, Government costs, etc., that would result if certain actions were taken. They call for expected prices, expected consumption, expected trade, and so on, conditional upon stated assumptions.

Other Studies

Statistical studies of demand developed rapidly in the 1920's and 1930's. This development was not limited to the Federal Government nor to the United States. Holbrook Working's early study on potatoes has already been mentioned. Dozens of studies dealing with other commodities were published in the 1920's.

In 1928, Warren and Pearson published a large collection of statistical studies of demand and supply for agricultural commodities.[14] This was the first of several major efforts to make systematic studies of demand for a large number of commodities and to publish them in one place.

European economists and statisticians also made a number of important studies in the 1920's and early 1930's. Among the most important were Hanau's study of hog prices in 1928,[15] a general study by Leontief in 1929,[16] and some interesting theoretical and statistical

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[12] Killough, Hugh B. WHAT MAKES THE PRICE OF OATS? U.S. Dept. Agr. Bul. 1351. 1925.

[13] Haas, G. C. and Ezekiel, M. J. B. FACTORS AFFECTING THE PRICE OF HOGS. U.S. Dept. Agr. Bul. 1440. 1926.

[14] Warren, G. F. and Pearson, F. A. INTERRELATIONSHIPS OF SUPPLY AND PRICE. Cornell Agr. Expt. Sta. Bul. 466, Ithaca, N.Y., 1928.

[15] Hanau, Arthur. DIE PROGNOSE DER SCHWEINPREISE. Institut für Konjunkturforschung, Berlin, 1928.

[16] Leontief, Wassily. EIN VERSUCH ZUR STATISTICHEN ANALYSE VON ANGEBOT UND NACHFRAGE. Weltwirtschaftlisches Archiv., XXX, 1929.

work by Roy in 1935.[17] Ezekiel published the first edition of his book on correlation analysis in 1930.[18] This for many years was the bible of those using least squares methods. It has recently been revised to include a short discussion of simultaneous methods.

Henry Schultz published his monumental work on demand in 1938.[19] This combined a review of economic theory with a large number of statistical studies. This book is well-known throughout the world.

Stone brought out in 1958 a book on demand in Great Britian which covered somewhat the same ground as that covered by Schultz' book in the United States.[20]

Several important textbooks have been published on agricultural prices. They include books by Thomsen and Foote,[21] Shepherd,[22] and Waite and Trelogan.[23] Foote also prepared a detailed handbook explaining the methods used in analyzing simultaneous equations of supply and demand.[24]

Concepts and Practical Applications

In practical applications, the interest is not generally in the true demand curves of economic theory. If some other curves will give closer estimates of prices or consumption, they are preferred. In such cases, there is no "problem of identification." Mr. Killough, for example, in his 1925 study discussed in some detail the factors affecting the price of oats. He did not bother to mention the true demand curves of economic theory, except in a footnote which read:

> "A suggestion has been made that these . . . curves do not exactly correspond to the economic concept of a demand curve."

The early critics were usually negative. Their comments were like that of Edgeworth, quoted at the beginning of this section, that it was impossible to derive a true demand curve from statistics.

Holbrook Working was more positive and more useful: [25]

> While it is natural that these statistical studies of demand and price should be commonly characterized as studies of the elasticity of demand, their true significance cannot properly be appreciated from this point of view. They should be considered rather as attempts to add to our knowledge of the more general and fundamental question of the factors determining prices. The concept of elasticity of demand, as developed from the law of diminishing utility, forms a part of a general scheme of approach to the subject of value into which the present statistical approach does not readily fit. Probably we are in the process of developing a somewhat different plan of approach to the theory of value which must be worked out along independent lines.

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[17] Roy, René. ÉTUDES ÉCONOMOÉTRIQUES. Sirely, Paris, 1935.

[18] Ezekiel, M. J. B. METHODS OF CORRELATION ANALYSIS. Wiley & Sons, editions of 1930 and 1941. Third edition, with Karl A. Fox, 1959.

[19] Schultz, Henry. THE THEORY AND MEASUREMENT OF DEMAND. Univ. Chicago Press, 1938.

[20] Stone, Richard. THE MEASUREMENT OF CONSUMERS' EXPENDITURE AND BEHAVIOUR IN THE UNITED KINGDOM. Cambridge Univ. Press, 1954.

[21] Thomsen, Frederick L. AGRICULTURAL PRICES. McGraw-Hill, 1936. Most recent edition with Richard J. Foote, 1952.

[22] Shepherd, Geoffrey S. AGRICULTURAL PRICE ANALYSIS. Iowa State Univ. Press, Ames. Editions of 1941, 1947, 1950, 1957, 1963.

[23] Waite, W. C. and Trelogan, H. C. INTRODUCTION TO AGRICULTURAL PRICES 1948 and AGRICULTURAL PRICE ANALYSIS 1951. Wiley & Sons.

[24] Foote, Richard J. ANALYTICAL TOOLS FOR STUDYING DEMAND AND PRICE STRUCTURES. U.S. Dept. Agr. Handb. 146,1958.

[25] Working, Holbrook. THE STATISTICAL DETERMINATION OF DEMAND CURVES. Jour. Econ. XXXIX, August 1925.

The economic profession has not followed the lead suggested in the last sentence of this quotation. It is high time that we develop new theories and concepts of value that are testable by statistical analyses. If statistical findings fail to confirm the theories inherited from our predecessors, should we struggle to invent elaborate methods to try to reconcile the facts with the theory? Rather like the physical scientists, we should modify theoretical concepts to make them fit the observed facts in the actual marketplace.

The first step in any statistical analysis should be to set up some sort of theoretical model describing how the markets for a commodity work. The model generally starts with a listing of factors that are believed to affect the supply, demand, and price of the commodity. Diagrams are often helpful in portraying various interrelationships. Finally, the model should be put into a form that can be fitted by statistical techniques to determine if it is consistent with the observed data. To set up a good model for measuring the demand for any commodity, the researcher must have an intimate understanding of the markets for that particular commodity. The routine fitting of the same model to cotton, beef cattle, and canned peas is poor research method.

Important articles by Elmer Working and by Haavelmo have spelled out in some detail the reasons why the usual least squares analysis and time series do not result in a true demand curve of economic theory.[26 27] Since those theoretical articles, various statistical methods have been developed to try to estimate true demand curves. Such methods are explained in detail in two reports of the Cowles Commission.[28]

Much of the literature on this subject is abstract. To some extent, however, the methods have been applied to concrete statistical problems, especially in the Department of Agriculture. Under the leadership of Richard J. Foote, about 10 bulletins were published, giving the results of structural analyses through simultaneous demand and supply equations for various farm products.

Without doubt, such statistical studies can give us valuable insight into the structure of agricultural markets; that is, they can help show us how a number of demand and supply functions work out simultaneously. So far, the structural equations have not been adequately tested in forecasting a dependent variable, such as price. But as Marschak showed, least squares should give the best unbiased forecasts, unless the basic structure of demand has changed.[29] Structural equations may be useful in forecasting the results of known changes in structure, such as a new support price.

Fox found that, in actual practice, least squares equations for agricultural products and feeds were practically identical with those obtained from the more elaborate simultaneous methods.[30] Wold has

[26] Working, Elmer. WHAT DO STATISTICAL DEMAND CURVES SHOW? Jour. Econ., XLI, 1927.

[27] Haavelmo, Trygve. THE STATISTICAL IMPLICATIONS OF A SET OF SIMULTANEOUS EQUATIONS. Econometrica, Vol. 11, Jan. 1943.

[28] Koopmans, T. C. ed. STATISTICAL DIFFERENCE IN DYNAMIC ECONOMIC MODELS. Wiley & Sons, New York, 1950.
Hood, W. C. and Koopmans, T. C., ed. STUDIES IN ECONOMETRIC METHOD. Wiley & Sons, New York, 1953.

[29] Marschak. Jacob. STUDIES IN ECONOMETRIC METHOD. Edited by N. C. Hood and T. C. Koopmans, Wiley, New York, 1953.

[30] Fox, Karl A. THE ANALYSIS OF DEMAND FOR FARM PRODUCTS. U.S. Dept. Agr. Tech Bul. 1081, 1953.

made some interesting theoretical and statistical studies of "recursive" relations which are very common in agriculture.[31] In a recursive situation, the current value of each variable depends upon previous values of other variables. For example, this year's price may depend on production decisions that were made last year, and influenced by last year's prices. Wold has shown that in fully recursive models, least squares simple equations give the best estimates, both of expected price and quantities, and also of the true curves of economic theory (assuming no errors of measurement). Harlow's study of hog prices used a recursive analysis.[32]

Future Studies

I hope that in the future we can build on the suggestion made by Holbrook Working back in 1925. Instead of sticking with the concepts of demand developed by Cournot, Walras, and Marshall, why not redefine demand as the expected quantities that would be purchased at given prices, with certain other things held constant? It is not practical in statistical work to hold everything else constant (as some followers of Walras would insist). For example, it is not feasible to hold prices of all other commodities and services constant. But in statistical work, we usually want to hold consumer income constant and perhaps also to hold constant the prices of a few of the leading substitute commodities.

I think there is too much concern with "the" elasticity of demand for a commodity. In practice, such elasticities commonly vary from market to market, from use to use, from grade to grade, from time to time, from one part of the curve to another . . . and so on, almost *ad infinitum*. I doubt if there is any such thing as "the" elasticity of demand for wheat, for example. There would always be at least two kinds of demand elasticity: the elasticity of expected consumption with respect to price and the elasticity of expected price with respect to the quantity sold. In general, these are not reciprocals of one another, because the two regression lines differ from one another.

The curves of expected purchases as a function of prices and of expected prices as a function of quantities sold probably should not be called demand curves—in order to avoid confusion. Back in the 1920's and 1930's, they were sometimes called "supply-price" curves, and "expected price-marketings" curves. Whatever name is most suitable, these curves are usually the ones that are needed in most practical work.

One of the most promising recent developments in demand analysis is Brandow's study, which is a synthesis of several statistical analyses resulting in a "demand matrix" exhibiting all the direct elasticities and cross elasticities of demand among many groups of farm products.[33] Brandow's matrix is proving very valuable in estimating the direct and indirect effects of proposed changes in farm programs. It also may serve as somewhat of a bridge between the partial-equilibrium

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[31] Wold, Herman and Juréen, Lars. DEMAND ANALYSIS. Wiley & Sons, New York, 1953.

[32] Harlow, Arthur. FACTORS AFFECTING THE PRICE AND SUPPLY OF HOGS. U.S. Dept. Agr. Tech. Bul. 1274,1962.

[33] Brandow, G. E. INTERRELATIONS AMONG DEMANDS FOR FARM PRODUCTS AND IMPLICATIONS FOR CONTROL OF MARKET SUPPLY. Penn. Agr. Expt. Sta. Bul. 680, Univ. Park, 1961.

analysis of Cournot-Marshall and the general equilibrium of Walras-Pareto-Hicks.

Wold's recursive models also should have a great future—both as tools of economic theory and as means of forecasting expected prices, consumption, and production.

To make real progress, the statistician, the economic theorist, and the mathematician must cooperate closely with one another. Pure economic theory is merely idle amusement unless it is tested and applied. On the other hand, the compilation of statistical data is of no value unless the data are used to test and to quantify theory.

2. Total Food Consumption Related to Retail Price and to Consumer Income

The desire of food is limited in every man by the narrow capacity of the human stomach.—ADAM SMITH in *The Wealth of Nations.*

This bulletin covers a variety of statistical studies. It does not attempt a comprehensive analysis of demands for all products. Rather, it gives examples of some of the wide variety of studies that are needed. Chapters 2 through 6 discuss a number of common problems in the statistical analysis of time series. Chapter 7 considers some implications of statistical findings to economic theory. Chapter 8 analyzes long-term demand. Chapters 9 and 10 apply statistical findings to problems of intermarket distribution and discriminative pricing.

This chapter starts with a simple graphic analysis of the demand for food as a whole. At the end of the chapter are some numerical results. The goal of this chapter is the determination of the total consumption of food in response to changes in food prices and in consumer incomes.

The Data

First to be decided is the way to measure total food consumption. Several measures are available.[1]

Total poundage of food or total caloric content are not of concern. Poundage and caloric content do not respond much to changes in consumer income, nor to changes in food prices. Moreover, the farmer and the consumer are interested in the quality of the diet, as well as poundage.

In one sense, Adam Smith was right that demand for food is limited. Today, we are not eating many more pounds of food than we did 20 or 30 years ago—but what changes have been made in the quality and variety of foods in our diets! And these changes are significant both to consumers and farmers.

We shall use the index of per capita food consumption. This is a price-weighted index. That is, the quantities of various foods are weighted by the retail prices of those foods in a base period. Thus, a pound of meat counts more than a pound of potatoes. The USDA Handbook states that this index is regarded as the best available measure of changes in the overall food consumption at the retail level. It

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[1] These measures are described in CONSUMPTION AND UTILIZATION OF AGRICULTURAL PRODUCTS, Vol. 5 of MAJOR STATISTICAL SERIES OF THE U.S. DEPARTMENT OF AGRICULTURE. Agr. Handb. 118. 1957. The measure used here is described on pp. 65 and 66 of the Handbook.

TABLE 2.1.—*Food consumption related to prices and incomes*

[1957–59=100]

Year	Retail food price [1]	Disposable income per capita [2]	Ratio of food prices to income	Food consumption per capita [3]
1926	57.6	35.3	163.2	90.1
1927	55.5	35.0	158.6	88.9
1928	54.9	35.4	155.1	88.9
1929	55.6	37.0	150.3	89.1
1930	52.9	32.7	161.8	88.7
1931	43.6	27.9	156.3	88.0
1932	36.3	21.1	172.0	85.9
1933	35.3	19.7	179.2	86.0
1934	39.3	22.3	176.2	87.1
1935	42.1	24.9	169.1	85.4
1936	42.5	28.0	151.8	88.5
1937	44.2	29.9	147.8	88.4
1938	41.0	27.4	149.6	88.6
1939	39.9	29.2	136.6	91.7
1940	40.5	31.2	129.8	93.3
1941	44.2	37.8	116.9	95.1
	(World War II years excluded)			
1947	81.3	64.0	127.0	99.9
1948	88.2	70.0	126.0	96.7
1949	84.7	68.9	122.9	96.7
1950	85.8	74.2	115.6	98.0
1951	95.4	80.0	119.2	96.1
1952	97.1	82.4	117.8	98.1
1953	95.6	85.8	111.4	99.1
1954	95.4	85.8	111.2	99.1
1955	94.0	90.0	104.4	99.8
1956	94.7	94.4	100.3	101.5
1957	97.8	97.8	100.0	99.9
1958	101.9	99.0	102.9	99.1
1959	100.3	103.2	97.2	101.0
1960	101.4	104.9	96.7	100.7
1961	102.9	107.3	95.6	100.8
1962	103.5	110.9	93.3	101.0
1963 [4]	105.0	117.0	89.7	101.8

[1] Bureau of Labor Statistics, food component of the Consumer Price Index (not deflated).
[2] U.S. Department of Commerce, income after taxes (not deflated).
[3] U.S. Department of Agriculture, retail price-weighted index.
[4] 1963 data are preliminary. They were not used in the analysis.

reflects the quantitative aspects of changes in consumer demand, but excludes the effects of demand on prices. But it does incorporate shifts from lower priced to higher priced foods.

The index of per capita food consumption is shown in the last column of table 2.1. This table omits data for the World War II years 1942–46. These years are omitted from all analyses in this report because the usual demand relationships were obscured by food rationing and price control.

The first two columns of table 2.1 show the indexes of retail food price and of per capita disposable income. The retail food price is the food component of the Consumer Price Index published by the U.S. Bureau of Labor Statistics. It is based upon prices reported by a large sample of retail food stores in 46 cities throughout the country. In constructing the index, the prices of the several foods are weighted by the quantities of these foods commonly bought by typical workingmen's families. The index of per capita disposable income is based upon the income and population estimates of the U.S. Department of Commerce. Disposable income is total income minus taxes.

We want to find the relationship between three variables: retail food price, per capita disposable income, and per capita food consumption. In this case, the problem can be simplified by "deflating" the index of food price by dividing it by the index of per capita disposable income. Such deflated prices (i.e., the index of food price divided by the index of per capita income) are shown in column 3 of table 2.1.

Deflation

This analysis has not followed the standard convention of deflating prices by dividing them by the Consumer Price Index. In many cases such standard deflation is logical, and works well. But I agree with Shepherd that no standard technique of deflation is applicable to all problems.[3] Especially when simplifying the analysis to two variables—the quantity consumed and some kind of deflated price—it may often be convenient to deflate by dividing prices by consumer income. This assumes that a doubling of food prices and a doubling of consumer income would not significantly change the quantity of food consumed. Such an assumption seems fairly reasonable—at least if all other prices were doubled, too—as demonstrated by the fact that the observations on figures 2.1 and 2.2 are clustered fairly closely around the freehand lines. When using consumer income as a deflator, a rise in income reduces the "real" price; that is, a smaller proportion of income is needed to purchase the same foods at the same nominal prices. Aside from savings, income is the sum of all prices multiplied by the respective quantities bought. Thus, when food prices are divided by consumer income, they are, in a sense, deflated by the weighted average of all prices.

Dot Charts

We want to find the relationship between the data in column 3 and column 4 of table 2.1. This can be done best with a dot chart, such as the one in figure 2.1. Properly used, the dot chart is a powerful tool of demand analysis. It has been too much neglected in recent years.

First, we plot on the chart the pair of observations for each year. We plot the food-consumption data on the horizonal (x) axis and the corresponding deflated price (i.e., price/income) data on the vertical (y) axis. For example, take the data for 1926—the first year shown in table 2.1. The index of per capita food consumption for 1926 was 90.1, and the index of deflated price was 163.2. So we measure off 90.1 units on the x-axis, and go up 163.2 units on the y-axis, locating the point marked 26. Similarly, the point marked 27 shows the two indexes for 1927 and so on.

Points for the years 1926 through 1941 are clustered rather closely around the line marked "prewar," and points for the years from 1948 through 1962 are clustered around the line marked "postwar" (except

[3] Shepherd, Geoffrey S. AGRICULTURAL PRICE ANALYSIS. Iowa State Univ. Press, Ames, pp. 121–131, 1963.

for the observation for 1947, when conditions reflected a holdover of wartime demand). These two lines were drawn in roughly, "eyeball method," without any of the distractions of mathematics. They are not necessarily the "lines of best fit" in any exact sense. One could have drawn a single line. Agriculture has progressively been producing more food per capita at lower "real" prices.

Note that this says nothing about cause and effect, nor about predetermination and postdetermination. It is purely arbitrary which data are plotted on the x-axis and which on the y-axis. Our data could just as well be plotted as shown in figure 2.2. Neither chart—and no statistical analysis—will demonstrate cause and effect.

Years ago, the convention in European countries was to draw dot charts like figure 2.2 with price on the x-axis and consumption on the y-axis. Presumably, this was because students thought of price changes as being the cause of consumption changes. In studying responses of consuming families, this seems most reasonable. To them, price changes are given data; their changes in purchases are the result.

But for many years, the convention in the United States has been to draw dot charts like figure 2.1 with consumption (or marketings) on the x-axis and price on the y-axis. Apparently, this is because U.S. students have commonly looked at another aspect of the problem. The quantity of potatoes, say, to be consumed was determined primarily by the size of the crop, and market prices had to be adjusted so the crop would move into consumption.

As I see it, most arguments about cause and effect are futile as far as statistical analysis is concerned. Regardless of the direction of cause and effect, regardless of any notions about which variable is "predetermined" or "exogenous," figures 1 and 2 are equally good. In fact, they are the same thing; the axes are simply reversed.

Comments on the Curves

Figure 2.1 shows that deflated food prices were highest in the depression years of the 1930's. Since then, deflated food prices have gradually dropped as per capita food consumption has increased. Remembering the sort of deflation used in this case, we might say that food prices have risen less than consumer incomes—and that this development has been associated with expanding per capita production and consumption.

The shift from the prewar to the postwar curve, although not large, is of considerable interest. It suggests that the postwar curve shifted slightly upward and to the right. This would mean that a given per capita consumption would be associated with a slightly higher deflated price in the postwar period than in the prewar period.

Similiar conclusions could be drawn from a study of figure 2.2. For example, figure 2.2 indicates that a given deflated food price in the postwar period would be associated with slightly higher per capita food consumption than in the prewar period. This is only stating the conclusions from figure 2.1 the other way around—which is natural because figure 2.2 is nothing but figure 2.1 turned around.

We have mentioned that the lines in figures 2.1 and 2.2 are not necessarily the lines of best fit. By fitting an appropriate mathematical curve, the statistician could doubtless gain a few decimal points of accuracy. But the fitting of a mathematical curve assures only that it

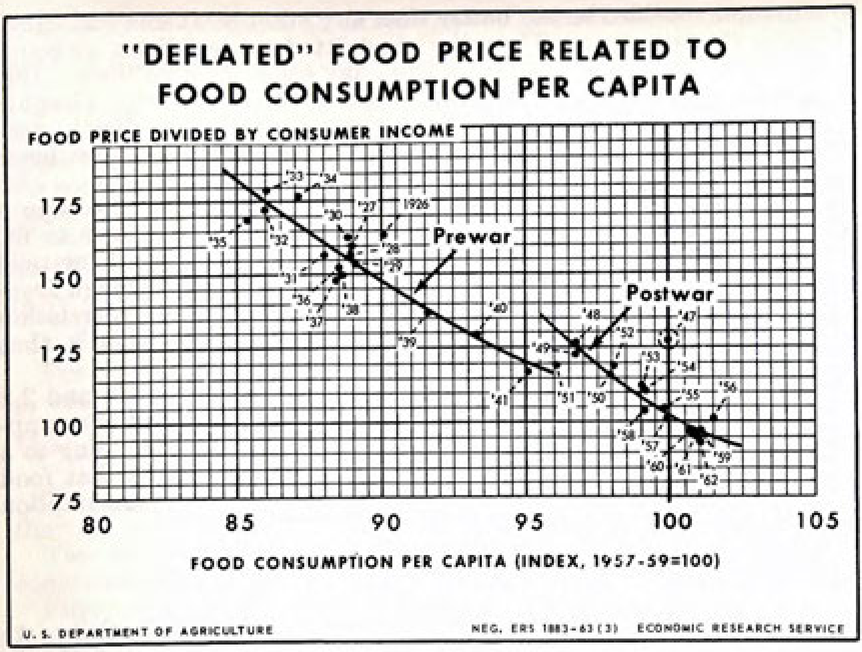

Figure 2.1

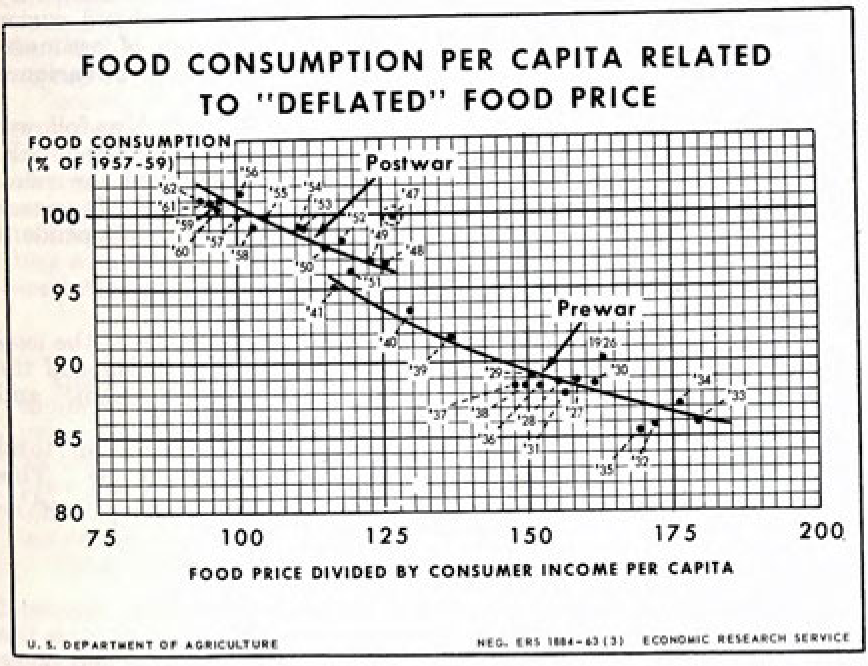

Figure 2.2

fits (in some specified sense) better than any other mathematical curve of *exactly the same type*. Thus, if we had fitted straight lines, we could be sure that they would fit better than any other straight lines. But the fit might not be so good as that of the freehand curves. Graphic analysis—especially the study of dot charts—avoids the deadly routine of always fitting straight lines (or straight lines in logarithms) whether or not they are appropriate.

If we want a mathematical curve at all, the proper time to compute it is after doing enough graphics to know what sort of curve to fit. Here we could get along well enough without computing mathematical lines. We could estimate price or consumption closely enough from the freehand curves. We do not really need to compute correlation coefficients. We can see that the correlations are high—that is, that the scatters around the lines are low.

The preliminary data for 1963 were plotted on figures 2.1 and 2.2 after the figures were drawn. Figure 2.2 shows that food consumption was slightly lower than the expected amount corresponding to a "deflated" food price of 91.2. Similarly, figure 2.1 shows that food prices were slightly lower than those expected with a food consumption index of 101.0.

Correlation Coefficients From Freehand Curves

If we want correlation coefficients, or coefficients of elasticity, we can estimate them from figure 2.1. The coefficient of correlation is the square root of the coefficient of determination. The coefficient of determination is the proportion of variance in the dependent variable explained by the independent variable. This can be computed from figure 2.1, because the amount of unexplained variance is shown by the vertical deviations of the observations from the freehand curve. Then vertical deviations are called residuals or errors of estimate. This unexplained variance can be substracted from the total variance to determine the explained variance.

The simplest way to estimate the correlation coefficient is as follows:

(1) Compute each residual or error of estimate, square the residuals, sum them, and divide by the number of observations to get the mean squared residual. The mean squared residual is the amount of *variance* remaining in the dependent variable *unexplained* by the independent variable.

(2) Compute the variance of the dependent variable—*total variance*.

(3) Compute the proportion that *unexplained variance* is of the *total variance* by dividing the mean squared residual by the variance of the dependent variable. This is called the "alienation coefficient" and may be represented by A^2_{xy}.

(4) Compute the proportion that explained variance is of total variance by substracting the "alienation coefficient" from one. This is the coefficient of determination and is represented by $r^2_{xy}=1-A^2_{xy}$.

Elasticity Coefficients From Freehand Curves

It is easy enough to compute elasticity coefficients from freehand curves. Let x and y be any two related variables. Then dy/dx is the slope of the line at any given point. Take a straight edge and place it so it is tangent to the curve at the given point. The slope of the tangent is the slope of the curve. And the elasticity of y with respect to x at the point y_k, x_k is $E_{yx}=\frac{dy}{dx}\frac{x_k}{y_k}$.

For example, we might use figure 2.2 to estimate "elasticities of demand", that is, the elasticity of per capita food consumption with respect to deflated food price. For the prewar period, we might take the point $x_k=150$, $y_k=89.7$. (Here, y_k is the value of the curve associated with $x_k=150$.) At that point, the slope of the curve is -0.149. (That is, y goes down 0.149 units for each increase of 1 unit of x.) So for the prewar period,

$$E_{qp}=-0.149\times\frac{150}{89.7}=-0.25. \tag{2.1}$$

For the postwar period the elasticity corresponding to $x_k=110$, $y_k=98.7$ is

$$E_{qp}=-0.154\times\frac{110}{98.7}=-0.17. \tag{2.2}$$

This indicates that the postwar demand for food was less elastic than the prewar demand.

Two comments might be made about the elasticities indicated in equations (2.1) and (2.2).

First, these elasticities are much lower than the elasticities usually found for individual foods. The elasticity of the aggregate demand for food as a whole is not a sort of an average of elasticities of demand for individual foods. When the price of an individual food changes, leaving prices of other foods unchanged, the consumer can easily substitute one food for another. Such substitution is impossible when the prices of all foods go up. While the elasticities indicated in equations (2.1) and (2.2) are lower than those usually found for individual foods, they are in line with the findings of many other studies of the aggregate demand for food as a whole.

Second, the drop in elasticity from a prewar figure of -0.25 to a postwar figure of -0.17 is not surprising. As incomes and levels of living increase, the demand for food is bound to become less and less elastic. In technical terms, this is because the marginal utility of money decreases. (This will be discussed in Chapter 7.) In less technical terms, when incomes are high, consumers are inclined to buy whatever foods they want, and are not much influenced by moderate changes in prices.

Mathematical Analysis

The relationships between food consumption, food price, and consumer income can be estimated more precisely by mathematical analysis. For this purpose, we do not need to deflate by dividing price by consumer income. Rather, we can measure the separate effects of price and of consumer income. But, as shown in table 2.2, I have followed the standard practice of deflating both price and consumer income by dividing each by the Consumer Price Index.

Thus, q is per capita food consumption

p is deflated food price (i.e., price/CPI)

y is per capita deflated consumer income (i.e., consumer income/population×CPI)

The general slope of the curves in figures 2.1 and 2.2 suggested a linear analysis in logarithms. Such an analysis gave these results:

$$(2.3)\quad \text{for 1927–41, } \log q = 1.98 - \underset{(0.05)}{0.24} \log p + \underset{(0.02)}{0.24} \log y. \quad (R^2 = 0.907)$$

$$\text{for 1948–62, } \log q = 2.19 - \underset{(0.15)}{0.24} \log p + \underset{(0.05)}{0.14} \log y. \quad (R^2 = 0.874)$$

The numbers in parentheses are the sampling errors of the coefficients above them. To be statistically significant, a coefficient should

TABLE 2.2.—*Indexes of food consumption, food price, and consumer income*

[1957–59=100]

Year	Food consumption per capita (q)	Food price [1] (p)	Consumer income [2] (y)
1927	88.9	91.7	57.7
1928	88.9	92.0	59.3
1929	89.1	93.1	62.0
1930	88.7	90.9	56.3
1931	88.0	82.3	52.7
1932	85.9	76.3	44.4
1933	86.0	78.3	43.8
1934	87.1	84.3	47.8
1935	85.4	88.1	52.1
1936	88.5	88.0	58.0
1937	88.4	88.4	59.8
1938	88.6	83.5	55.9
1939	91.7	82.4	60.3
1940	93.3	83.0	64.1
1941	95.1	86.2	73.7
	(World War II years excluded)		
1948	96.7	105.3	82.1
1949	96.7	102.0	83.1
1950	98.0	102.4	88.6
1951	96.1	105.4	88.3
1952	98.1	105.0	89.1
1953	99.1	102.6	92.1
1954	99.1	101.9	91.7
1955	99.8	100.8	86.5
1956	101.5	100.0	99.8
1957	99.9	99.8	99.9
1958	99.1	101.2	98.4
1959	101.0	98.8	101.8
1960	100.7	98.4	101.8
1961	100.8	98.8	103.1
1962	101.0	98.4	105.5
1963 [3]	101.8	98.4	107.9

[1] Retail prices of Bureau of Labor Statistics, deflated by dividing by Consumer Price Index.

[2] Per capita disposable income, deflated by dividing by Consumer Price Index.

[3] 1963 data are preliminary. They were not used on the analysis.

be at least twice its sampling error. So the coefficient of log p in the postwar equation is not significant.

A most interesting statistical fact is that in the prewar equation, the coefficients of log p and of log y are equal, but of opposite signs. This justifies the kind of deflation used in our graphic analysis. The first equation in (2.3) can be written

$$q=95.5\ p^{-0.24}\ y^{0.24}=95.5\ (p/y)^{-0.24}.$$

So price divided by income has a sound statistical basis—at least in the prewar period.

The coefficients of log p in this analysis are elasticities of expected consumption with respect to retail prices (holding income constant). This is often called the price elasticity, but it is not the elasticity of price. These elasticities are −0.24 in each period. Our graphic analysis (equations 2.1 and 2.2) indicated elasticities of −0.25 in the prewar period and −0.17 in the postwar period. These results are remarkably similar, especially when we note the sampling error associated with the −0.24 in the second equation of (2.3).

Equations (2.3) also give the elasticities of food consumption with respect to income (holding price constant)—often called income elasticity. They are +0.24 in the prewar period and +0.14 in the postwar period.

The price elasticities of −0.24 mean (approximately) that each 1 percent increase in price would be associated with a reduction of −0.24 percent in food consumption (consumer income held constant). No change is seen here from prewar to postwar.

The income elasticity of +0.14 in the postwar period means that each 1 percent increase in consumer income was associated with an increase of 0.14 percent in food consumption (price held constant). This was much less elastic than in the prewar period, when a 1 percent increase in income was associated with an increase of 0.24 percent in food consumption.

To dig deeper into the interrelationships between consumption, price, and income, I tried some equations like

$$\log q=a+b\log p+c\log y+d\log p\log y. \tag{2.4}$$

The high intercorrelation between log p and log y made the sampling errors of the regressions rather large. But adding together the data for the two periods and computing (2.4) for the composite, I found

$$\log q=3.49-\underset{(.37)}{1.00}\log p-\underset{(.41)}{0.72}\log y+\underset{(.21)}{0.49}\log p\log y. \tag{2.5}$$

$(R^2=0.977)$

Although the coefficient of log y is not fully significant, this equation is interesting. It gives the elasticity of q with respect to p,

$$E_{qp}=-1.00+0.49\log y, \tag{2.6}$$

and an elasticity of q with respect to y,

$$E_{qy}=-0.72+0.49\log p. \tag{2.7}$$

Equation (2.6) indicates that demand for food gets more and more inelastic with respect to price as incomes rise. This is in accord with Harrod's theory, but is the opposite of the theory proposed by Bowley and Allen.[4] [5] Increasing inelasticity of demand for food could lead to increasing instability in agriculture.

Equation (2.7) indicates that food consumption gets more elastic (or less inelastic) with respect to income as food prices rise and less elastic (or more inelastic) as food prices fall. Both of these results seem reasonable and in accord with observed facts.

Since Marshall, economists generally use the term "demand curve" to mean the curve showing how the consumption of a commodity varies with the price of that commodity. They often use the term "Engel curve" to mean the curve showing how the consumption of a commodity varies with consumer income. The term comes from Ernst Engel, who made some of the earliest household-budget studies in the 19th century. "Engel's law" states that as incomes rise, the proportion of income spent for food drops.[6] This law is in line with the finding that the demand for food is inelastic with respect to income—in fact, that is one way of stating Engel's law.

Conditional Expectations

Equations (2.3) and (2.5) were based upon historical data through 1962. How well do they fit the data for 1963?

The preliminary data for 1963 were:

$$q=101.8,\ p=98.4,\ y=107.9.$$

The corresponding logarithms (to the base 10) are:

$$\log q=2.0075,\ \log p=1.99300,\ \log y=2.03302.$$

Inserting the 1963 data for log p and log q into (2.3), we get what I shall call the conditional expected value of log q. It is 1.99630. Translating this back to ordinary numbers, the conditional expected value of q in 1963 was 99.2. The actual preliminary figure for q was 101.8, so the conditional expectation was 2.6 percent too low. In other words, there was an error of -2.6 percent.

The main purpose here is to test how well—or how poorly—the historically based equations fit the 1963 data. In this case, the fit is only fair.

In similar manner, we can insert the preliminary 1963 data for log p and log y into (2.5). This gives us a conditional expectation of $q=104.4$. Comparing this to the preliminary figure of 101.8, we see that equation (2.5) gives an error of $+2.6$ percent. Curiously, this is the negative of the error resulting from equation (2.3). It happens that an average of the two expectations would exactly hit the 1963 data but I don't recommend such an average.

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[4] Harrod, R. F. THE TRADE CYCLE. Clarendon Press, Oxford, 1936.

[5] Bowley, A. L. and Allen, R. G. D. FAMILY EXPENDITURE. King, London, 1935.

[6] Engel, Ernst. DIE LEBENKOSTEN BELGISCHER ARBEITER-FAMILIEN FRÜHER UND JETZT. C. Heinrich, Dresden, 1895. Engel's statement of the law (page 26) was, "je ärmer eine Familie ist, einen desto grösseren Antheil von den Geramm-tausgaben muss sie zur Beschaffung der Nahrung aufwenden."

The conditional expectations shown here and elsewhere in this bulletin are not forecasts for 1963. Actually, they were made early in 1964 after preliminary 1963 data were available. But they are similiar in principle to most practical economic forecasts. Such forecasts are usually based upon some sort of historical relation, say of X_1 to X_2, X_3, ----------, X_n. Then, in one way or another, the forecaster decides upon what values to assume for X_2, X_3, ----------, X_n. And if the "structure" has not changed, his "best" estimate of X is found by inserting the assumed values of X_2, X_3, ----------, X_n in his historical equation and estimating X_n. Technically, this is the conditional expectation of X_1. It is the value of X_1 to be expected (in the probability sense) *if* the assumed values of X_2, X_3, ----------, X_n were precisely correct.

Actually, it would have been possible some time during 1963 to make rough estimates of p and y, and, therefore, to make a forecast of q. But equations like (2.3) and (2.5) are useful for many "projections" that are not forecasts. They let us estimate the expected values of q corresponding to any assumed values of p and y. This, of course, assumes no change in structure. If there has been a known change in structure, any estimating equations need to be adjusted. This is true whether the equation is obtained from ordinary least squares or from some other method.

3. Retail Price and Farm Price[1]

> Farm-level demand for domestic food is less elastic than retail demand for foods—much less elastic if marketing margins are large.—George Brandow in *Interrelations Among Demands for Farm Products* . . .

In primitive societies the original producer usually sells directly to the consumer. Thus, the farmer or the fisherman in a primitive country gets the retail price. But in modern, highly specialized countries, the farmer commonly gets less than one-half the retail price of food. This is because of the enormous expense for transporting, processing, storing, wholesaling, and retailing.

Demand theory often overlooks the spread between prices at the farm and prices in retail stores. This is quite unrealistic. Certainly in our statistical work we must take account of this spread. The nature of this spread is of much importance to the farmer. Generally, it tends to make demand less elastic (more inelastic) at the farm than in the city.

Theory of Derived Demand

There are two conflicting notions concerning the relationship between demands at farm and retail levels. Some people are firmly convinced that food prices are made at the farm—and that retail

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[1] Robert E. Olson, William H. Waldorf, and Forrest Scott provided data for this chapter and made a number of useful suggestions concerning the analysis.

Also, I have used ideas developed in the following papers:

Bunkers, E. W., and Cochrane, W. W. ON THE INCOME ELASTICITY OF FOOD SERVICES. Rev. Econ. and Statis., May 1957.

Burk, Marguerite C. SOME ANALYSES OF INCOME FOOD RELATIONSHIPS. Jour. Amer. Statis. Assoc., Dec. 1958.

Daly, Rex F. DEMAND FOR FARM PRODUCTS AT RETAIL AND FARM LEVEL. Jour. Amer. Statis. Assoc., Sept. 1958.

Ogren, Kenneth E. FARM-RETAIL SPREADS FOR FOOD PRODUCTS. U.S. Dept. Agr. Misc. Pub. 741, 1957.

prices are made up of the farm price plus various charges for processing and distribution. In the short run, this may often be true. Buyers of hogs at the farm level, for example, may notice a decline in marketings; so they may increase the price of hogs at the farm while the retailer is still selling his accumulated supply of pork at the old retail price. Thus, there is often a lag between changes in price at the farm and in city stores.

But in the long run, I think that consumer demand is controlling. It is based upon the wants and preferences of consumers, together with income, prices, and supplies of competing commodities and other factors. This being true, in the long run, food prices are determined at the retail market by what the consumer can and will pay for what is offered. The price at the farm must equal the retail price minus all charges for transporting, processing, storing, wholesaling, and retailing. This theory is explained well by Thomsen and Foote.[2]

A complete theory of demand would have to explain the factors that influence retail prices and price spreads between the farmer and the consumer. Here we shall ask only how price spreads are related to quantities marketed and to consumer income. Actually, it is not consumer income, as such, that affects price spreads. Rather, it is wages and other costs. But wages and other costs are highly correlated with consumer income.

Percentage vs. Absolute Spreads

The effects of price spreads between the farmer and the consumer depend partly on the size of the spread. They also depend partly upon whether the spreads are percentages of the retail food dollar or whether they are absolute amounts. The mathematics of this is explained in Appendix 1. Briefly, if the spreads were a constant percentage of the retail price, the "flexibilities" of retail prices and farm prices would be equal. (Price flexibility was Moore's term for the elasticity of price with respect to quantity.) On the other hand, if the spreads were absolute amounts in dollars and cents, the prices would be more flexible at the farm than at retail. This is important. Increased output raises gross income if, and only if, expected price at the farm is inflexible (that is, if the elasticity of farm price with respect to quantity is less than 1.0).

Inflexible prices correspond to elastic demand, and flexible prices to inelastic demand. The terms price elasticity and income elasticity seem confusing to me. They do not mean the elasticity of prices and the elasticity of income. As commonly used, price elasticity is shorthand for the elasticity of consumption with respect to price, and income elasticity means the elasticity of consumption with respect to income.

Many studies of this matter in the Department of Agriculture suggest that the price spreads are neither constant percentages nor constant absolute amounts, but somewhere in between the two. In such cases, the farm price is more flexible than the retail price.

Results of Statistical Study

The following results are based upon data in table 3.1. The table provides data on retail food prices, consumer income, and food con-

[2] Thomsen, Frederick L., and Foote, Richard J. AGRICULTURAL PRICES. McGraw-Hill, New York, pp. 51–54, 1952.

sumption from 1926 through 1941 and from 1948 through 1962. Note that there are four different price series. The first column is the same retail price index that was used in Chapter 2. The next three columns are prices of the so-called market basket. This represents the prices of fixed quantities of foods that are bought by typical urban families with moderate incomes in the United States. So far as possible, these are the same foods from year to year, keeping changes in services, packaging, and so on, at a minimum. This market basket is priced every month according to the retail prices gathered by the Bureau of Labor Statistics. The Department of Agriculture computes the price of the farm equivalent (for example, the farm price of the wheat and other ingredients in a loaf of bread). The difference between the prices in column 2 and in column 3 is the price spread.

TABLE 3.1.—*Data on market basket food prices, consumer income, and per capita food consumption*

Year	Retail price index 1957–59=100 a (p)	Retail price [1] b (p)	Farm equivalent [1] c (p)	Price spread [1] d (p)	Per capita disposable income 1957–59=100 (y)	Food consumption per capita 1957–59=100 (q)
		Dol.	*Dol.*	*Dol.*		
1926	57.6	634	259	375	35.3	90.1
1927	55.5	614	247	367	35.0	88.9
1928	54.9	617	256	361	35.4	88.9
1929	55.6	617	255	362	37.0	89.1
1930	52.9	597	227	370	32.7	88.7
1931	43.6	481	167	314	27.9	88.0
1932	36.3	403	125	278	21.1	85.9
1933	35.3	392	125	264	19.7	86.0
1934	39.3	442	148	281	22.3	87.1
1935	42.1	491	187	291	24.9	85.4
1936	42.5	495	197	298	28.0	88.5
1937	44.2	514	210	304	29.9	88.4
1938	41.0	466	177	289	27.4	88.6
1939	39.9	450	170	280	29.2	91.7
1940	40.5	451	177	274	31.2	93.3
1941	44.2	494	215	279	37.8	95.1
	(World War II years excluded)					
1948	88.2	982	497	485	70.0	96.7
1949	84.7	928	435	493	68.9	96.7
1950	85.8	920	432	488	74.2	98.0
1951	95.4	1,024	497	527	80.0	96.1
1952	97.1	1,034	482	552	82.4	98.1
1953	95.6	1,003	445	558	85.8	99.1
1954	95.4	986	421	565	85.8	99.1
1955	94.0	969	395	574	90.0	99.8
1956	94.7	972	390	582	94.4	101.5
1957	97.8	1,007	401	606	97.8	99.9
1958	101.9	1,064	430	634	99.0	99.1
1959	100.3	1,040	398	642	103.2	100.0
1960	101.4	1,053	407	646	104.9	100.7
1961	102.9	1,060	406	654	107.3	100.8
1962	103.5	1,068	410	658	110.9	101.0

[1] Annual costs for the typical family.

None of the data in this table were deflated. A number of multiple regression equations were run out on the electronic computer, together with all the standard errors and various measures needed to test reliability of results. The regression equations, the standard errors of the regression coefficients, and the squared correlations are shown in tables 3.2 and 3.3. In general, the correlations were quite high, and most of the regression coefficients were significant, at least twice their respective standard errors.

Footnote 3 at the bottom of table 3.2 shows the prices that would have been indicated if the 1963 preliminary data for consumer income and for food consumption had been inserted in the estimating equations. Of course, these are not price forecasts for 1963. They were made in March 1964. Technically, they are conditional expectations of prices

TABLE 3.2.—*Regressions of log price (p) upon log income [1] (y) and log consumption [2] (q)*

Price measure [3]	Regressions
Retail price index:	
1926–41	estimated log $p_a = 7.3467 + \underset{(0.0841)}{1.0183} \log y - \underset{(0.5588)}{3.6871} \log q$; $R^2 = 0.923$
1948–62	estimated log $p_a = 4.8094 + \underset{(0.0671)}{0.5658} \log y - \underset{(0.6076)}{1.9700} \log q$; $R^2 = 0.920$
Retail price, market basket:	
1926–41	estimated log $p_b = 8.6976 + \underset{(0.0689)}{1.0183} \log y - \underset{(0.4820)}{3.8408} \log q$; $R^2 = 0.946$
1948–62	estimated log $p_b = 7.6123 + \underset{(0.0740)}{0.5050} \log y - \underset{(0.6701)}{2.8029} \log q$; $R^2 = 0.830$
Farm price, market basket:	
1926–41	estimated log $p_c = 7.4567 + \underset{(0.0984)}{1.4507} \log y - \underset{(0.6890)}{3.7448} \log q$; $R^2 = 0.952$
1948–62	estimated log $p_c = 12.6329 + \underset{(0.1572)}{0.1376^*} \log y - \underset{(1.4238)}{5.1449} \log q$; $R^2 = 0.738$
Farm-retail spread:	
1926–41	estimated log $p_d = 8.7550 + \underset{(0.0908)}{0.7955} \log y - \underset{(0.6359)}{3.8117} \log q$; $R^2 = 0.855$
1948–62	estimated log $p_d = 2.8925 + \underset{(0.0606)}{0.7696} \log y - \underset{(0.5488)}{0.8189^*} \log q$; $R^2 = 0.978$

Numbers in parentheses are standard errors of the regression coefficients immediately above them.

*These coefficients are not statistically significant, because they are less than twice their standard errors. All other coefficients in the table are statistically significant.

[1] Disposable personal income per capita, 1957–59=100.

[2] Per capita food consumption, price-weighted index, 1957–59=100.

[3] The preliminary 1963 data were: $y=117.0$ and $q=101.8$. The actual values of p_a, p_b, p_c, and p_d were 105.0, 1,078, 394, and 684 respectively, while their conditional expectations were 106.7, 1,072, 388, and 692 respectively. This represents a percentage error for p_a, p_b, p_c, and p_d of +1.6, −0.6, −1.5, and +1.2 respectively.

in 1963; that is, they are the prices that would be expected if consumer income were 117.0 and if per capita food consumption were 101.8.

But such equations would enable us to make conditional forecasts if we had good indications of consumer income and food consumption for a future year. Franklin Fisher makes a good distinction between unconditional forecasts and conditional forecasts.[3] Actually, almost all economic forecasts are conditional; that is, they are forecasts based upon estimated or assumed values of the independent variables.

The regression equations are all given in logarithms. This is partly because a preliminary graphic analysis suggested that the curves were somewhat concave rather than linear. Also, the logarithmic form of the equations is handy if we want to compare elasticities. In fact, in the logarithmic equations, regression coefficients themselves are

TABLE 3.3.—*Regressions of log consumption*[1] *(q) upon log price (p) and log income*[2] *(y)*

Price measure	Regressions
Retail price index:	
1926–41	estimated log q=1.9459 $-0.2037 \log p_a + 0.2320 \log y$; $R^2=0.869$ (0.0325) (0.0257)
1948–62	estimated log q=2.1043 $-0.2370 \log p_a + 0.1852 \log y$; $R=0.868$ (0.0731) (0.0299)
Retail price, market basket:	
1926–41	estimated log q=2.1864 $-0.2161 \log p_b + 0.2368 \log y$; $R^2=0.911$ (0.0271) (0.0210)
1948–62	estimated log q=2.3469 $-0.2116 \log p_b + 0.1458 \log y$; $R^2=0.900$ (0.0506) (0.0157)
Farm price, market basket:	
1926–41	estimated log q=1.9343 $-0.1854 \log p_c + 0.2991 \log y$; $R^2=0.839$ (0.0341) (0.0402)
1948–62	estimated log q=2.1459 $-0.1013 \log p_c + 0.0598 \log y$; $R^2=0.882$ (0.0280) (0.0148)
Farm-retail spread:	
1926–41	estimated log q=2.1662 $-0.1926 \log p_d + 0.1794 \log y$; $R^2=0.860$ (0.0321) (0.0200)
1948–62	estimated log q=2.0788 $-0.1991^* \log p_d + 0.2279 \log y$; $R^2=0.792$ (0.1281) (0.0897)

Numbers in parentheses are standard errors of the regression coefficients immediately above them.

*Not statistically significant. All other coefficients in table are statistically significant.

[1] Per capita food consumption, price-weighted index (1957–59=100).

[2] Disposable personal income per capita (1957–59=100).

[3] Fisher, Franklin M. THE PLACE OF LEAST SQUARES IN ECONOMETRICS: COMMENT. Econometrica, Vol. 30, No. 3, July 1962.

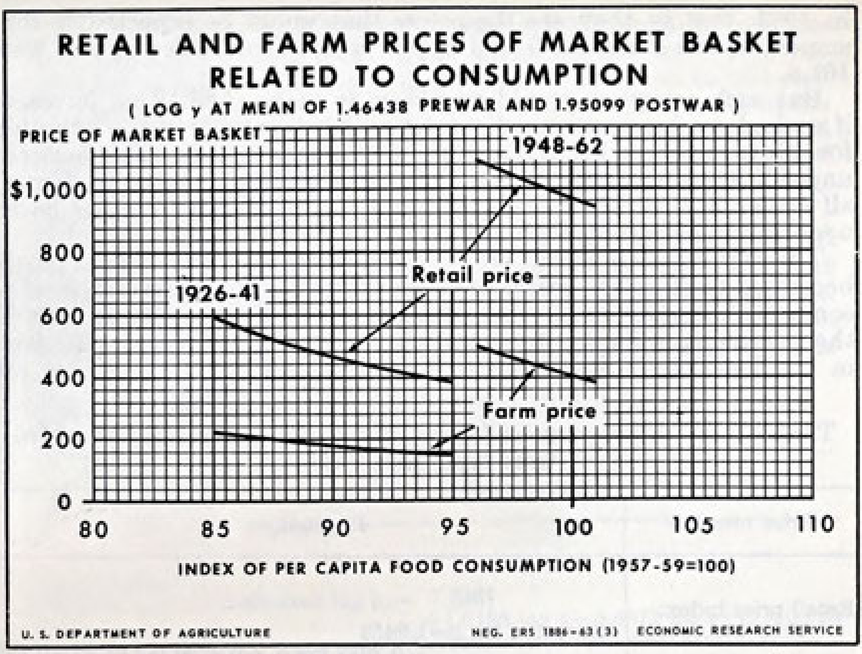

Figure 3.1

elasticities. For example, in the first equation in table 3.2 the elasticity of expected retail price with respect to consumer income is 1.0183, and the elasticity of expected retail price with respect to the quantity sold and consumed is −3.6871.

In table 3.2 the logarithm of price is the dependent variable and the logarithms of income and quantity are treated as independent variables. In table 3.3 the logarithm of quantity is taken as dependent, while the logarithms of price and income are treated as independent. Thus, table 3.2 gives the closest possible estimates of the expected logarithms of prices. Table 3.3, on the other hand, gives the closest possible estimates of the expected logarithms of quantities. None of these equations are true demand curves. These equations are for estimating expected food prices and expected food consumption. Actually, there is a third regression equation which would estimate the expected consumer income associated with stated values of food consumption and food price. But I think the two regression equations shown are of greater interest.

The elasticity of expected consumption with respect to retail price is −0.2037 if we use the retail price index as a measure, and it is −0.2161 if we use the retail price of the market basket as a measure.

An analysis of the equations in these two tables indicates some interesting and important changes since World War II. These changes can perhaps be seen most easily in figures 3.1 and 3.2.

Some Changes

Figure 3.1 shows the net relation of price of the market basket to the index of per capita food consumption. It is a net relation because

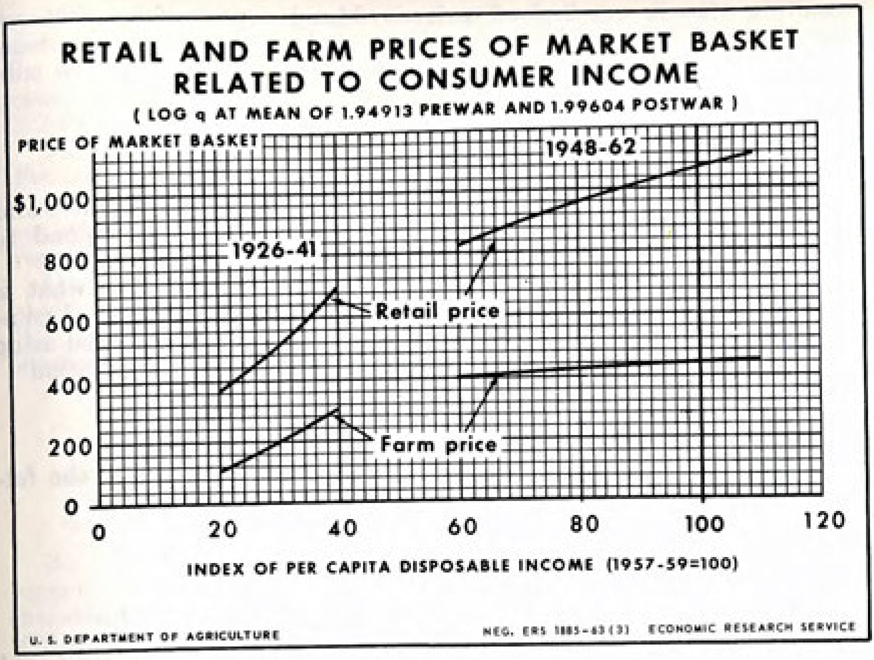

Figure 3.2

consumer income was held constant. In the prewar period it was held constant at the prewar mean. In the postwar period it was held constant at the postwar mean, which was, of course, much higher. This explains the change in the level of the lines.

In the prewar period, the elasticity of expected food prices with respect to quantity sold and consumed was about the same at retail and at the farm. Figure 3.1 suggests, for example, that in the prewar period the price spread tended to be nearly a constant percentage of the retail price. But in the postwar period, this situation seems to have changed significantly. The price spread seems to have been a more nearly constant absolute amount. As a result, prices were much more flexible at the farm than at retail. This refers, of course, to flexibility with respect to the quantity of food sold and consumed.

Figure 3.2 shows an even more striking change in the farm-to-retail price relationships with respect to income. In the prewar years, farm prices were substantially more flexible than retail prices with respect to consumer income. In the postwar years, farm prices were much less flexible than retail prices with respect to consumer income. Nevertheless, the elasticity in terms of farm prices was not statistically significant. Figure 3.2 shows, for example, that retail prices in the postwar years rose substantially as incomes increased. It shows also that farm prices were affected very little by the increase in consumer income.

This is simply because a large part of the price spread is made up of wages, and because wages are a large part of consumer income. Wages have increased substantially since World War II. They have pushed up the price spread so much that the farmer has gotten very little advantage from the increase in consumer income and the resulting rise in retail food prices. Many reports from research agencies and from trade sources have spoken of the steadily rising demand for food since World War II. The demand has risen substantially at retail, but little at the farm.

A "True" Demand Relationship

Tables 3.2 and 3.3 give the closest estimates of expected prices and of expected consumption. They do not necessarily correspond to the true demand curves of economic theory.

One method of estimating a "true" demand equation is what is sometimes called "orthogonal regression." As an experiment, I computed an orthogonal regression, using the 1948 to 1962 data and using the index of retail prices as the price variable. I got the equation

$$-37.847 \log p + 24.504 \log y - 101.097 \log q = 0.$$

This equation could be written in several forms, including the following:

$$\begin{aligned} -1.00000 \log p + 0.64745 \log y - 2.67120 \log q &= 0 \\ 1.54452 \log p - 1.00000 \log y + 4.12523 \log q &= 0 \\ -0.37436 \log p + 0.24238 \log y - 1.00000 \log q &= 0. \end{aligned}$$

The first and third equations immediately above can be compared with the first equations in tables 3.2 and 3.3.

Appendix 2 discusses the orthogonal regression and describes the numerical computation. In computing the above equations, I assumed that there were errors of observation and errors of specification in all three variables. I also assumed that the ratio of the error to the standard deviation was the same for each of the three variables. If this had been true, and if the errors were strictly uncorrelated with one another and with the variables themselves, the observed scatter could have been explained entirely by errors in each of the variables, representing about 2.2 percent of their respective standard deviations. That is, the true values of p, y, and q could have fallen exactly upon the regression plane.

For this example, I chose one of the highest correlations. In such cases the ordinary least squares regressions are not far different from one another. Also, it is not hard in such cases to visualize the possibility that the true values of all the data could lie exactly on an n–1 dimensional plane. In other words, one could well imagine that the deviations from the plane were due to errors, either in measurement or in specification.

Statisticians should know a great deal more than they do about errors in published data. If they had accurate estimates of such errors, they might make more use of something like orthogonal regressions. It is not necessary to assume that the relative errors in all variables are equal to one another. If the statistician can estimate the relative magnitude of the errors in the different variables, he can use some of the kinds of regressions developed by Frisch.[4]

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[4] Frisch, Ragnar. STATISTICAL CONFLUENCE ANALYSIS BY MEANS OF COMPLETE REGRESSION SYSTEMS. Universitetets Økonomiske Institutt. Oslo, Norway, 1934. Especially note part II.

Although orthogonal regressions and similar estimates of "true" demand equations may be very interesting, I do not know what to do with one after I compute it. For practical purposes, I would much rather have the simple regression equations shown in tables 3.2 and 3.3.

The orthogonal regression does have the virtue of consistency. For example, the elasticity of price with respect to consumption is equal to the reciprocal of the elasticity of consumption with respect to price if both elasticities are figured from the same orthogonal regression.

Consistency is often overrated, especially when it means averaging things that are essentially different. A hunter has a single-shot rifle. Two geese fly overhead. If the hunter is practical, he will aim at a single goose, not at a point halfway between the two.

4. Some Individual Foods

> The theory of prices is based—or at least is supposed to be based—on observation of the actual behaviour of prices.—OSKAR MORGENSTERN in foreword to GERHARD TINTNER's *Price and The Trade Cycle*

So far, we have discussed the demand for food as a whole. Of course, there is no such commodity as food. Food is made up of hundreds of commodities. Some, like potatoes, are very simple. Others, like bread, are very complex. When we lump all food commodities into a single index, we may cover up many important and interesting relationships. We shall now consider the demand for several individual foods.

Data and Analysis

One of our main purposes is still to compare the demand at the retail level with demand at the farm level; that is, we want to know how retail prices respond to changes in quantities and consumer income, and we want the same sort of information for prices of various food commodities at the farm level. This is a subject which has not been adequately analyzed either from the standpoint of theory or statistics. It is a subject that is important to the farmer. For this purpose, my colleagues William H. Waldorf and Forrest Scott made available detailed data on prices and price spreads for 44 individual foods. With the help of two other colleagues, Martin E. Abel and Hyman Weingarten, I had a simple, standard analysis run for all these 44 foods, using the electronic computer.

Such routine mass production methods have both advantages and disadvantages. They enable us to get enormous amounts of statistical results which are comparable between commodities. On the other hand, any analysis which uses the same equations for all commodities is likely to overlook essential features in the markets for the individual foods. The market for rolled oats is simply not the same thing as the market for chickens or for carrots. Both for theoretical purposes and for such applications as price forecasting and program appraisal, I would strongly prefer special analyses by economists with intimate knowledge of the production, processing, distribution, marketing, and consumption of each particular commodity they are studying.

But very simple demand models are often satisfactory for some of the perishable foods which involve very little processing. This is especially the case when supplies of these commodities are practically fixed in the short run.

This chapter will analyze the demand for five fresh fruits and vegetables, and also the demand for beef and milk. The analysis covers only the postwar period 1948 through 1962. In each case, linear functions were assumed.

$$
\begin{aligned}
&r=a+bq+cy \\
(4.1)\qquad &\underline{s=d+eq+fy} \\
&f=(a-d)+(b-e)\,q+(c-f)y,
\end{aligned}
$$

where r, s, f represent the expected retail price, the expected farm-to-retail price spread, and the expected farm price (r–s); where q represents the given, or assumed, per capita consumption of the particular food; where y represents the given, or assumed, per capita consumer income; and where the other letters represent constants to be determined by statistical analysis. In the case of fluid milk, a linear time trend was added to equations (4.1).

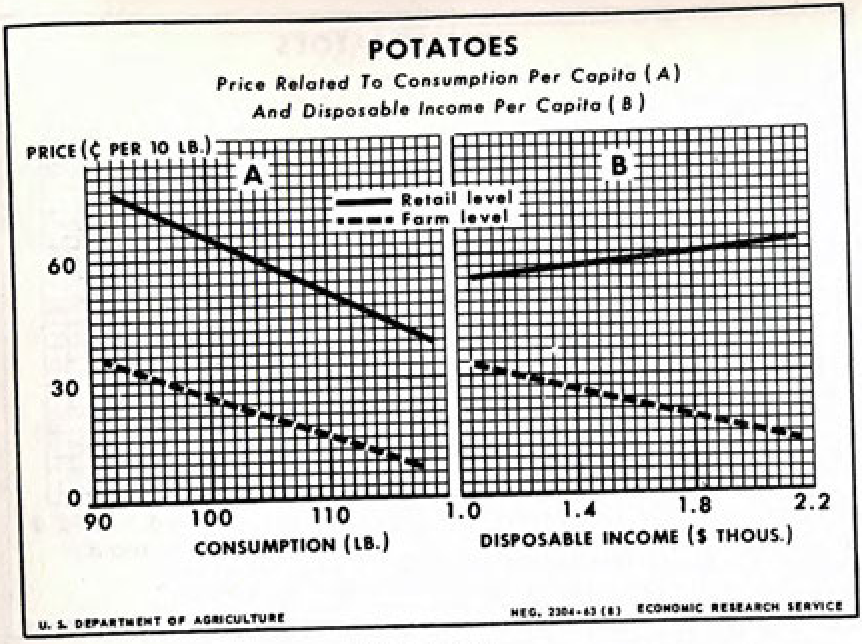
Note that this analysis assumes linear relationships between prices, quantities, and consumer incomes—*not* linear relationships between the logarithms of these variables. This was because I wanted to show certain relationships between retail prices, price spreads, and farm prices. Obviously, farm price equals retail price minus the spread. If the equations for retail prices and for price spreads are linear, the equation for farm price is also a simple linear equation. As a practical matter, either linear equations in the absolute numbers or linear equations in the logarithms of the numbers would fit the data reasonably well within the observed range.

The Diagrams and Tables

Mr. Waldorf, Mr. Scott, and their able assistants helped me review the results of the analyses for the 44 commodities. We chose seven commodities which seemed reasonably satisfactory. Five of the seven are fruits and vegetables. All seven are very simple foods that are sold with little processing and are not involved significantly either in the export or import markets.

The results for these seven foods are shown on the following pages. The same standard form was used for each commodity. Take figure 4.1 for example. It gives the results for potatoes. The data on retail food prices, farm-retail price spread, consumption per capita and disposable income per capita are shown in the table at the bottom of the page, covering the years 1948 through 1962. The regression equations, the squared multiple correlations, and the price flexibilities are shown in the middle of the page. The estimating equations for $\hat{r}$ and for $\hat{s}$ were computed by least squares. The equation for the expected farm price, $\hat{f}$, was obtained by subtracting $\hat{s}$ from $\hat{r}$. The numbers in parentheses are the standard errors of the regression coefficients immediately above them.

The price flexibilities are elasticities of prices with respect to quantities and with respect to income. More specifically, $F_{\hat{r}q \cdot y}$ is the flexibility of the expected retail price with respect to quantity consumed, holding income constant. The other three price flexibilities are to be interpreted in a similar manner. These flexibilities can be



Regression equations	Square of multiple correlation	Price flexibilities	
$\hat{r} = 197.918 - 1.448q + .00751y$ $(.469)\quad(.00698)$	$R^2_{\hat{r}.qy} = .6424$	$F_{\hat{r}q.y} = -2.5666$	$F_{\hat{r}y.q} = .2121$
$\hat{s} = 32.354 - .365q + .02605y$ $(.132)\quad(.00197)$	$R^2_{\hat{s}.qy} = .9654$		
$\hat{f} = 165.564 - 1.083q - .01854y$		$F_{\hat{f}q.y} = -5.2771$	$F_{\hat{f}y.q} = -1.4389$

Means: $\bar{r} = 59.0000$ $\bar{s} = 37.5333$ $\bar{f} = 21.4667$ $\bar{q} = 104.6000$ $\bar{y} = 1666.0000$

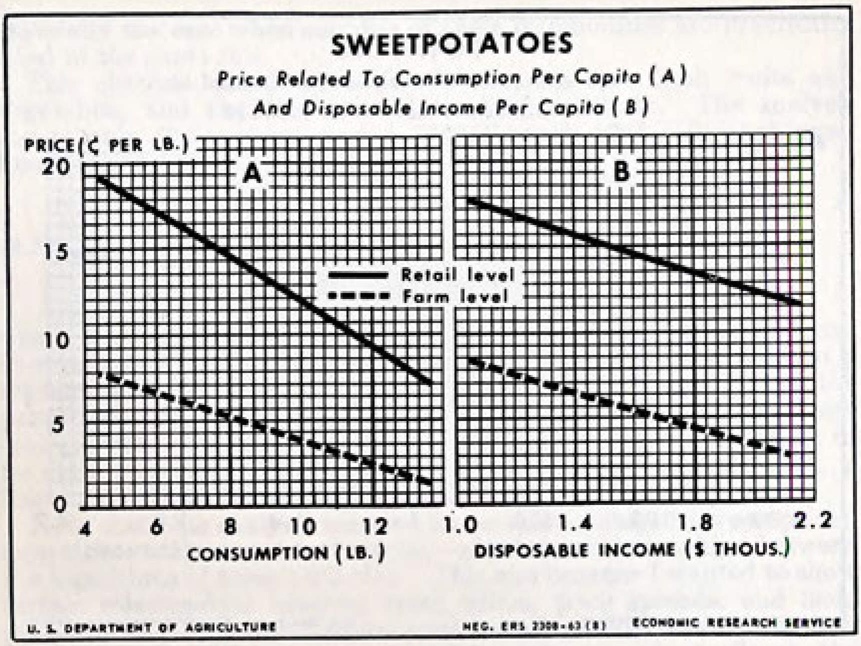
Year	Retail price per 10 lb.	Farm-retail spread per 10 lb.	Consumption per capita 1/	Disposable income per capita	Year	Retail price per 10 lb.	Farm-retail spread per 10 lb.	Consumption per capita 1/	Disposable income per capita
	Cents	Cents	Pounds	Dollars		Cents	Cents	Pounds	Dollars
					1956	67.7	41.5	99	1,742
1948	54.1	26.4	105	1,291	1957	57.1	41.4	106	1,804
1949	53.0	28.2	110	1,271	1958	62.6	43.9	101	1,826
1950	44.8	27.1	106	1,363	1959	63.3	44.1	101	1,904
1951	47.2	26.4	113	1,473	1960	71.8	47.5	102	1,934
1952	73.7	35.8	101	1,520	1961	62.9	46.6	103	1,960
1953	53.2	34.7	106	1,582	1962	63.2	46.2	103	2,098
1954	52.0	34.2	106	1,582					
1955	56.4	37.0	107	1,660					

1/ Potatoes used for all purposes except canned and frozen. Farm weight.

Figure 4.1

computed from the regression coefficients and the means. For example, flexibility of the expected retail price of potatoes with respect to quantity, holding income constant, is $-1.448 \times \frac{104.6}{59.0} = -2.57$. The flexibility of -2.566 shown below the potato diagram used data taken to more decimal places.

The reciprocals of price flexibility are often taken to represent elasticities of demand. Thus, a price flexibility of -2.5 is often used as the equivalent of a demand elasticity of -0.4. I prefer to use the price flexibilities themselves rather than their reciprocals. If, for



Regression equations	Square of multiple correlation	Price flexibilities
$\hat{r} = 33.854 - \underset{(.422)}{1.321q} - \underset{(.00339)}{.00533y}$	$R^2_{\hat{r}.qy} = .5656$	$F_{\hat{r}q.y} = -.7643$ $F_{\hat{r}y.q} = -.6274$
$\hat{s} = 14.066 - \underset{(.212)}{.593q} - \underset{(.00170)}{.00015y}$	$R^2_{\hat{s}.qy} = .7272$	
$\hat{f} = 19.788 - .728q - .00518y$		$F_{\hat{f}q.y} = -1.1476$ $F_{\hat{f}y.q} = -1.6617$

Means: $\bar{r} = 14.1533$ $\bar{s} = 8.9600$ $\bar{f} = 5.1933$ $\bar{q} = 8.1867$ $\bar{y} = 1666.0000$

Year	Retail price per lb.	Farm-retail spread per lb.	Consumption per capita 1/	Disposable income per capita	Year	Retail price per lb.	Farm-retail spread per lb.	Consumption per capita 1/	Disposable income per capita
	Cents	Cents	Pounds	Dollars		Cents	Cents	Pounds	Dollars
1948	11.5	6.8	11.5	1,291	1956	12.8	8.7	7.8	1,742
1949	12.7	7.7	11.7	1,271	1957	14.7	9.7	7.6	1,804
1950	10.5	6.5	12.1	1,369	1958	15.8	10.5	6.5	1,826
1951	12.2	7.5	8.1	1,473	1959	13.8	9.7	7.4	1,904
1952	18.7	10.8	7.3	1,520	1960	13.7	9.1	6.3	1,934
1953	16.6	9.6	8.0	1,582	1961	16.1	10.4	5.7	1,980
1954	13.6	8.5	8.1	1,582	1962	15.8	10.3	6.3	2,052
1955	13.8	8.6	8.4	1,660					

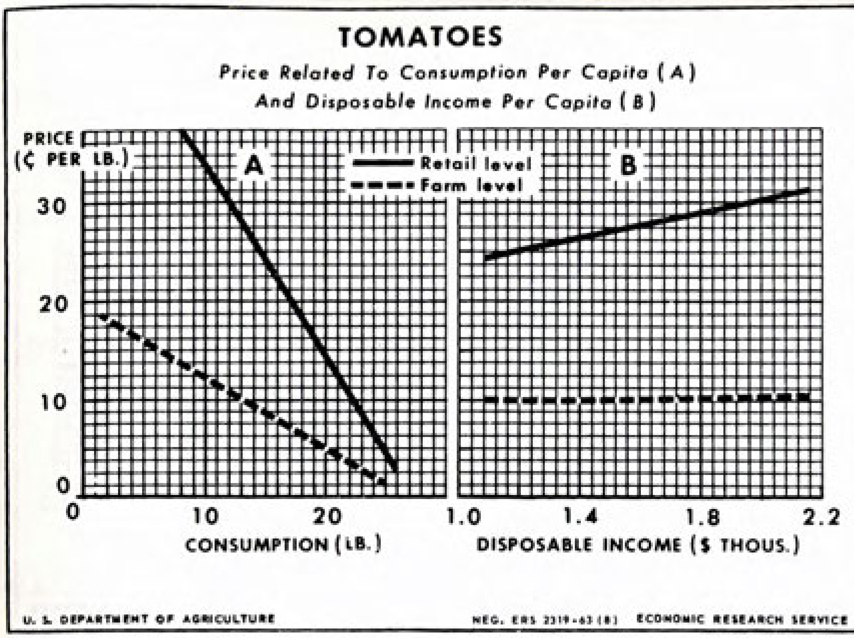
1/ Farm weight.

Figure 4.2

any reason, the elasticity of demand is wanted, I would prefer to use the other regression equations, using quantities as the dependent variables; then the elasticity of demand with respect to price is approximately the expected percentage change in quantity associated with a 1 percent change in price.

The diagrams at the top of each chart show retail prices and farm prices as functions of quantities consumed and as functions of disposable income.

The left-hand diagram marked A relates prices to consumption. In this case, consumer income is held constant at the mean $1,066 for



Regression equations	Square of multiple correlation	Price flexibilities	
$\hat{r} = 42.582 - 1.907q + .00595y$ (1.020) (.00197)	$R^2_{\hat{r}.qy} = .7291$	$F_{\hat{r}q.y} = -.8525$	$F_{\hat{r}y.q} = .3599$
$\hat{s} = 21.574 - 1.070q + .00593y$ (.734) (.00142)	$R^2_{\hat{s}.qy} = .7901$		
$\hat{f} = 21.008 - .837q + .00002y$		$F_{\hat{f}q.y} = -1.0302$	$F_{\hat{f}y.q} = .0033$

Means: $\bar{r} = 28.5231$ $\bar{s} = 18.1615$ $\bar{f} = 10.3616$ $\bar{q} = 12.7538$ $\bar{y} = 1725.2307$

Year	Retail price per lb.	Farm-retail spread per lb.	Consumption per capita 1/	Disposable income per capita	Year	Retail price per lb.	Farm-retail spread per lb.	Consumption per capita 1/	Disposable income per capita
	Cents	Cents	Pounds	Dollars		Cents	Cents	Pounds	Dollars
1950	24.4	15.1	12.9	1,369	1957	29.5	19.6	12.6	1,804
1951	27.3	15.5	13.3	1,473	1958	32.1	20.5	11.9	1,826
1952	27.1	16.1	13.1	1,520	1959	29.6	19.1	12.9	1,904
1953	27.5	17.7	12.8	1,582	1960	31.6	20.1	12.6	1,934
1954	26.4	17.1	12.9	1,582	1961	28.7	19.1	12.7	1,980
1955	27.4	18.5	13.4	1,660	1962	29.7	19.0	12.4	2,052
1956	29.5	18.7	12.3	1,742					

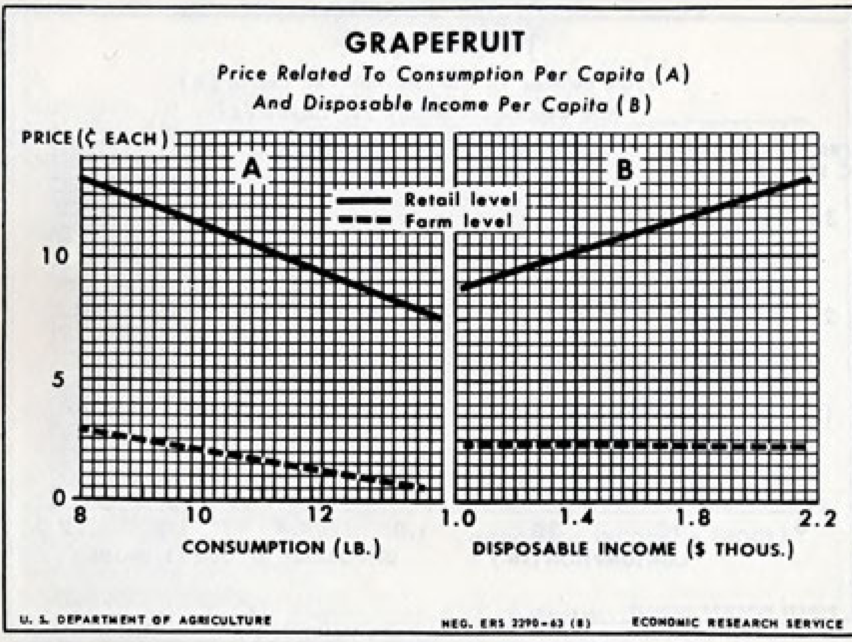
1/ Farm weight.

Figure 4.3

the period. The right-hand side marked B shows the relation of prices to consumer income, holding quantity constant at the mean of 104.6 pounds per capita.

Some Tentative Findings

No sweeping conclusions should be drawn from a study of seven commodities in the short period since World War II. But these analyses suggest certain implications that deserve further study.



Regression equations	Square of multiple correlation	Price flexibilities
$\hat{r} = 14.439 - .936q + .00370y$ $(.250)\quad(.00108)$	$R^2_{\hat{r}.qy} = .8306$	$F_{\hat{r}q.y} = -.8223$ $F_{\hat{r}y.q} = .5452$
$\hat{s} = 7.998 - .499q + .00370y$ $(.191)\quad(.00083)$	$R^2_{\hat{s}.qy} = .8300$	
$\hat{f} = 6.441 - .437q + .00000y$		$F_{\hat{f}q.y} = -2.0671$ $F_{\hat{f}y.q} = .0000$

Means: $\bar{r} = 11.3067$ $\bar{s} = 9.2067$ $\bar{f} = 2.1000$ $\bar{q} = 9.9333$ $\bar{y} = 1666.0000$

Year	Retail price, each	Farm-retail spread, each	Consumption per capita 1/	Disposable income per capita	Year	Retail price, each	Farm-retail spread, each	Consumption per capita 1/	Disposable income per capita
	Cents	Cents	Pounds	Dollars		Cents	Cents	Pounds	Dollars
1948	7.5	6.7	12.3	1,291	1956	10.5	8.6	10.5	1,742
1949	10.2	8.0	10.9	1,271	1957	11.2	9.1	9.5	1,804
1950	11.9	9.1	8.2	1,369	1958	12.9	10.1	8.7	1,826
1951	10.6	6.7	10.3	1,473	1959	13.3	10.7	9.1	1,904
1952	10.7	8.3	10.5	1,520	1960	14.4	11.7	9.6	1,934
1953	10.1	8.2	9.7	1,582	1961	13.5	11.2	9.4	1,980
1954	9.7	6.2	11.0	1,582	1962	13.6	11.2	8.6	2,052
1955	10.0	8.3	10.7	1,660					

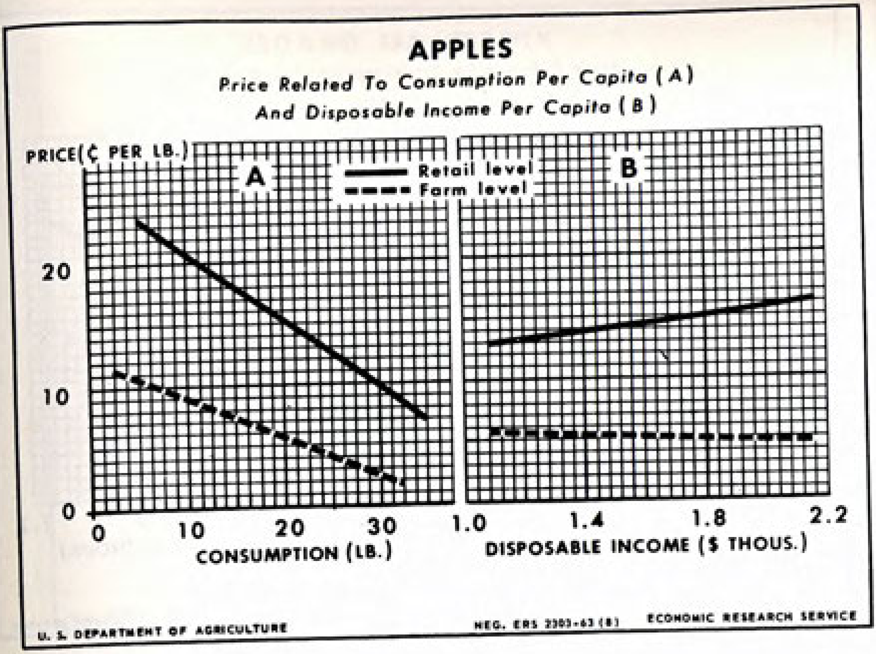
1/ Farm weight.

Figure 4.4

Price Flexibility With Respect to Quantity

Table 4.1 summarizes the flexibilities of retail and farm prices with respect to quantities consumed.

The striking fact brought out by this table is that prices of all the commodities are more flexible at the farm level than at the retail level. This is not surprising to anyone familiar with the statistics of agricultural prices. However, it does run counter to some common notions about the nature of price spreads.



Regression equations

$$\hat{r} = 21.442 - \underset{(.113)}{.542q} + \underset{(.00113)}{.00282y}$$

$$\hat{s} = 8.516 - \underset{(.070)}{.212q} + \underset{(.00073)}{.00331y}$$

$$\hat{f} = 12.926 - .330q - .00049y$$

Square of multiple correlation

$$R^2_{\hat{r}.qy} = .8830$$

$$R^2_{\hat{s}.qy} = .8936$$

Price flexibilities

$$F_{\hat{r}q.y} = -.8074 \qquad F_{\hat{r}y.q} = .3242$$

$$F_{\hat{f}q.y} = -1.4254 \qquad F_{\hat{f}y.q} = -.1637$$

Means: $\bar{r} = 14.4667$ $\bar{s} = 9.4800$ $\bar{f} = 4.9867$ $\bar{q} = 21.5400$ $\bar{y} = 1666.0000$

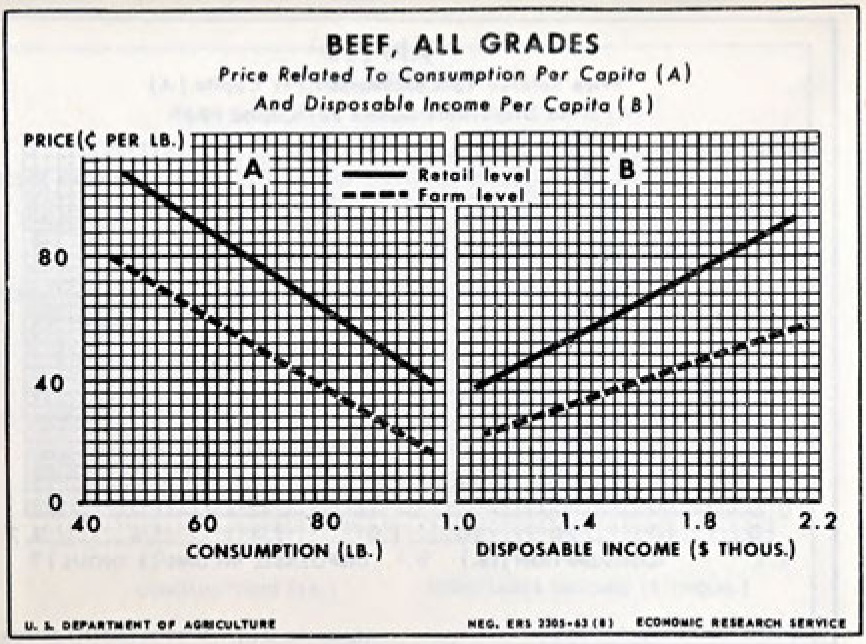
Year	Retail price per lb.	Farm-retail spread per lb.	Consumption per capita 1/	Disposable income per capita	Year	Retail price per lb.	Farm-retail spread per lb.	Consumption per capita 1/	Disposable income per capita
	Cents	Cents	Pounds	Dollars		Cents	Cents	Pounds	Dollars
1948	11.4	7.5	26.3	1,291	1956	15.1	10.1	18.9	1,742
1949	12.1	7.9	24.7	1,271	1957	16.4	10.9	19.3	1,804
1950	11.6	7.6	22.7	1,369	1958	14.7	10.6	22.6	1,826
1951	10.7	7.1	25.7	1,473	1959	14.2	10.0	23.0	1,904
1952	14.7	9.2	21.6	1,520	1960	16.2	10.5	20.1	1,934
1953	15.7	9.5	20.9	1,582	1961	17.3	11.1	18.6	1,960
1954	15.3	9.6	20.0	1,582	1962	16.5	10.8	19.1	2,052
1955	15.1	9.8	19.6	1,660					

1/ Apples from commercial areas only. Farm weight.

Figure 4.5

Relationship of Price Spreads to Quantity

Three different assumptions are commonly made about the nature of the spread between farm price and retail price. A common assumption is that the farmer gets about a constant percentage of retail price. If this were the case, the price flexibilities at the farm and at retail would be the same. This is definitely contradicted in the findings presented in table 4.1.



Regression equations	Square of multiple correlation	Price flexibilities
$\hat{r} = 79.219 - 1.299q + .05296y$ $(.101) \quad (.00446)$	$R^2_{\hat{r}.qy} = .9331$	$F_{\hat{r}q.y} = -1.4442 \quad F_{\hat{r}y.q} = 1.2879$
$\hat{s} = 2.608 - .096q + .01887y$ $(.073) \quad (.00322)$	$R^2_{\hat{s}.qy} = .8842$	
$\hat{f} = 76.611 - 1.203q + .03409y$		$F_{\hat{f}q.y} = -2.1919 \quad F_{\hat{f}y.q} = 1.3585$

Means: $\bar{r} = 68.5067$ $\bar{s} = 26.7000$ $\bar{f} = 41.8067$ $\bar{q} = 76.1733$ $\bar{y} = 1666.0000$

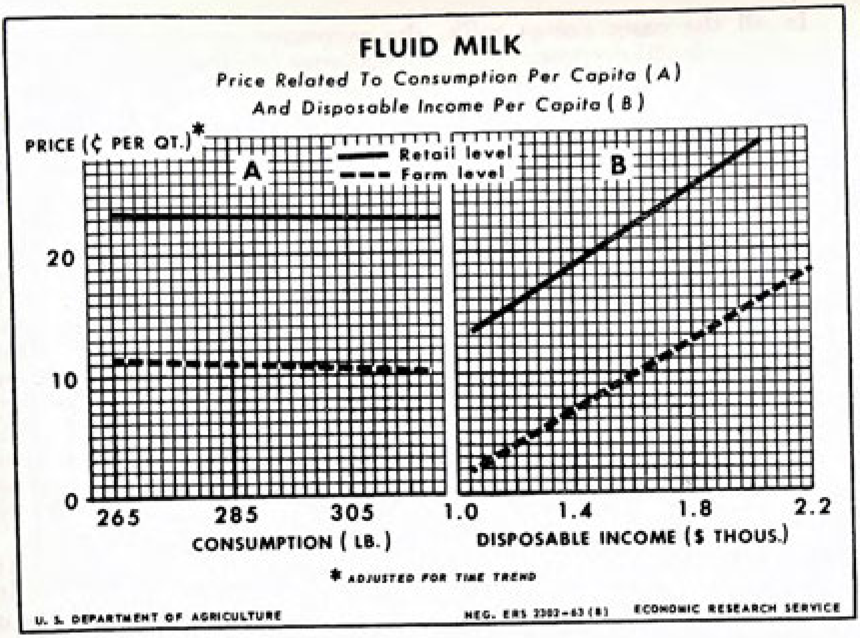
Year	Retail price per lb.	Farm-retail spread per lb.	Consumption per capita 1/	Disposable income per capita	Year	Retail price per lb.	Farm-retail spread per lb.	Consumption per capita 1/	Disposable income per capita
	Cents	Cents	Pounds	Dollars		Cents	Cents	Pounds	Dollars
1948	69.5	22.3	63.1	1,291	1956	57.7	26.3	85.4	1,742
1949	63.3	20.2	63.9	1,271	1957	61.9	27.6	84.6	1,804
1950	69.8	21.3	63.4	1,369	1958	72.5	30.2	80.5	1,826
1951	81.5	23.1	56.1	1,473	1959	74.4	31.7	81.4	1,904
1952	79.0	26.7	62.2	1,520	1960	72.6	33.3	85.2	1,934
1953	61.7	25.8	77.6	1,582	1961	71.1	32.2	88.0	1,980
1954	60.0	24.3	80.1	1,582	1962	73.6	30.0	89.1	2,052
1955	59.0	25.5	82.0	1,660					

1/ Carcass weight.

Figure 4.6

TABLE 4.1.—*Price flexibilities with respect to quantity*

Commodity	At retail level	At farm level
Potatoes	−2. 57	−5. 28
Sweetpotatoes	−0. 76	−1. 15
Tomatoes	−0. 85	−1. 03
Grapefruit	−0. 82	−2. 06
Apples	−0. 81	−1. 43
Beef	−1. 44	−2. 19
Milk	−0. 07	−0. 56



Regression equations	Square of multiple correlation	Price flexibilities
$\hat{r} = 1.872 - .0059q + .01641y - .544t$ $(.0223)\ (.00641)\ (.369)$	$R^2_{\hat{r}.qyt} = .9239$	$F_{\hat{r}q.yt} = -.0752$ $F_{\hat{r}y.qt} = 1.1825$
$\hat{s} = 2.138 + .0145q + .00225y + .273t$ $(.0080)\ (.00231)\ (.133)$	$R^2_{\hat{s}.qyt} = .9892$	
$\hat{f} = -.266 - .0204q + .01416y - .817t$		$F_{\hat{f}q.yt} = -.5579$ $F_{\hat{f}y.qt} = 2.1884$

Means: $\bar{r} = 23.1200$ $\bar{s} = 12.3400$ $\bar{f} = 10.7800$ $\bar{q} = 294.800$ $\bar{y} = 1666.0000$ $\bar{t} = 8.0000$

Year	Retail price per qt.	Farm-retail spread per qt.	Consumption per capita 1/	Disposable income per capita	Year	Retail price per qt.	Farm-retail spread per qt.	Consumption per capita 1/	Disposable income per capita
	Cents	Cents	Pounds	Dollars		Cents	Cents	Pounds	Dollars
1948	21.1	9.8	295	1,291	1956	23.3	12.7	306	1,742
1949	20.2	9.9	296	1,271	1957	24.2	13.3	303	1,804
1950	19.8	9.9	293	1,369	1958	24.5	13.8	298	1,820
1951	22.1	10.9	298	1,473	1959	24.7	14.0	292	1,904
1952	23.2	11.3	300	1,520	1960	25.3	14.4	287	1,934
1953	22.8	11.7	298	1,582	1961	25.4	14.5	278	1,980
1954	22.4	12.0	299	1,582	1962	25.3	14.6	276	2,052
1955	22.5	12.3	303	1,660					

1/ Whole milk

Figure 4.7

Another fairly common assumption is that the price spread tends to be a constant number of cents a pound. The regression equations shown under the several diagrams suggest that this assumption may be somewhat nearer the truth than the assumption of constant percentage markups. Note that in three of the seven cases (tomatoes, beef, and milk) the regression coefficient showing the effect of quantity upon price spread was less than twice its standard error. This is often taken as an indication that the coefficient is not significantly different from zero. If the coefficient were zero, it would indicate the constant absolute markup.

In all the cases except milk, the regression equations indicate a negative relation between quantity and spread—that is, as the quantity increases and price comes down, the spread also comes down, at least somewhat.

Warren and Pearson, as the result of a general study of prices of farm products, came to the general conclusion:

> "When there is a large crop, the farm price is reduced more cents per bushel than is the retail price. It costs more cents per bushel to get the cheap crop to the consumer than to get the high-priced crop to him." [1]

The findings reported here are in line with the conclusions drawn from a number of statistical studies made in the former Bureau of Agricultural Economics to the effect that price spreads decline somewhat with increased quantities, that they tend to be somewhat between percentage and absolute amounts, and that they are probaby somewhat closer to absolute amounts than percentages.

Theory alone is not a sure guide on this matter. There are theoretical reasons for expecting price spreads to come down in some cases with increases in quantity, but there are also theoretical reasons for expecting them to go up in other cases with increases in quantity.

In a general way, price spreads are based upon costs—at least in the long run. Average costs may go up or down with increases in quantity. For example, processing costs may go up if an industry is operating at near capacity and has to pay higher wages for overtime work or has to use less efficient equipment to handle excess supplies. But the average costs of both processing and retailing may go down if an industry has unused capacity or labor that is not fully used.

In any case, retailers do not generally allocate costs carefully between individual food commodities. A retailer tries to recover his costs for a whole department, such as fruits and vegetables, but his markup on potatoes may be quite different from that on peaches, for example.

If price spreads for some food commodities tend to widen when supplies increase, as Warren and Pearson found, the explanation may be that there is less competition in buying when supplies are large. If, in addition, the food commodity is one which tends to sell at a fairly standard retail price, then the spread obviously becomes wider with larger quantities.

Price Flexibility With Respect to Income

Table 4.2 shows for each of the seven commodities the price flexibility with respect to income.

Price flexibilities at retail are all positive except in the case of sweetpotatoes. The "income elasticity" for white potatoes probably has been increased by new processed products. Most sweetpotatoes are still sold unprocessed. This indicates, as expected, that the prices of most foods increase as consumer incomes increase. But three of the price flexibilities with respect to income at the farm level are negative and two are zero. Thus, only two of the seven price flexibilities with respect to income are positive at the farm level. Only in two cases out of seven did higher consumer income tend to raise the price to the farmer.

---

[1] Warren, G. F. and Pearson, F. A. INTERRELATIONSHIPS OF SUPPLY AND PRICE. Bul. 466, p. 144. Cornell Univ., 1928.

This is suggestive of what happened in the years following World War II. As consumer incomes increased, the retail prices of most foods went up; the price spreads also went up; and the farmer got little or no benefit from the rising retail prices. For some commodities he actually took lower prices, because the price spreads increased more than retail prices.

TABLE 4.2.—*Price flexibilities with respect to income*

Commodity	At retail level	At farm level
Potatoes	0. 21	−1. 44
Sweetpotatoes	−0. 63	−1. 63
Tomatoes	0. 36	0. 00
Grapefruit	0. 55	0. 00
Apples	0. 32	−0. 16
Beef	1. 29	1. 36
Milk	1. 18	2. 19

Relationship of Price Spreads to Income

Consumer income in the postwar period has been highly correlated with wage rates and with other marketing costs. Thus, when we use consumer income in the equation to estimate the price spread, we are using it as sort of a proxy for costs. Naturally, we would expect the price spread to widen as processing and marketing costs increase. The tables below the diagrams indicate that this happened in six of the seven cases. The only exception was sweetpotatoes, for which the coefficient was practically zero and clearly nonsignificant.

The widening of price spreads as consumer incomes increase causes a fanning out from left to right of the two lines on part B of the diagrams.

Economic Analysis of Price Spreads

The results in this chapter are obviously very general. They seem to explain fairly well the consumer demand for simple, unprocessed perishable foods. They give at least a fair indication of the forces affecting the aggregate price spreads for these foods. Thus, they help explain demand at the farm level.

Both Brandow and Stine, in reviewing this chapter, commented on the need for a thorough and detailed study of the wide variety of economic forces that affect price spreads at every stage of marketing from the farmer to the consumer. Such a study would need to cover a wide variety of farm products. It would need to go into such matters as costs, methods of ratemaking, the nature of competition, the structure of agricultural markets, contract selling, vertical integration, time lags, and a great variety of other matters.

Until recent years, the U.S. Department of Agriculture's work on price spreads was limited mainly to measuring what the spreads are, and publishing detailed statistics. The Congress has been very interested in this kind of statistical measurement. In the past few years, attention has been shifting more to economic analysis of the forces affecting price spreads. Such studies are needed very much to aid in understanding the demand for farm products, especially at the farm level.

Statistics alone are not enough. We need to know what the statistics mean. Especially, we need to know why price spreads change from time to time—and the effects of such changes upon the farmer, the middleman, and the consumer.

Such studies deserve the work of a number of able economists over a period of several years.

5. Some Shifts in Demand

> . . . it is possible to resort to simplified methods of multiple correlation requiring little time or labor and yielding results of considerable practical value.—Louis H. Bean in *a Simplified Method of Graphic Curvilinear Correlation*.

We have noted that routine, mechanical analyses of large masses of data are likely to overlook many essential features of the demand for any particular good. The student interested in practical results will do well to plot the data and make appropriate graphic analyses. The art of graphic analysis was well developed back in the 1920's and 1930's. In these days of automatic computation, many mathematicians have neglected it. Fortunately, many commodity economists interested in really understanding what makes the prices of pork, potatoes, and bread still make good use of graphic analysis.

In this chapter, we shall consider only meats. But similar analyses might well be made for other foods.

Postwar Demand for Meats

Most of the analyses in Chapter 4 were based upon the assumption that there were no significant trends or shifts in demand from 1948 through 1962. We had to abandon this assumption in a few cases. But in these exceptional cases we assumed that there was a steady trend in demand throughout the period. This, too, was an outright assumption which needs to be checked.

A case in point is the postwar demand for meats. Our previous analyses assumed that the relationship between per capita consumption and price of each individual meat was constant throughout the postwar period. We shall now look at the data to see whether this assumption needs to be modified.

The necessary data are in table 5.1, which exhibits the deflated retail prices of beef, pork, lamb, veal, and chicken, together with the per capita consumption of each of these meats.

Deflation

Before proceeding to a graphic analysis of the data, it might be appropriate to ask why the table shows deflated rather than actual prices. There is no magic about deflation. And there is nothing sacred about dividing prices by the consumer price index. As a matter of fact, I made two different graphic analyses: one using actual retail prices and the other using deflated retail prices. The one using deflated prices (that is, prices divided by the CPI) seemed to turn out better. The data lined up more regularly around the indicated curves.

This is, of course, a purely empirical observation. It is also reasonable to think that some form of deflated prices is likely to give better

TABLE 5.1—*Per capita consumption and deflated prices of selected meats, 1948–63*

Year	Beef		Pork		Lamb		Veal		Chicken	
	Consumption per capita [1]	Price per pound [2]	Consumption per capita [1]	Price per pound [2]	Consumption per capita [1]	Price per pound [2]	Consumption per capita [1]	Price per pound [2]	Consumption per capita [1]	Price per pound [2]
	Pounds	*Cents*	*Pounds*	*Cents*	*Pounds*	*Cents*	*Pounds*	*Cents*	*Pounds*	*Cents*
1948	63.1	82.9	67.8	67.6	5.1	77.8	9.5	77.1	18.3	75.4
1949	63.9	76.3	67.7	61.5	4.1	82.4	8.9	75.7	19.6	71.8
1950	63.4	88.3	69.2	60.4	4.0	84.2	8.0	81.1	20.6	68.0
1951	56.1	90.0	71.9	60.6	3.4	86.7	6.6	87.6	21.7	66.0
1952	62.2	85.4	72.4	57.3	4.2	86.2	7.2	86.3	22.1	65.0
1953	77.6	66.2	63.5	62.9	4.7	70.0	9.5	68.7	21.9	62.8
1954	80.1	64.1	60.0	63.7	4.6	71.0	10.0	65.8	22.8	56.4
1955	82.0	63.2	66.8	54.6	4.6	69.0	9.4	65.8	21.3	58.7
1956	85.4	60.9	67.3	51.4	4.5	68.3	9.5	63.6	24.4	50.4
1957	84.6	63.1	61.1	57.6	4.2	69.9	8.8	65.5	25.5	47.6
1958	80.5	72.0	60.2	60.5	4.2	74.1	6.7	76.1	28.2	45.8
1959	81.4	73.3	67.6	52.8	4.8	69.6	5.7	79.8	28.9	41.4
1960	85.2	70.4	65.2	51.6	4.8	67.6	6.2	77.8	28.2	41.4
1961	88.0	68.3	62.2	53.3	5.1	63.3	5.7	77.3	30.3	37.0
1962	89.1	69.8	64.0	52.9	5.1	67.1	5.5	79.5	30.2	38.6
1963 [3]	95.2	67.8	64.9	50.5	4.9	68.0	5.0	79.2	30.6	37.6

[1] Carcass weight equivalent.
[2] Divided by consumer price index (1957–59=100).
[3] 1963 data are preliminary and were not used in the analysis.

results than actual prices in any period where there is a decided change in the general price level. For example, take the retail price of lamb. It rose 8.4 percent from 1948 through 1962. But the consumer price index (that is, the general level of retail prices) rose 25.8 percent during the same period. Thus, the "real" price of lamb was lower in 1962 than in 1948. The concept of real prices is important—just as is the concept of real wages, for example. An increase in wages does not mean much if it is offset by a corresponding rise in the cost of living. In a similar way, an increase in the price of any food is nominal only if the percentage increase is equal only to the percentage increase in the general level of retail prices.

In Chapter 4, we included consumer income as a separate variable. In other words, prices were adjusted for changes in consumer income. Here, we adjusted prices of several meats to changes in the general level of prices by dividing meat prices by the consumer price index. We could have used some other kind of deflation, but in this case, dividing prices by the CPI seemed to work well.

Graphic Analysis

The five separate parts of figure 5.1 show the data for beef, pork, lamb, veal, and chicken. The simplest case seems to be that of chicken, which is shown in the lower right-hand corner of the chart. All the data for chicken in the entire period 1948 through 1962 seemed to line up closely around the smooth line which is shown. True, there was a fairly regular drop in price throughout the period and a corresponding regular increase in consumption. But apparently this was a movement along the same "demand" curve (or quantity-price curve).

In the four other cases, it seemed necessary to draw two demand curves. For example, the data for beef in the period 1948 through 1957 line up closely around a smooth line, as indicated. The corresponding data for the period 1958 through 1962 line up closely around a higher line. This indicates that the demand for beef shifted upward in the latter part of the period. In the period 1958 through 1962, consumers paid higher deflated prices than they paid earlier for the same per capita quantities. Similar analyses indicate downward shifts in the demands for pork, lamb, and veal. These shifts came earlier than the shift in the demand for beef.

Comparison of the different sections of figure 5.1 suggests that these shifts in demand for individual meats may have been due in part to shifts in supplies of competing meats. This is particularly true for beef and pork. The upward shift in demand for beef came several years after a reduction in pork marketings. The drop in the demand for pork occurred simultaneously with the increase in per capita supplies of beef. I do not know how to explain the difference in timing of the shifts in demand for beef and pork. Logically, one might have expected the shifts to have occurred simultaneously. Figure 5.1 shows when the shifts happened, but it does not explain why they took place. Such things as outdoor grills have been suggested as possible causes.

Abrupt Shifts vs. Gradual Trends

The analysis in figure 5.1 suggests abrupt shifts in the demand for beef, pork, lamb, and veal. Such shifts could be explained partly by substantial changes in the marketings of competing meats. For

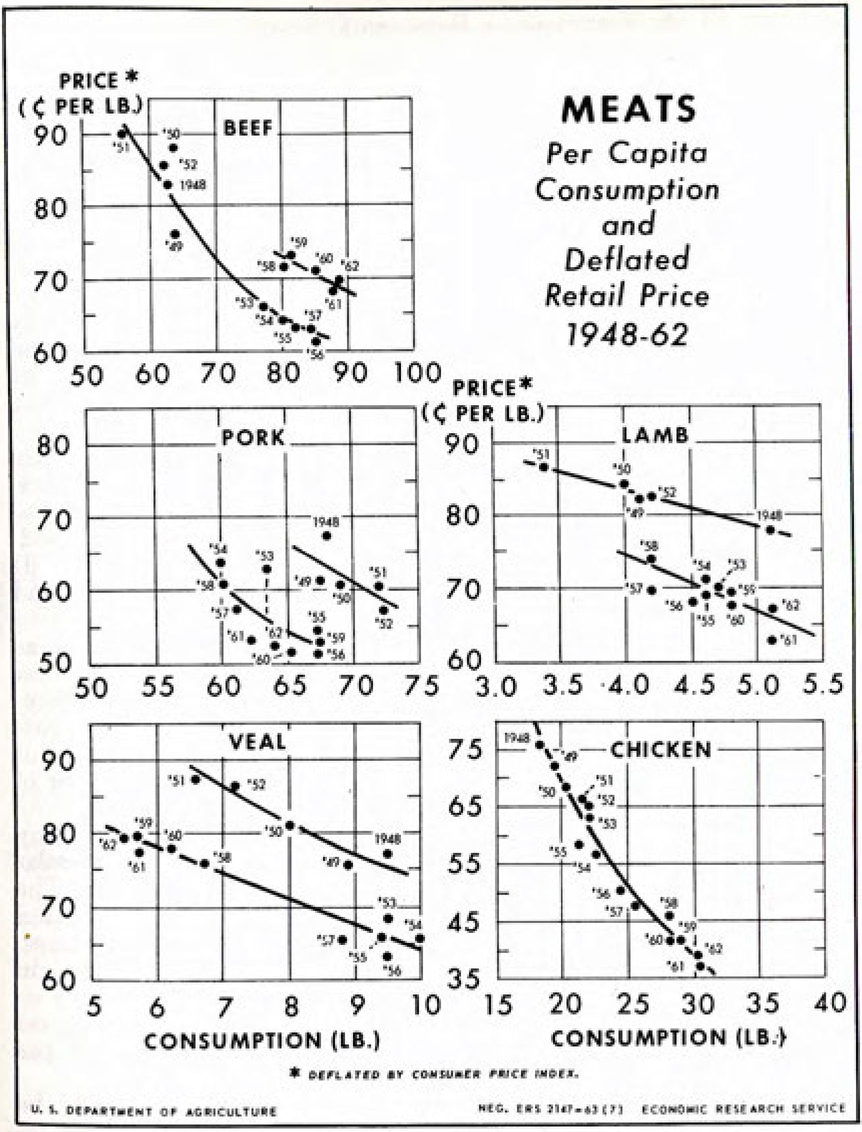


Figure 5.1

example, the sharp increase in beef marketings from 1952 to 1954 may relate to the drop in hog marketings in the same 2 years. In Chapter 6 we shall consider the demand for each meat as a function of the per capita quantities of that meat, and also of competing meats. The shifts in demand can be mostly explained by changes in marketings and by changes in consumer income. Income has increased gradually, and has resulted in a gradual upward trend in the demand for meats as a whole.

Abrupt shifts in demand in past years do not help us forecast the future unless we can explain why the shifts occurred, and thus forecast future shifts.

6. Competition Between Different Foods

. . . competing commodities like beef and pork, or corn and oats, are of particular interest today when the various plans for relieving farmers are under discussion.—HENRY SCHULTZ in *The Theory of Measurement of Demand.*

The price of any food may be affected by hundreds of variables—for example, by supplies of all other goods and services. The reverse is also true: The consumption of any food may be affected by hundreds of factors, including the prices of all other goods and services. In view of this, it may seem presumptuous for a statistician to try to explain past prices—or to forecast future prices—by considering only a few variables. And for the same reason, many mathematicians and economic theorists have doubted the possibility of measuring by statistical means the relations between prices, consumption, and income.

True, it is impossible to measure the effect of every factor that may have influenced price in the past, or that will do so in the future. Good statistical analyses of time series never result in perfect correlations. In practice, there are always unexplained "errors", or "residuals." A good statistician studies his residuals with some care. If he can discover a systematic relationship between the residuals and some other variable, he can improve his analysis. But he can never reduce all residuals to zero except by such reprehensible practices as using as many variables as observations. That is probably what Edgeworth meant in the statement quoted at the beginning of Chapter 1. We cannot hope to get the exact "true" demand curve from statistics, simply because we shall always have residual errors. But we can often get equations that give estimates of expected price, or of expected consumption.

Chapter 4 showed that most of the variation in the prices of many foods was associated with changes in the quantity of the particular good offered for sale and with changes in consumer income. The lines shown on the diagrams in that chapter were not "true" demand curves because not all the observations lay exactly on the lines. There were residuals around the lines. The equations did not explain all the past variations in prices. Certainly, they would not give us perfect forecasts of the future. But they would give fairly good estimates of the expected prices associated with any assumed per capita consumption and per capita consumer income.

Chapter 5 showed that our explanation of meat prices could be improved by taking account of time trends; i.e., of shifts in the relations of expected prices to per capita consumption. Chapter 6 will demonstrate that further improvements can be made by considering the per capita supplies of competing meats.

We are not concerned here with any technical definition of competition in terms of marginal utility. Those interested in the theory and the mathematics of such a definition should consult Hicks.[1] Important as this concept is in the pure theory of demand, we are concerned here with a much simpler idea. In a practical marketing sense, beef competes with pork if increased supplies of beef result in lower prices of pork. This sort of competition can be measured statistically

[1] Hicks, J. R. VALUE AND CAPITAL. Pp. 11–52 and the mathematical appendix, pp. 303–328. Clarendon Press, Oxford, 1939.

without any refined assumptions about the nature of utility surfaces. It is in this sense that we proceed to consider competition between different meats.

Competition Among Beef, Pork, and Chickens

As an example of this problem, take meats again.[2] This is a small part of the general question of interrelationships among demands for all foods. The general question is discussed in Appendix 3. A study of Figure 5.1 suggests that changes in beef supplies affect pork prices. Thus, when beef supplies increased in the mid-1950's, the supply-price curve for pork dropped. To investigate this matter in detail, I made an analysis of prices of beef, pork, and chickens. It is in table 6.1.

TABLE 6.1.—*Relationships between expected price and per capita consumption and disposable consumer income for beef, pork, and chicken*

A. Correlation matrix

$$
\begin{array}{c}
 \\ X_1 \\ X_2 \\ X_3 \\ X_4
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} X_1 & X_2 & X_3 & X_4 & Y_5 & Y_6 & Y_7 \end{array} \\
\left[\begin{array}{ccccccc}
1.0000 & -0.7213 & 0.7620 & 0.8080 & -0.3844 & 0.1567 & -0.8519 \\
-.7213 & 1.0000 & -.4837 & -.5048 & .4460 & -.6551 & .5407 \\
.7620 & -.4837 & 1.0000 & .9569 & .2627 & .2240 & -.9613 \\
.8080 & -.5048 & .9569 & 1.0000 & .1063 & .1775 & -.9455
\end{array}\right]
\end{array}
=[a:b]
$$

X_1 = Beef consumption per capita
X_2 = Pork consumption per capita
X_3 = Chicken consumption per capita
X_4 = Disposable consumer income per capita
Y_5 = Retail price of beef
Y_6 = Retail price of pork
Y_7 = Retail price of chicken

B. Intermediate computations

$$
\begin{bmatrix}
12.0893 & 4.6020 & 9.8750 & -17.6206 \\
4.6020 & 3.0937 & 3.7691 & -6.0398 \\
9.8750 & 3.7691 & 19.9317 & -25.7425 \\
-17.6206 & -6.0398 & -25.7425 & 37.8802
\end{bmatrix} = a^{-1}
$$

$$
\begin{bmatrix}
-1.8744 & -2.0351 & -0.6438 \\
-.0413 & -1.5330 & -.1605 \\
.3831 & -1.0248 & -1.1964 \\
1.3460 & 2.1508 & .6772
\end{bmatrix} = a^{-1}b
$$

C. Regression equations (based on standard deviation units)

$$
\begin{aligned}
y_5 &= \underset{(0.2559)}{-1.8744x_1} \underset{(0.1294)}{-0.0413x_2} \underset{(0.3285)}{+0.3831x_3} \underset{(0.4529)}{+1.3460x_4} \qquad R^2=0.9458 \\
y_6 &= \underset{(0.4433)}{-2.0351x_1} \underset{(0.2242)}{-1.5330x_2} \underset{(0.5692)}{-1.0248x_3} \underset{(0.7847)}{+2.1508x_4} \qquad R^2=.8375 \\
y_7 &= \underset{(0.1855)}{-0.6438x_1} \underset{(0.0938)}{-0.1605x_2} \underset{(0.2382)}{-1.1964x_3} \underset{(0.3283)}{+0.6772x_4} \qquad R^2=.9715
\end{aligned}
$$

D. Alternative regression equation for beef

$$
y_5 = \underset{(0.1497)}{-1.9352x_1} \qquad \underset{(0.1497)}{+1.7847x_4} \qquad R^2=.9337
$$

---

[2] Throughout this bulletin I have used average prices for all cuts of meat. Actually, various cuts of beef, pork, and lamb sell at very different prices.

TABLE 6.1.—*Relationship between expected price and per capita consumption and disposable consumer income for beef, pork, and chicken*—(Continued)

E. Standard deviations and means

	X_1	X_2	X_3	X_4	Y_5	Y_6	Y_7
Standard deviations:	11.1000	3.9222	4.0019	251.2898	7.4561	3.5228	8.2491
Means:	76.1733	65.7933	24.2667	1666.0000	68.5067	54.6200	51.3400

F. Regression equations (based on original units)

$$Y_5' = \underset{}{79.3100} \underset{(0.1066)}{-1.2999X_1'} \underset{(0.0044)}{+0.0529X_4'}$$

$$Y_6' = 166.0721 \underset{(0.1407)}{-0.6459X_1'} \underset{(0.2014)}{-1.3769X_2'} \underset{(0.5010)}{-0.9021X_3'} \underset{(0.0110)}{+0.0301X_4'}$$

$$Y_7' = 132.8033 \underset{(0.1378)}{-0.4784X_1'} \underset{(0.1973)}{-0.3375X_2'} \underset{(0.4909)}{-2.4661X_3'} \underset{(0.0108)}{+0.0222X_4'}$$

Part A of the table is a matrix of zero-order correlation coefficients. The matrix is partitioned into two submatrices: a is the submatrix of correlations among the independent variables (consumption of the three kinds of meat and consumer income). Submatrix b shows the correlations between the independent variables and the dependent variables. The dependent variables are the ones whose values are given, or assumed.

Part B first shows the inverse of a; that is, a^{-1}. Then it shows $a^{-1}b$. The three columns of $a^{-1}b$ are the three regressions in terms of standard deviation units—often called the "betas." This way of computing the regressions is similar to that explained by Fisher.[3] The detailed computations followed the general pattern of Friedman and Foote.[4] Similar computations are explained in Ezekiel and Fox.[5]

Part C of the table repeats the regressions given by the columns of $a^{-1}b$ in Part B, and adds the standard errors of regression coefficients (shown in parentheses below the corresponding coefficient) and the squared multiple correlation coefficients. The squared standard error of β_{ik} is

$$S^2\beta_{ik} = \frac{(1-R_i^2)}{n-m} c_{kk},$$

where c_{kk} is the k^{th} diagonal element of a^{-1}, n is the number of observations, and m is the total number of variables (including the dependent).

The first regression—that for beef price—is peculiar. The regressions of beef price on pork supply and on chicken supply are both nonsignificant. Moreover, the regression of beef price on chicken supply is positive, which does not seem at all possible. This result is doubtless due to the strong upward trends in chicken supplies and in consumer income. For these reasons, I computed an alternative regression for beef prices. It is shown in Part D. I strongly prefer it to the first equation in Part C. With this change, all the coefficients

[3] Fisher, R. A. STATISTICAL METHODS FOR RESEARCH WORKERS. Pp. 129–166. Oliver and Boyd, Edinburgh and London, 12th ed., 1954.

[4] Friedman, Joan and Foote, R. J. COMPUTATIONAL METHODS FOR HANDLING SYSTEMS OF SIMULTANEOUS EQUATIONS. U.S. Dept. Agr. Agr. Handb. 94, 1957.

[5] Ezekiel, M. J. B. and Fox, K. A. METHODS OF CORRELATION AND REGRESSION ANALYSIS. Pp. 489–530. Wiley & Sons, New York, 1959.

make sense, and most of them are statistically significant. The correlations are satisfactory.

How well do the equations in part F of table 6.1 fit the 1963 data? The preliminary 1963 data, per capita, were:

Beef consumption	$X_1=$ 95.2
Pork consumption	$X_2=$ 64.9
Chicken consumption	$X_3=$ 30.6
Consumer income	$X_4=$\$2,127

Inserting these values into the equations, we get the following retail prices in cents per pound and percentage errors in expectations:

	1963 preliminary	*Conditional expectations*	*Percentage error*
Beef, X_5	67.8	68.1	+0.4
Pork, X_6	50.5	51.6	+2.2
Chicken, X_7	37.6	37.1	−1.3

Implications

This analysis implies that beef prices are not affected significantly by changes in supplies of pork, nor by changes in supplies of chickens. On the other hand, it suggests that pork prices and chicken prices are both affected by changes in supplies of other meats. This seems to run counter to the findings of Fox.[6] Using prewar data and first differences of logarithms, Fox concluded that pork prices were practically independent of variations in supplies of other meats, while beef prices were not. This should make us cautious about accepting too readily the equations shown in part F of table 6.1. They need further testing, including actual forecasting.

In any case, if pork competes with beef, it seems only logical to expect beef to compete with pork. If chicken competes with beef, beef probably competes with chicken. And if pork competes with chicken, chicken doubtless competes with pork. Thus, if we have demand equations like

$$
\begin{aligned}
Y_5&=a_{11}x_1+a_{12}x_2+a_{13}x_3+a_{14}x_4\\
Y_6&=a_{21}x_1+a_{22}x_2+a_{23}x_3+a_{24}x_4\\
Y_7&=a_{31}x_1+a_{32}x_2+a_{33}x_3+a_{34}x_4,
\end{aligned}
\tag{6.1}
$$

we should expect a_{12} and a_{21} to have the same signs; likewise a_{13} and a_{31}, and a_{23} and a_{32}.

But this is not all. If the consumer spends a very small proportion of his income on any of the three commodities, we should expect a_{12} and a_{21} to be approximately equal to one another; i.e., $a_{12}\approx a_{21}$. Likewise, we should expect $a_{13}\approx a_{31}$ and $a_{23}\approx a_{32}$. The reasons for this are given in Appendix 3.

A Symmetric Matrix

If we assume that the real matrix of coefficients a_{ij} to the right of 6.1 is symmetric, we can fit it by the method indicated in Appendix 4.

[6] Fox, Karl A. THE ANALYSIS OF DEMAND FOR FARM PRODUCTS. U.S. Dept. Agr. Tech. Bul. 1081. Pp. 41–46. 1953.

Applying that method to the same beef-pork-chicken data as used before, we get the symmetric equations

$$
(6.2)\quad
\begin{aligned}
Y_5 &= 124.62 - 1.5349X_1 - 0.4788X_2 - 0.4460X_3 + 0.0619X_4 \\
Y_6 &= 139.26 - 0.4788X_1 - 1.2027X_2 - 0.3101X_3 + 0.0231X_4 \\
Y_7 &= 123.99 - 0.4460X_1 - 0.3101X_2 - 2.3871X_3 + 0.0238X_4.
\end{aligned}
$$

Compare these equations with those in part F of table 6.1 Don't these seem more likely plausible, acceptable? I think so.

Of course, the estimated prices, as computed from equations (6.2) are less highly correlated with the actual prices than are the estimates from part F of table 6.1. This is because the equations in part F were computed to maximize the correlations. So we sacrifice some of the high correlations to gain what seems to be a more logical set of equations. Which set would give the better results in actual forecasting? We cannot be sure. But the high intercorrelations among independent variables can make any regressions untrustworthy—even though the multiple correlations are very high.

7. Marginal Utility and Indifference

> Le fait qui permet d'établir un lien entre la théorie abstraite de l'économie pure et les phénoménes économiques concrets, est . . . la proportionalité des prix et des utilités marginales au point de l'équilibre du marché.[1]—RAGNAR FRISCH in *Sur un Probleme d' Economie pure.*

It sometimes seems as if there is a yawning chasm between pure abstract economic theory and the actual facts of economic life as seen in the marketplace and as recorded in economic statistics. Ricci pointed this out in a famous article analyzing the contributions of Pareto and his brand of mathematical economic theory.[2] After paying tribute to the great accomplishments in pure logic, Ricci went on to say that the theory remains abstract and intangible, and that there is "no bridge" between the pure theory of Pareto and nine-tenths of the problems with which the economist is usually concerned.

The chasm between pure economic theory and practical economic analysis is extremely wide and deep. Some economists prefer to live on one side and some on the other side. They have little opportunity to communicate with one another, and sometimes they seem to have little desire to do so. I think this is extremely unfortunate. The statistician and economist engaged in analyzing practical problems could gain a great deal of insight by understanding basic theory. Also, a good theorist needs to do more than sit in an armchair and think deep thoughts. He needs to observe what happens in the real world of economics. Even the fanciest mathematics, filled with Greek letters, will not give him this sort of information. There is a crying need for closer understanding and cooperation between pure theorists and those who apply theory to practical problems of business and politics. This is getting to be more and more difficult. It is something like trying to build a bridge over the Grand Canyon.

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[1] That is, equilibrium market prices are proportional to marginal utilities; this fact enables us to establish a relationship between abstract economic theory and concrete economic phenomena.

[2] Ricci, Umberto. GIORNALE DEGLI ECONOMISTI. P. 43. Rome. Jan.–Feb. 1924.

But, the quotation from Frisch at the head of the chapter says that a bridge between theory and economic phenomena can be built upon the basis of the proportionality of prices and the marginal utilities when the market is in equilibrium.

What did Frisch mean by proportionality? Assume that a given consumer is rational and that he spends his income in a free market without any rationing. In other words, he can buy as much or as little of each good or service as he chooses. How will he spend his money? He will allocate his expenditures among the different goods and services in such a way that he gets the same marginal utility (or marginal satisfaction) from a dollar's worth of each good or service. Otherwise, he is not rational. For example, if he gets more marginal satisfaction from a dollar's worth of beef than from a dollar's worth of pork, he should obviously buy more beef and less pork. He should obviously continue to make such adjustments as long as the marginal satisfactions are unequal.

Remember that we are talking about a consumer's own preferences, whether or not they seem appropriate to us. We may think a friend spends too much money on liquor, on gambling, or on the opera. We may think he saves too little, or too much. This is entirely beside the point. In our analysis, we accept the old Roman doctrine, *de gustibus non disputandum*. Whatever a consumer's preferences are, if he is rational he will try to allocate his spending so that he gets the same marginal satisfaction out of a dollar's worth of each good or service.

Most bridges are useful in both directions. Near my boyhood home in Massachusetts is a bridge over the Connecticut River. You can use this bridge to go from Hadley to Northampton. You can also use it to go from Northampton to Hadley. There are certain advantages to be obtained by going in either direction. The Northamptonites can go to Hadley for asparagus and onions. The Hadleyites can go to the movies in Northampton.

So it is with the bridge suggested by Frisch. In one direction you can go from pure economic theory to statistical measurement and application. In the other direction you can use statistical measurements to infer important things about theory.

By using this bridge, modern econometricians are beginning to build a new economics of welfare. For example, such writers as Hicks,[3] Samuelson,[4] Strotz,[5] and Tolley and Gieseman[6] have shown how we can get useful measurements of marginal utility.

Appendix 3 discusses some of the implications of ulility theory to the demand for related commodities. Bernouilli used the principles of utility to analyze certain phenomena about gambling and insurance. Brandow used some of the theoretical principles derived from utility analysis for setting up a demand matrix for agricultural products. His demand matrix has proved to have many practical applications. These are examples of using the bridge to go from pure theory to practical application.

---

[3] Hicks, J. R. VALUE AND CAPITAL. Oxford Univ. Press, London, 1939.

[4] Samuelson, P. A. FOUNDATIONS OF ECONOMIC ANALYSIS. Harvard Univ. Press, Cambridge, 1955.

[5] Strotz, R. H. THE EMPIRICAL IMPLICATIONS OF A UTILITY TREE. Econometrica 24: 269–280, 1957.

[6] Tolley, G. S. and Gieseman, R. W. CONSUMER DEMAND EXPLAINED BY MEASURABLE UTILITY CHANGES. Econometrica, Vol. 31, No. 3, July 1963.

Using the bridge established by Frisch and others, we should be able to start with statistical measurements and explore their theoretical implications. Thus, we can start with statistical measurements of the interrelationships between prices, consumption, and consumer income. We can deduce from these certain implications about marginal utility, or at least about indifference function. Any such estimates of marginal utility or indifference can have very important practical applications. They could give us an income tax structure graduated so that each income group made roughly the same sacrifice in terms of disutility. They could perhaps give us a better way of estimating changes in welfare to be expected from proposed changes in public programs, such as price supports, production controls, and the diversion of surpluses.

There have been many attempts in recent years to "rehabilitate" the Dupuit-Marshall concept of consumer surplus. Marshall was careful to point out that this concept applied only when one could assume that the marginal utility of money was constant. Instead of rehabilitating consumer surplus, perhaps we might substitute analyses based upon quantitative estimates of changes in the marginal utility of money.

The Marginal Utility of Money

We do not need here an elaborate theory of marginal utility. Readers particularly interested in the subject are advised to read the writings of Frisch on this subject. The general idea is that a rational consumer would spend his money in such a way that he gets the same marginal utility from a dollar's worth of each commodity.

If u_m represents the marginal utility of money to a consumer, if u_k represents the marginal utility of a unit of commodity k, and if p_k represents the price of commodity k, the bridge between the observed statistics and the theory can be written

$$\frac{u_k}{p_k} = u_m. \tag{7.1}$$

In this equation, $\frac{u_k}{p_k}$ is simply the marginal utility of a dollar's worth of commodity k. The equation is a logical relation and should hold that the consumer is completely rational and that he has full information. In actual practice, of ccurse, (7.1) may not be an exact equation, but rather an approximation.

Equation, or approximation, (7.1) holds good at all times and places. For example, letting the subscripts 1 and 2 represent two periods of time, we could write the equations for these two periods in the form

$$\frac{u_{k.1}}{p_{k.1}} = u_{m.1} \text{ and } \frac{u_{k.2}}{p_{k.2}} = u_{m\,2.2}. \tag{7.2}$$

Moreover, we can divide one equation by the other. Suppose we divide the second equation of (7.2) by the first. This gives us

$$\frac{u_{m.2}}{u_{m.1}} = \frac{u_{k.2}}{p_{k.2}} \frac{p_{k.1}}{u_{k.1}}. \tag{7.3}$$

Finally, suppose that in a particular situation we can assume that $u_{k.1}=u_{k.2}$. In such a case, equation (7.3) obviously has become simply

$$\frac{u_{m.2}}{u_{m.1}}=\frac{p_{k.1}}{p_{k.2}}. \tag{7.4}$$

Equation (7.4) says that the ratio of the marginal utilities of money in the two periods in this case equals the ratio of reciprocals of the prices of commodity k. This is, of course, very abstract. Can we make it concrete? Suppose the k^{th} good is all foods. Perhaps we can assume that the utility or satisfaction obtained from foods is practically independent of the utility from other commodities. Then, if we compare two periods of time when the consumer is getting the same quantities of food (and when his wants are the same), he must also be getting the same marginal utilities from food. In such a case, equation (7.4) would be applicable. We could use it to measure the relative marginal utility of money in the two periods.

I first tried an analysis of post-World War II data for consumer income, food prices, and food consumption. However, there was such a high inverse correlation in this period between deflated consumer income and deflated food prices that there was not enough scatter left to make a good analysis. For that reason, I decided to use pre-World War II data covering the period 1927 through 1941. This period includes the business boom that reached a climax in 1929. It includes the depression of the early 1930's. And it includes the recovery from the depression to the beginning of World War II. In time of depression, we expect money to be tight and its marginal utility to be high. In time of prosperity, we expect the reverse. In many ways, 1927 through 1941 is an ideal period to test the method of analysis. Of course, for many practical purposes we would rather have an up-to-date analysis, but I hope the study of the 1927–41 period may be of interest nonetheless.

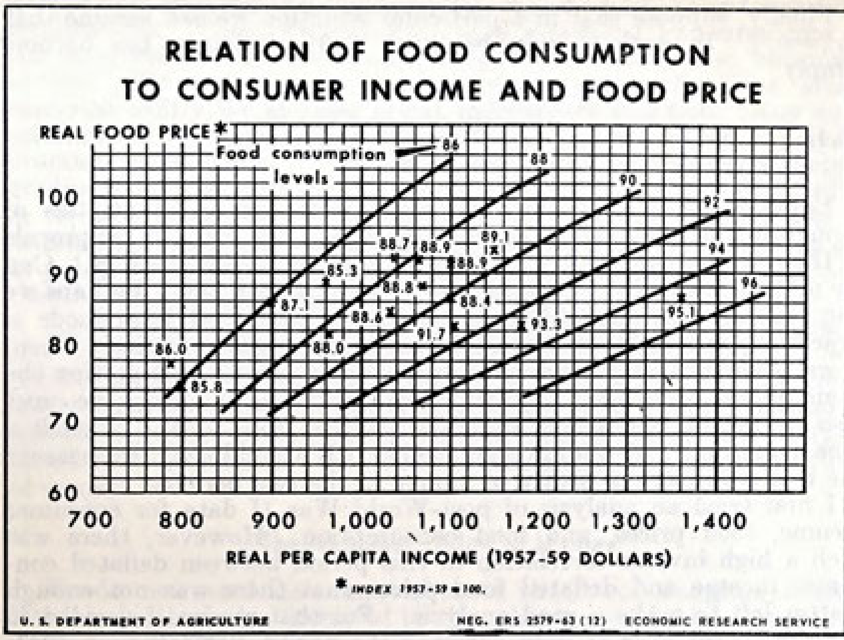
A Statistical Example

Figure 7.1 presents and analyzes the annual data from 1927 through 1941 for per capita real income, real food price, and per capita food consumption. The real incomes and real prices are in terms of 1957–59 dollars; that is, the data in current dollars were divided by the consumer price index, with the base 1957–59=100.

The analysis attempts to measure relative changes in the marginal utility of real income (that is, income measured in dollars of constant purchasing power) to a typical consumer who has the national average income and buys the national average amount of food.

The concept of a typical consumer is extremely important if we are to have a bridge between pure theory and practical application. All economists know that it is not possible to compare the utilities obtained by different individuals. This is simply because wants and preferences vary from one person to another. But in many practical applications we are concerned with the welfare of typical persons or typical families, whose wants probably do not change much from year to year.

If we had records of the income of a typical family, a record of the amounts of food bought at each income, and the price of food in each



Basic Data

Year	Real income per capita in 1957-59	Real food price in 1957-59	Index of per capita food consumption 1957-59=100
	Dollars	Dollars	Dollars
1927	1,066	91.7	88.9
1928	1,094	92.0	88.9
1929	1,142	93.1	89.1
1930	1,038	90.9	88.7
1931	972	82.3	88.0
1932	819	76.3	85.8
1933	807	78.3	86.0
1934	882	84.3	87.1
1935	960	88.1	85.3
1936	1,070	88.0	88.5
1937	1,102	88.4	88.4
1938	1,031	83.5	88.6
1939	1,112	82.4	91.7
1940	1,180	83.0	93.3
1941	1,359	86.2	95.1

Figure 7.1

situation, we would look for combinations of income and food price that would induce the typical consumer to buy the same fixed quantity of food all the time. Then, assuming that the family got the same marginal utility from the same amount of food, we could estimate the relative marginal utility of income at different times by equation (7.4). The relative marginal utility of money would be proportional to the reciprocal of food prices. We can do somewhat the same thing by imagining a typical consumer who gets the national average income and buys the national average amount of food.

In the present case, we can estimate how the marginal utility of a typical family in the United States varied with its income by interpreting the slopes on the lines of figure 7.1. For example, take the isoquant labeled 90. Any point on this isoquant indicates a combination of real per capita income and real food price, resulting in a per capita food consumption index of 90. At any of these points, we assume in our analysis that the marginal utility of food was constant. Therefore, we can use equation 7.4 to estimate changes in the marginal utility of money. Reading from isoquant 90, we find that the food price associated with an income of $1,000 was 79; while the food price associated with an income of $1,200 was 94. Equation (7.4) lets us interpret what this means in terms of relative marginal utility. Specifically, we find that when income rose from $1,000 to $1,200, the marginal utility of money dropped to 79/94 or 84 percent—a drop of 16 percent. Similar computations could be made for other incomes, using any of the isoquants in figure 7.1.

Each observation is plotted on the diagram by first locating the point corresponding to the combination of real income and food price in a given year, and adding the index of per capita food consumption for that year. For example, the observation for the year 1927 was plotted by first locating the point corresponding to $1,066 income and an index of real food prices of 91.7, then writing beside that point the index of per capita food consumption, which was 88.9. The observations for each of the other years were plotted in a similar manner.

We need a three-dimensional analysis to find the relationship between the three variables—income, food price, and food consumption. Such a relationship can be shown graphically by means of isoquants. These are similar to contour lines drawn on a map. Such contour lines can be drawn graphically the same way a curve is drawn through a number of observations on a two-dimensional dot chart. The only difference is the interpolation in three dimensions instead of two. This is what the surveyor does when he makes a contour map.

My first analysis of the data in figure 7.1 was entirely by graphics. Later, I computed a mathematical fit. The isoquants shown in figure 7.1 were obtained from the mathematical equation. The equation was chosen for two reasons: First, it appeared to be logical (for reasons discussed later); second, it fit the data well. The equation was:

$$(7.5)\qquad q = 66.492 - \underset{(0.103)}{1.085}\left(\frac{100}{p}\right) + \underset{(0.040)}{0.424}\left(\frac{100 \log y}{p}\right).$$

$$R^2 = 0.905$$

The numbers in parentheses are the standard errors of the regression coefficients immediately above. Both regression coefficients are clearly statistically significant. The squared coefficient of multiple correlation is 0.905. The fit is evidently very good.

Equation (7.5) is an ordinary least squares equation, treating q as the dependent variable.

In this analysis we are interested in the relation between food price and consumer income when food consumption is held constant. For this purpose we can rewrite equation (7.5) in the following form.

$$(7.6)\qquad \frac{100}{p} = \frac{q - 66.492}{0.424 \log y - 1.085}.$$

We want to use equation (7.1) to measure relative marginal utility of money. Using (7.1) and (7.6), we can estimate the marginal utility of money along any isoquant (i.e. where q is constant), as

$$(7.7)\qquad U_m=\frac{k\ (q-66.492)}{0.424\log y-1.085}=\frac{\text{constant}}{\log y-2.560}.$$

Equations similar to (7.7) have a long history in the literature of economics and mathematics. Daniel Bernouilli[7] (one member of the famous Swiss family of mathematicians) suggested as early as 1738 that the marginal utility of money was $\frac{\text{constant,}}{y-a}$ where a is "the minimum of existence"; that is, with an income of less than a, there would be no utility at all. As incomes increased from a, utility would rise, but at a decreasing rate.

Most economists and mathematicians interested in utility agree that total utility would rise rapidly as income increased above the minimum of existence, but that the rate of increase would decline as incomes increased and the curve of total utility would flatten out and be almost horizontal for the highest incomes. The same reasoning indicates that the marginal utility of money would be infinite at the minimum of existence, would drop off as incomes increased, and would approach zero for the largest incomes. Either Bernouilli's curve or Frisch's curve, indicated in (7.7), would meet these conditions. Frisch presented five logical conditions that he believed should be met by a curve of marginal utility of money. In addition to the conditions already mentioned, Frisch was concerned with what he has called "money flexibility." This does not mean a rubber dollar. It means the flexibility[8] of the marginal utility of money with respect to income.

Frisch indicated that the money flexibility is greater than unity for small incomes, but that as incomes increase, money flexibility decreases and approaches zero as incomes become very large. Bernouilli's equation does not meet the last of these specifications. His formula would indicate a flexibility greater than unity for all incomes. For that reason, I have used the same type of equation that Frisch used in his 1926 study.

The flexibility of marginal utility of money with respect to income is obtained by differentiating (7.7) with respect to y, and multiplying the result by y/u_m. This gives us

$$(7.8)\qquad Fu_m\cdot r=\frac{-0.434}{\log y-2.560}.$$

According to (7.8), the logarithm of the minimum of existence is 2.56. This would indicate that $363 per capita annual income was the minimum of existence. Of course, this is a very great extrapolation, since the lowest annual average income used in this analysis was about $800. The minimum of existence probably could be located more accurately if separate studies were made of the marginal utility of money among very poor families. Also, it would be desirable to

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[7] Bernouilli, Daniel. SPECIMEN THEORIAE NOVAE DE MESURA SORTIS. Commentarii Acadamiae Scientiarum Imperialis Petropolitanae, Vol. 5, 1738.

[8] That is, $Fu_m\cdot r=\frac{du_m}{dr}\cdot\frac{r}{u_m}$.

have similar studies of the marginal utility of money for very rich families. Actually, the measurements of marginal utility given here are accurate only within the range of observations, which is between \$800 and \$1,400 a year per capita real income in 1957–59 dollars.

Equation (7.8) indicates the flexibilities:

Per capita income	*Money flexibility*
\$800	−1. 27
\$1,000	−. 99
\$1,200	−. 84
\$1,400	−. 74

Similar estimates of money flexibility could have been made from a graphic analysis without any mathematical equation. Such estimates would be based on the slopes of one of the isoquants corresponding to several different income levels. Also, one could try other forms of an equation relating food consumption, food price, and income. For example, an equation which would meet all of Frisch's original tests would be similar to (7.5), except that it would use log log r in place of log r. I did not fit such an equation, because (7.5) seems to fit well enough.

An Indifference Surface for Beef and Pork

In trying to measure marginal utility, we had to assume that the satisfactions obtained from food were independent of the satisfactions from other goods and services. This is a rather heroic assumption, and perhaps needs to be taken with a grain of salt. We can justify the methods we used on two grounds: First, it seems logical to think that the assumption is approximately true, although probably not exact; second, a number of studies by Frisch, using many different commodities in place of food, all gave somewhat similar results. But in what follows, we are not concerned with measuring marginal utility, but with finding combinations of quantities of pork and beef that seem to be indifferent to the typical consumer who buys the average quantities and who has an average income.

There is a great deal of literature about indifference surfaces. Almost all of it starts with an assumed indifference surface and deduce what demand functions and supply functions would be implied. To my knowledge, very few people have attempted to start with market data and find the indifference functions that are implied by the quantities purchased and their prices. Yet, this is just what we need if we are to make any practical use of indifference functions, or even if we are to use such functions to help us understand how the market operates.

I have attempted to derive an indifference surface for beef and pork. It is based on data in table 7.1 The first two columns in the table show per capita consumption of beef and pork in the United States from 1948 through 1962. The third column, q_3, is the per capita consumption of all goods and services other than beef and pork. It is found by starting with the per capita disposable income, subtracting the expenditures for beef and pork, and dividing the remainder by the consumer price index. This gives us the deflated expenditures for everything except beef and pork. In this sense, it represents consumption of all other things. The fourth column, r, is the ratio of retail beef prices to retail pork prices. (The fifth column will be explained a little later.)

TABLE 7.1.—*Data for indifference surface*

Year	Annual per capita consumption		Consumer income [1] (q_3)	Actual price ratio [2] (r)	Adjusted price ratio [3] (r')
	Beef (q_1)	Pork (q_2)			
	Pounds	*Pounds*	*Dollars*		
1948	63. 1	67. 8	1, 615	1. 226	1. 529
1949	63. 9	67. 7	1, 592	1. 241	1. 584
1950	63. 4	69. 2	1, 703	1. 379	1. 594
1951	56. 1	71. 9	1, 744	1. 485	1. 657
1952	62. 2	72. 4	1, 785	1. 490	1. 607
1953	77. 6	63. 5	1, 847	1. 052	1. 079
1954	80. 1	60. 0	1, 817	1. 006	1. 053
1955	82. 0	66. 8	1, 924	1. 158	1. 120
1956	85. 4	67. 3	2, 003	1. 185	1. 081
1957	84. 6	61. 1	2, 006	1. 095	. 997
1958	80. 5	60. 2	1, 960	1. 190	1. 120
1959	81. 4	67. 6	2, 040	1. 388	1. 259
1960	85. 2	65. 2	2, 057	1. 364	1. 197
1961	88. 0	62. 2	2, 083	1. 281	1. 104
1962	89. 1	64. 0	2, 144	1. 319	1. 090

[1] Per capita disposable income less expenditures for beef and pork, deflated by the consumer price index.

[2] Ratio of retail beef price to pork price.

[3] The same ratio corrected for the effect of q_3. Specifically,

$$\log r' = \log r - 1.452289\ (\log q_3 - 3.274239),\ \text{or}$$
$$\log r' = \log r + 4.755141 - 1.452289 \log q_3.$$

The first step in the analysis was to run an ordinary regression equation in logarithms, using log r as the dependent variable, since r is the variable to be explained. It turned out to be

$$(7.5)\quad \log r = -4.788588 - \underset{(0.310)}{0.85546} \log q_1 + \underset{(0.441)}{0.955203} \log q_2 + \underset{(0.398)}{1.452289} \log q_3.$$

The numbers in parentheses are standard errors of the regression coefficients immediately above. The squared correlation coefficient was 0.800.

Now, we come back to column 5 of table 7.1; r is the price ratio adjusted for variations in q_3. The mean of log q_3 was 3.274239. The formula for the corrected price ratio is given in footnote 5 of the table.

The adjusted price ratios r' are estimates of what the price ratios would have been with varying amounts of beef and pork (i.e., varying q_1 and q_2, but with expenditures for all other goods and services held constant). I will use these adjusted price ratios to make inferences about the shape of a *partial* indifference surface for beef and pork—that is, *a set of isoquants connecting various combinations of beef and pork to which the typical consumer would be indifferent* (assuming constant amounts of other things).

This use of indifference curves differs from those found elsewhere. Edgeworth, and many other early writers on indifference, discussed cases in which the consumer spent his entire income for the two goods studied—say, for beef and pork, or for foods and nonfoods. This enabled them to work in only two dimensions. Hicks and some other modern economists make a similar simplification by considering combinations of one commodity and other things grouped together.

I searched for combinations of beef and pork that would apparently have been equally satisfactory to the typical consumer, *always assuming that he could have bought the same amounts of other goods and services.* These indifference combinations of beef and pork will be inferred from the adjusted price ratios, r'. The price ratios are the "bridge" between objective statistical analysis and the pure theory of subjective indifference.

First, we plot the data for q_1, q_2, and r' for each year, as in figure 7.2. In 1948, for example, q_1 was 63.1, q_2 was 67.8, and r' was 1.529. We locate the point (63.1, 67.8), and label it 48 to identify the year. Through this point we draw a line sloping downward 1.529 units on the x-axis for each unit on the y-axis. A transparent triangle and straight edge are very useful in drawing such lines. Similarly, we locate the (q_1, q_2) points and the price-ratio slopes for all the other years.

What do these lines mean? Take 1948, for example. If the price ratio were 1.529, the typical consumer could have bought any combination of beef and pork lying along the straight line (extended as far as he pleased in either direction). Any of the combinations along that line would have cost the same amount of money and would have left the consumer as much to spend on other things. Actually, the typical consumer bought 63.1 pounds of beef and 67.8 pounds of pork. He did so of his own free will, *because he preferred that combination to the others on the straight line.*

This is the key to indifference analysis. We can infer certain things about preferences from the actual responses of consumers to prices. More precisely, we can infer that there is an indifference curve tangent to the straight line through each observed combination (q_1, q_2), and that each such line is concave downward. We know that no two indifference curves can cross one another.

With these simple principles in mind, it is easy to interpolate a series of graphic curves in a diagram like that in figure 7.2. Like any statistical problem with actual data, the conditions will not be met exactly—the fit will not be perfect. But it will be close enough for practical purposes—that is, the adjusted price ratios, r', will be approximately equal to the slopes of the indifference lines passing through a given (q_1, q_2) combination.

For precise measurement, there is merit in fitting a mathematical surface to the data. The isoquants (contour lines) of such a surface should fit the data in the sense described above. Appendix 5 explains a mathematical equation that I used to fit the surface in figure 7.2. But we need not spend time on the mathematical fit here. The principles are the same, whether the indifference lines in figure 7.2 are interpolated graphically, or are computed on a calculating machine.

I have drawn five indifference curves through figure 7.2. Of course, any number could have been drawn. The five curves are numbered in Roman numerals. The analysis does not indicate which combinations are preferred—only which are indifferent. But the consumer's position is obviously improved as he goes from combinations on curve I to those on II, and to those on the higher curves, since he can get

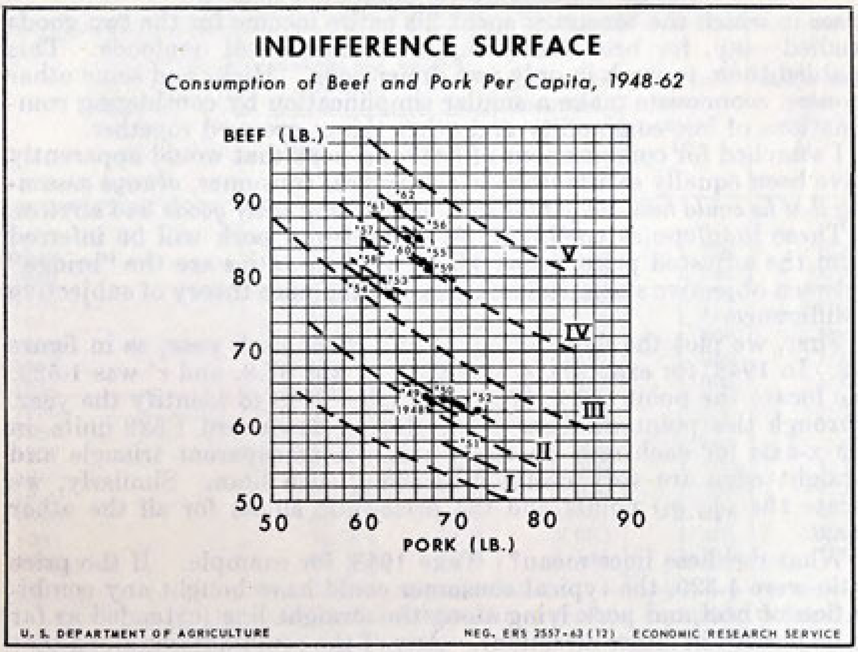

Figure 7.2

more beef and more pork on the higher curves. But there is no attempt in this analysis to measure the gain, either in total utility or in marginal utility. The satisfactions obtained from combinations of curve II are not necessarily twice as great as those on curve I—they are simply greater. How much greater we do not know. This is no different from measuring how hot it is by a thermometer. We do not necessarily feel twice as warm when the thermometer reads 60° F. as when it reads 30°. We are simply warmer. (Advertising claims of a certain soap making clothes 9.2 percent brighter, 28.6 percent fluffier, or 1.67 percent better smelling may well be considered with some suspicion.)

One final comment should be made on the indifference lines in figure 7.2. These lines are only slightly curved—that is, they are almost straight lines. If they were straight lines, they would indicate that beef and pork were perfect substitutes for one another. They obviously are good substitutes—at least for many people. The small degree of curvature indicates, as we would expect, that the typical consumer does not consider them perfect substitutes. He will buy more pork and less beef if, and only if, pork becomes less expensive relative to beef. But the main point is that this analysis indicates that only small changes in price ratios are needed to induce rather substantial adjustments in consumption. Some mathematicians might wonder whether the relative flatness of the indifference lines in figure 7.2 might not be due to the particular mathematical equation that was used. The answer is that any mathematical equation that fits the data would give the same results—as anyone can see by studying the slopes of the actual price ratios in figure 7.2.

8. Long-Run Demand for Cotton

> For time is required to enable a rise in the price of a commodity to exert its full influence on consumption.—ALFRED MARSHALL in *Principles of Economics.*

Many statistical studies are designed to measure the short-run relationship between the consumption and price of a commodity. These include studies based upon first differences and upon deviations from trend. They also include the measurement of the net relationship between consumption and price, based upon a multiple regression including time as one of the independent variables. Other studies, including simple relationships between actual consumption and price over time, mix the short-run and long-run relationships in unknown proportions.

As indicated in the above quotation, Marshall noted the distinction between short-run and long-run demand. Mighell and Allen wrote an excellent paper on the subject.[1] They pointed out:

> We have developed neither the theory nor the methodology for estimating what quantity of any product will presently be taken by the consumers if the price has definitely fallen to a level 10 percent lower relative to other prices and consumers have reason to believe it is there to stay.

Elmer Working's study in 1954 was one of the first attempts to make statistical measurements of long-run and short-run demand.[2] A study by Nerlove in 1958 proposed a different statistical method of measuring long-run demand, based upon a particular form of distributed lags.[3] A paper by Tomek and Cochrane in 1962 discussed the concept of long-run demand, and used the method outlined by Nerlove to estimate both the short-run and long-run demand for meats.[4]

The statistical results obtained by Tomek and Cochrane differ from those of Elmer Working. But they both indicate that the demand for meat is more elastic (or less inelastic) in the long run than in the short run. There is a need for more theoretical work—and especially for more statistical measurement—in this area. Almost all the statistical studies have attempted to estimate short-run demand functions. For practical purposes, we need good estimates of long-run demand functions. This is especially true when we are considering the probable long-run effects of any farm program—for example, programs to support prices, restrict output, or divert surpluses from normal channels of trade.

A Case in Point—Cotton

We need more studies of long-run demand for many farm products. Here we shall consider cotton as an example. Cotton was chosen partly because there has been some controversy about its elasticity of demand in the long run.

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[1] Mighell, R. L. and Allen, R. H. DEMAND SCHEDULES—'NORMAL' AND 'INSTANTANEOUS'. Jour. Farm Econ., XXI—3, p. 555, August 1939.

[2] Working, Elmer J. DEMAND FOR MEAT. Univ. Chicago Press, Chicago, 1954.

[3] Nerlove, Marc. DISTRIBUTED LAGS AND DEMAND ANALYSIS FOR AGRICULTURAL AND OTHER COMMODITIES. U.S. Dept. Agr. Handb. 141, June 1958.

[4] Tomek, William G. and Cochrane, Willard W. LONG-RUN DEMAND: A CONCEPT, AND ELASTICITY ESTIMATES FOR MEATS. Jour. Farm Econ., Vol. XLIV, August 1962.

Some think American cotton is losing the domestic market to rayon and other manmade fibers because of high cotton prices and reduced prices of manmade fibers. This view has been endorsed by the National Cotton Council of America and has been supported by statistical studies of Horne and McCord.[5] Yet, most of our standard analyses indicate that the short-run domestic demand for American cotton is highly inelastic. An elasticity of −0.3 is commonly used, and is supported by a study of Lowenstein.[6] A recent study [7] found a still more inelastic demand of −0.14, when adjusted to hold constant the consumption of noncellulosic fibers. An elasticity of −0.3 would mean, roughly, that a 10 percent increase in the price of cotton would reduce domestic consumption by only 3 percent. This would seem to be a profitable deal for the cotton farmer. In fact, it might seem to his advantage to set the price as high as possible.

But the three studies mentioned are not in conflict with each other. All of them recognize two main facts: (1) the short-run domestic demand for American cotton is very inelastic; but (2) the long-run domestic demand is much less inelastic—and perhaps elastic. This is because mills will gradually shift from cotton if the competing fibers have a continued price advantage over several years. Also, the final consumer will gradually shift from cotton clothing to clothing made from substitutes if the price ratios encourage the shift.

Thus, it is quite possible that the short-run domestic demand for American cotton is highly inelastic, while the long-run demand is elastic. But none of the statistical studies has yet measured the long-run elasticity. This is a key datum needed in analyzing agricultural policy. I do not claim to have anything like a final answer, but this chapter may have some bearing on a practical question of economics and politics. In any case, it explores a method which is somewhat similar to Elmer Working's, but which uses a "distributed lag" somewhat similar to those developed by Irving Fisher [8] and by Marc Nerlove.[9]

The Data and an Estimating Equation

A rise in the price of cotton has only a small direct, immediate effect upon cotton consumption. But indirectly, and over a period of years, it increases the production and consumption of rayon and noncellulosic fibers—which, in turn, affect the consumption of cotton.

The following analysis is based upon two ratios: (1) the mill consumption of cotton divided by the mill consumption of rayon and acetate, and (2) the price of Strict Middling 1$\frac{1}{16}$-inch cotton divided by the price of rayon staple. The data are shown in table 8.1. My colleague, James R. Donald, helped me get appropriate data and advised me on the analysis in this chapter.

The price and consumption ratios are shown graphically in figure 8.1. Since 1933, there has been a striking increase in the ratio of cotton prices to rayon prices. There has also been a sharp decrease

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[5] Horne, M. K., Jr. and McCord, F. A. PRICE AND TODAY'S MARKETS FOR U.S. COTTON. National Cotton Council of America, Memphis, Sept. 1962.

[6] Lowenstein, Frank. FACTORS AFFECTING THE DOMESTIC MILL CONSUMPTION OF COTTON. U.S. Dept. Agr. Agr. Econ. Res., IV–2, p. 50, April 1952.

[7] Donald, J. R., Lowenstein, F. and Simon, M. S. THE DEMAND FOR TEXTILE FIBERS IN THE UNITED STATES. U.S. Dept. Agr. Tech. Bul. 1301, Nov. 1963.

[8] Fisher, Irving. OUR UNSTABLE DOLLAR AND THE SO-CALLED BUSINESS CYCLE. Jour. Statis. Assoc. 20. 1925.

[9] See footnote 3, page 57.

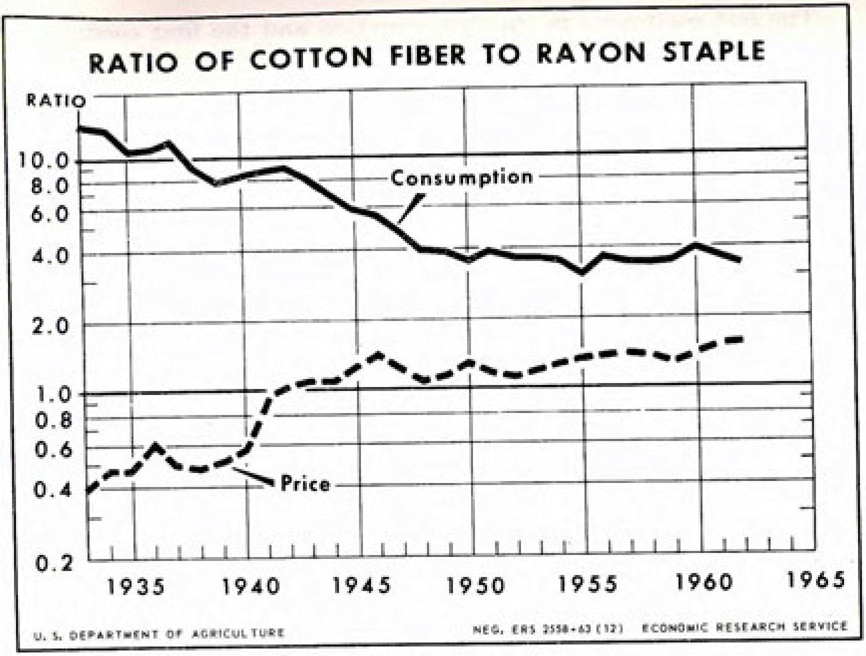


Figure 8.1

in the ratio of cotton consumption to rayon consumption. But, neither the rise in the price ratio nor the drop in the consumption ratio has been entirely regular. There have been many ups and downs, especially in the price ratio. A close study of the two lines indicates that changes in the price ratio do not have a large immediate effect upon the consumption ratio—rather, there is a lag. Moreover, the lag does not appear to be for a definite period—such as 3 years or 5 years, for example. Rather, it appears to be spread out over several years. In other words, the consumption ratio seems to respond not to the price ratio in any one year, but to the price ratios over several past years.

To investigate this further, I used the 3-year averages shown in table 8.1. The following two alternative estimating equations are based upon these 3-year averages. The difference between these two equations is simply in the assumed lags. Equation (8.1) uses price ratios centered 3 years, 6 years, and 9 years previous to the current year, t. Equation (8.2) uses the ratios centered on the current year, 3 years before, and 6 years before.

$$(8.1)\quad Q_t = 11.70 - \underset{(0.70)}{4.28}\, P_{t-3} - \underset{(0.77)}{2.08}\, P_{t-6} - \underset{(0.52)}{0.23}\, P_{t-9},\ (R_2 = 0.95)$$

and

$$(8.2)\quad Q_t = 11.32 + \underset{(0.63)}{0.73}\, P_t - \underset{(0.69)}{4.79}\, P_{t-3} - \underset{(0.48)}{2.21}\, P_{t-6},\ (R_2 = 0.97),$$

where P_t is the current 3-year average price ratio.

Q_t is the current 3-year average consumption ratio, and

P_{t-3}, P_{t-6}, P_{t-9}, are price ratios centered 3, 6, and 9 years before the current year.

The last coefficient in the first equation and the first coefficient in the second equation are statistically nonsignificant. They indicate only that the true coefficients are probably close to zero.

TABLE 8.1.—*Consumption and price ratios: cotton and rayon*

Year	Consumption ratios [1]		Price ratios [2]		Year	Consumption ratios [1]		Price ratios [2]	
	Annual	3-year averages	Annual	3-year averages		Annual	3-year averages	Annual	3-year averages
1933	14. 06	14. 47	0. 390	0. 370	1948	3. 89	4. 16	1. 064	1. 138
1934	13. 50	12. 73	. 476	. 445	1949	3. 86	3. 74	1. 135	1. 169
1935	10. 64	11. 64	. 468	. 521	1950	3. 47	3. 72	1. 309	1. 206
1936	10. 74	11. 12	. 619	. 525	1951	3. 82	3. 66	1. 174	1. 203
1937	11. 96	10. 54	. 489	. 528	1952	3. 68	3. 72	1. 125	1. 164
1938	8. 87	9. 58	. 478	. 493	1953	3. 65	3. 63	1. 194	1. 190
1939	7. 91	8. 33	. 512	. 521	1954	3. 57	3. 47	1. 251	1. 261
1940	8. 21	8. 30	. 575	. 675	1955	3. 09	3. 43	1. 337	1. 316
1941	8. 77	8. 68	. 937	. 847	1956	3. 63	3. 39	1. 359	1. 364
1942	9. 07	8. 63	1. 028	1. 018	1957	3. 45	3. 50	1. 397	1. 373
1943	8. 04	7. 97	1. 091	1. 068	1958	3. 43	3. 45	1. 362	1. 335
1944	6. 79	6. 90	1. 085	1. 139	1959	3. 46	3. 62	1. 246	1. 332
1945	5. 86	6. 05	1. 242	1. 241	1960	3. 97	3. 68	1. 387	1. 382
1946	5. 49	5. 39	1. 396	1. 284	1961	3. 62	3. 63	1. 514	1. 477
1947	4. 72	4. 70	1. 215	1. 225	1962	3. 31	------	1. 529	------

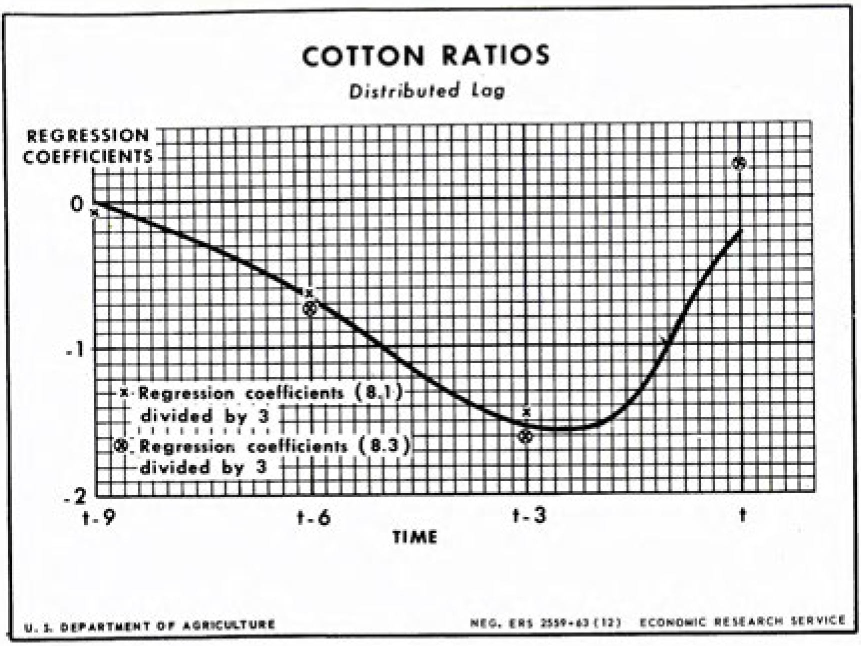
[1] Mill consumption of cotton and of rayon and acetate.
[2] Price of SM $1\frac{1}{16}$ inch cotton divided by price of rayon staple.

All ratios are computed from data in STATISTICS FOR COTTON. U.S. Dept. Agr. Statis. Bul. 329. Table 13, p. 12; table 232, p. 208, 1963.

Distributing the Effects Over Time

While either equation (8.1) or (8.2) gives a very high squared correlation, the correct equation doubtless would distribute the effects more evenly over a period of years, rather than staying at one level for 3 years and then jumping abruptly to another. Such a distributed effect can be visualized in figure 8.2. First the regression coefficients in equations (8.1) and (8.2) were each divided by 3 to put them on an annual basis. Then they were plotted on the diagram, and a smooth curve was drawn through them, except that at the extreme right of the curve, I disregarded the nonsignificant positive coefficient. It seems unreasonable to believe that the immediate effect of a rise in the price ratio would be a rise in the consumption ratio. I have assumed, in drawing the curve, that the immediate effect is small, but negative.

The table shown in the lower part of figure 8.2 shows the meaning of the curve. The first column is simply the values of the curve, reading backwards; that is, from right to left. For example, at time t (the current year) the price ratio would be weighted −0.25; for year t–1 the weight would be −1.00; and so on. The second column gives cumulative weights; for example, for year t–1 the cumulative weight is −0.25−100=−1.25; and so on. By the year t–8, the



Basic Data

Time	Regression coefficient (8.1) divided by 3	Regression coefficient (8.3) divided by 3	Weights	Cumulative weights	Cumulative elasticities of the price ratio of 1.17 and consumption ratio of 5.05	Weights as a percentage
t		+0.24	-0.25	-0.25	-0.06	3.2
t-1			-1.00	-1.25	- .29	12.6
t-2			-1.55	-2.80	- .65	19.5
t-3	-1.43	-1.60	-1.55	-4.35	-1.00	19.6
t-4			-1.32	-5.67	-1.31	16.6
t-5			-1.00	-6.67	-1.54	12.6
t-6	- .69	- .74	- .67	-7.34	-1.70	8.4
t-7			- .40	-7.74	-1.79	5.0
t-8			- .20	-7.94	-1.84	2.5
t-9	- .08		0	-7.94	-1.84	0

Figure 8.2

cumulative weight has risen to −7.94. This apparently measures the full long-run effect of the price ratio upon the consumption ratio.

What does this imply in terms of elasticity? The mean price ratio was 1.17 and the mean consumption ratio was 5.05. So the long-run elasticity *of the consumption ratio* with respect to *the price ratio* was

$$E_{qp} = -7.94\,\frac{(1.17)}{5.05} = -1.84. \tag{8.3}$$

This elasticity can be distributed among the 9 years. Simply multiply each cumulative weight in column 2 by 1.17/5.05. This gives column 3 which indicates an immediate elasticity of −0.06, a cumulative elasticity after 1 year of −0.29, and a final cumulative elasticity of −1.84.

Of course, these elasticities are based upon quantity *ratios* and price *ratios*. They are not conceptually the same as elasticities based upon actual quantities and actual prices. They are somewhat similar to elasticities based upon "deflated" quantities and prices. They may help bridge the gap between short-run and long-run concepts of demand. The commonly accepted short-run elasticity of −0.3 is based upon an analysis in which consumption was lagged 6 months after prices. Figure 8.2 indicates an elasticity of −0.29 after 1 year. It also strongly confirms the idea that the long-run domestic demand for cotton is elastic. If the price ratio were increased 10 percent, the immediate effect upon the consumption ratio would be insignificant. But if the price ratio were raised 10 percent *and held at the higher level for 9 or 10 years*, the consumption ratio would apparently drop by 0.6 percent immediately, by 2.9 percent in 1 year, by 6.5 percent in 2 years, and so on, until it reached a level about 18 percent below where it was originally.

The final column in the table in the lower part of figure 8.2 gives percentage weights for each year, obtained by dividing each weight in column 1 by −7.94. These percentage weights would be appropriate for computing a weighted moving average of the price ratios. Such a moving average could, for example, be plotted in figure 8.1 to smooth the irregular bumps and dips in the year-to-year data.

The "long-run demand elasticity" used here reflects changes in the output of competing fibers and also technological improvements in the qualities of both cotton and other fibers. It is not the only possible concept of long-run demand elasticity, but it is useful for some purposes.

The method used here to distribute effects over time is more like the method used by Irving Fisher than the one used by Marc Nerlove. Nerlove assumed a particular mathematical function, similar to a "decay curve" in physics. Like Fisher, I have not assumed any particular distribution function. Rather, I have tried to find one that seems to fit the observed data.

An Estimate for 1963

If the annual price ratios are weighted by the numbers shown in the fourth column of the table at the bottom of figure 8.2, the weighted sum for 1962 is −10.867. When this publication was written, the actual average ratio between cotton and rayon prices for the 1963–64 crop year was not known. But the average ratio in the first 5 months was 1.41, and I assumed that ratio for the whole crop year. The weighted sum of price ratios for 1963 was −11.132. The difference between these two sums is −0.265, or, say, −0.26, indicating a drop of 0.26 point in the cotton-rayon consumption ratio for 1963. This would mean a 1963 consumption ratio of 3.05, compared with 3.31 for 1962.

When the above statement was written, the 1963 consumption ratio was not available, so this was really a conditional forecast. Since then, the consumption ratio for 1963 has been estimated at 2.81.

9. Distribution Among Markets: Some Graphs

It often happens that a monopolist finds it possible and profitable to sell a single commodity at different prices to different buyers.—Joan Robinson in *The Economics of Imperfect Competition*

One of the most common problems in agricultural marketing is how to distribute a given quantity of some commodity among several markets. In the case of perfect competition, the invisible hand of Adam Smith solves this problem by itself. But when there are any departures from competition, due to organized selling by farmers or dealers, or due to governmental programs, the distribution of a crop among markets becomes a matter of deliberate choice. And the distribution among markets often may make a substantial difference in returns to farmers (or to others), especially when it is possible to sell at different prices in the different markets—thus taking advantage of differences in the elasticities of demand.

In what follows, I assume that the quantity to be distributed is fixed in the short run. This is a very common situation in agriculture, especially in the case of perishables, where the quantity to be distributed in the short run is determined by production.

Place, Time, Form, and Person

Markets are commonly separated in at least four ways: by place, by time, by form, and by person. Sometimes, they are separated by combinations of these. For example, some of the wheat is sold in the domestic market, and some in the foreign market. Some of it is sold this year, and some held over until next year. Some is sold as wheat, and some is made into flour and other products.

One typical problem is the distribution by place—that is, to different cities, regions, or countries. Suppose there is a given supply of wheat in the United States. Some of it will be sold in the domestic market, some for export. These may be considered as two markets for wheat. By some means or other, the existing supply must be allocated between these two markets. Import duties, import quotas, and many other devices are commonly used to keep the export market more or less separate from the domestic market.

Another common problem is distribution through time. Part of the late potato crop is sold in the fall, part in the winter, and part in the spring. These markets are naturally separated, although not necessarily independent of one another. But time moves only in one direction. Supplies can be carried forward in time, but not backward. Thus, distribution through time differs from distribution through space.

Many farm commodities are sold in various forms. For example, milk is sold as fluid milk, cream, butter, cheese, and other products. Each of these can be considered a market for milk. The existing milk supply must be divided in some way among the different forms. These, again, are naturally separate markets. Once milk has been made into butter or cheese, it is not likely to be transferred to some other product. (A minor exception is the transfer of butter to butter oil.)

Still another problem is the distribution to different groups of persons, such as income groups. This includes such problems as whether to have public food programs, such as school lunches and food distribution to needy families—and if so, what kinds and sizes of programs. This problem has welfare aspects and also farm income aspects. These programs have safeguards to prevent resale of the foods distributed, thus keeping the markets separated.

Even a uniform commodity (that is, one of the same grade and quality) can be distributed by place, by time, by form, and by person. In addition, there are many other interesting and important problems in agricultural marketing which involve larger or smaller differences in quality. One of these is the distribution by grade. A similar problem is the determination of amounts to be sold under different brands, packages, etc. Chamberlin presented an illuminating analysis of "product differentiation", the common practice of making small differences in the quality of goods in order to sell them at different prices.[1]

Two Kinds of Distribution

In principle, there are two main ways to distribute a given quantity among several markets. First, the quantity may be distributed so as to get equal net prices to the producer from all markets. (Here, net price means price minus all costs of processing and distribution.) Second, the distribution may be aimed at making marginal net returns the same from all markets. Here, marginal net return from any market means roughly the change in net returns to the producer if he sells one more unit in that market (including any indirect effects, such as the effect of greater shipments to market A upon the price in market B).

Perfect competition would result in equal net prices from all markets. For if the net price in market A were higher than in market B, for example, some of the shipments to market B would be diverted to market A. Under perfect competition, such diversions would continue until the net prices in the two markets were equal. In the case of geographic markets, such a distribution has come to be called "spatial equilibrium" in many recent papers.[2] In some old textbooks on marketing, such a distribution, whether by place or by time, was commonly called "orderly marketing." It was sometimes assumed that this distribution gave the producer the greatest income. It is

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[1] Chamberlin, Edward. THE THEORY OF IMPERFECT COMPETITION. Harvard Univ. Press, Cambridge, 1933.

[2] Baumol, W. J. SPATIAL EQUILIBRIUM WITH SUPPLIES PREDETERMINED. Mimeographed. U.S. Dept. Agr. Bur. Agr. Econ., 1952.

Beckman, M. A CONTINUOUS MODEL OF TRANSPORTATION. Econometrica, Vol. 20, pp. 643–60, 1952.

Enke, S. EQUILIBRIUM AMONG SPATIALLY SEPARATED MARKETS: SOLUTION BY ELECTRIC ANALOGUE. Econometrica, Vol. 19, pp. 40–48, 1951.

Fox, K. A. A SPATIAL EQUILIBRIUM MODEL OF THE LIVESTOCK FEED ECONOMY. Econometrica, Vol. 21, pp. 547–66, 1952.

Henry, W. R. and Bishop, C. E. NORTH CAROLINA BROILERS IN INTERREGIONAL COMPETITION. Agr. Econ. Inf. Series 56, North Carolina State College, 1957.

Judge, G. G. A SPATIAL EQUILIBRIUM MODEL FOR EGGS. Bul. 318, Conn. Agr. Expt. Sta., Storrs, 1956.

Koopmans, T. C. OPTIMUM UTILIZATION OF THE TRANSPORT SYSTEM. Econometrica, Vol. 17, pp. 136–46, Supplement, July 1949.

Mosak, J. L. GENERAL EQUILIBRIUM THEORY IN INTERNATIONAL TRADE. Cowles Commission Monograph No. 7, Bloomington, Ind., 1949.

true, of course, that any small, individual producer could maximize his net income by selling in whatever market offered the highest net price. But for an industry as a whole, the purely competitive distribution would seldom, if ever, maximize the net returns (that is, total returns minus marketing costs) of producers. Chapter 10 will discuss two statistical examples and will compare distributions under competition with those under discriminating monopoly.

To get the highest possible net income to producers, distribution would have to be such as to equalize marginal net returns from the different markets. The reason for this is simple. If the marginal returns from market A are higher than those in market B, the returns from the two markets together will be increased by diverting a small amount from market B to market A. As long as the marginal net returns are unequal in any pair of markets, such diversions would increase net returns from all markets together. Thus, a necessary condition for maximum income from all markets is that marginal net returns be the same in each market.

Except in extremely unlikely cases, this process of equalizing marginal net returns would unequalize net prices. It would result in selling the commodity at different net prices in different markets. This is what economists call price discrimination. They even sometimes call it monopolistic price discrimination, since it would be impossible to charge different net prices if there were perfect competition. These terms have a bad connotation to many people.

This is not the place to discuss in any detail the social consequences of price discrimination. Yet, it might be well to note that not all forms of price discrimination are necessarily bad. Price discrimination ordinarily benefits certain groups and harms others. Whether a particular form of discrimination is socially desirable or undesirable depends upon who gets the benefit and who is harmed and how much. Many forms of discrimination in favor of needy people have been generally accepted. A much broader defense of price discrimination was made by Dupuit.[3] Robinson presents a discussion of the social aspects of discrimination.[4]

Charts

Figure 9.1 presents two diagrams that are useful in analyzing a wide variety of distribution problems in markets independent of one another. When markets are independent, the quantity consumed in a market depends upon the price in that market and is not influenced by prices in other markets. Most of the modern discussions of price discrimination have been limited to cases of independent markets. For example, Robinson made it clear when she presented her analysis that, "In the following argument we shall only consider cases in which the demand curve in each separate market is independent of prices charged in the other markets."

Diagram A at the top of figure 9.1 shows assumed demand curves in three markets. They are labeled I, II, and III. The prices are net to the producer. For example, the demand for any given quantity in market II may actually be the same as that in market I in terms of

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[3] Dupuit, Jules. DE L'UTILITÉ ET DE SA MESURE. (A collection of Dupuit's writings.) La Riforma Sociale, Torino, Italy, 1933.

[4] ROBINSON, JOAN. THE ECONOMICS OF IMPERFECT COMPETITION. Macmillan, London, 1938.

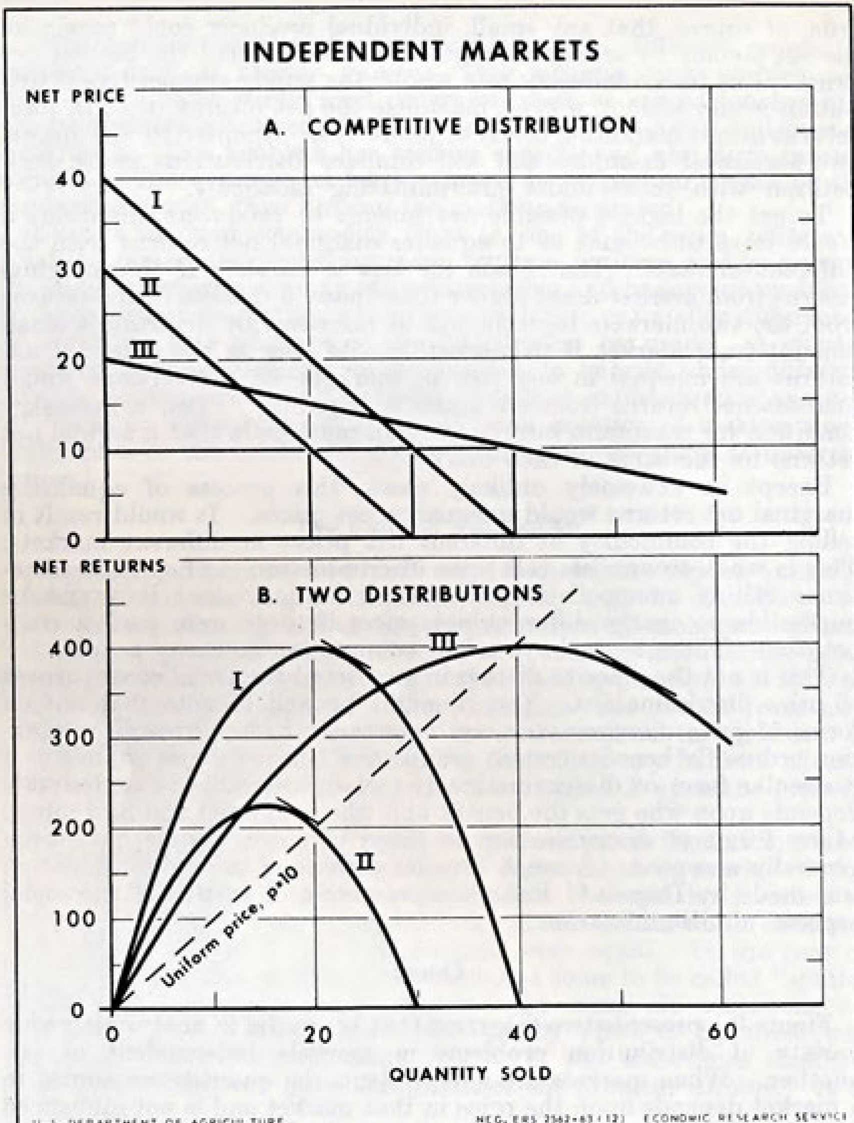

Figure 9.1

delivered prices, but there may be a difference of $10 in the net prices to producers, due to a difference in the freight rate. To make the diagram easier to read, I have assumed only three markets and linear demand functions. But a similar diagram could be drawn for any number of markets and for demand curves of any shape.

Suppose the producer has 90 units of the commodity to distribute among these three markets. If he sets a uniform price of 20, he would sell 20 units in market I, 10 units in market II, and nothing in market III. This obviously would not work because he could sell a total of only 30 units. He would have to lower his price until he could dispose of 90 units. To do this at a uniform price, he would have to lower the price to 10. Then he would sell 30 units in market I, 20 in market II, and 40 in market III. His net returns from the three markets together would be 900.

Part B at the bottom of figure 9.1 presents the same data in a different form. Instead of showing the net price associated with each quantity, part B shows net returns (that is, quantity times net price) associated with each quantity. Economists have become accustomed to working with demand curves. Yet, there are many advantages to working with returns curves. And whenever the demand curve is given, it is a simple matter to complete the corresponding returns curve.

When dealing with returns curves such as those in part B, a uniform price would be indicated by a straight line, such as the dashed line shown on the diagram. The dashed line indicates a uniform price of 10; that is, it shows that 10 units would give a return of 100; 20 units would give a return of 200, etc. At a different uniform price we would have a different dashed line. For example, if the price were 5 instead of 10, the line would be one-half as steep as the line shown. The price is indicated by the angle between the dashed line and the x-axis.

Reading from the dashed line, you can derive the same results as were obtained in part A; that is, a uniform price of 10 would dispose of 30 units in market I, 20 in market II, and 40 in market III. The total returns from the three markets would be 900, as before.

The marginal net return at any point on one of these curves is simply the slope of the tangent to the curve at that point. Assume a uniform price of 10 and note the slope of the returns curves at the point where the dashed line crosses them. The slope of the returns curve in market I is sharply downward; that in market II is slightly downward; and that in market III is level. (That is, the slope is zero.) This indicates that returns from three markets together would be increased by shifting to market III a part of the 30 units going to market I, and part of the 20 units going to market II. We look for points on three curves where the slopes are equal and where we can dispose of the total of 90 units. These points indicate a distribution of 22½ units to market I, 17½ to II, and 50 to III. The net return from the three markets together is 987½ under this distribution. Remember that it was 900 under the competitive distribution.

In this case, the marginal net return in each of the markets is −5. A monopoly that could adjust both distribution and output would make a gain by reducing total shipments somewhat. It would sell 20 units in I, 15 in II, and 40 in III, making total marketings of 75 units and increasing the net returns to 1,025. But ordinarily, agriculture supplies more than the most profitable amount of most farm products. This means that the farmer usually produces to the point where marginal net returns are negative. And once having produced a supply, it all generally gets marketed one way or another.

A minor point that might be noted incidentally is that the distribution that maximizes net returns results in a price differential of 5 between markets I and II. The difference in the level of the two demand curves is 10, and we assume that the 10 is equal to the freight charges. The analysis indicates that, with linear demand functions such as we have shown, it would pay the producer to "absorb" half the freight charges. Actually, he would raise the price in nearby

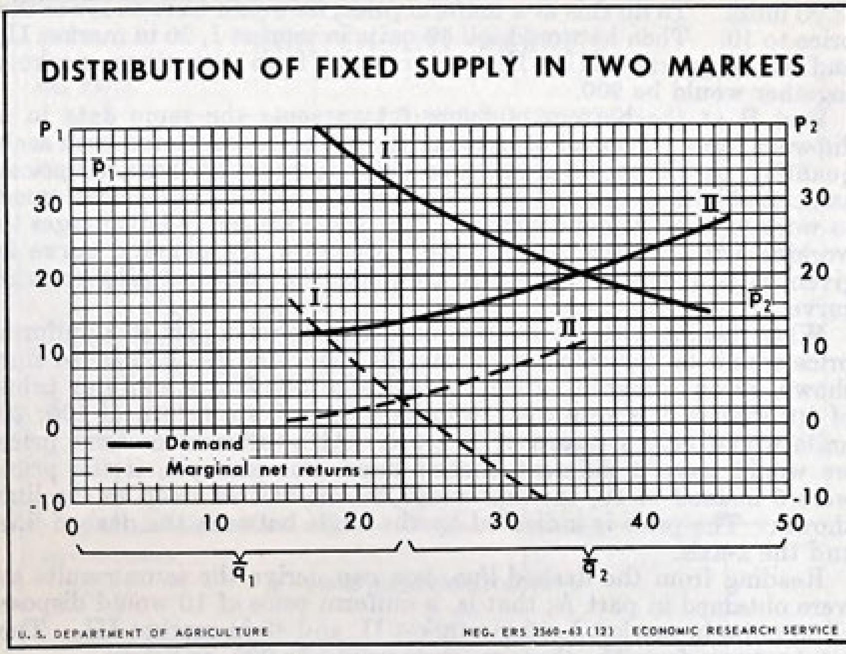

Figure 9.2

markets and lower it in distant markets, compared to purely competitive pricing.

Figure 9.2 is useful in all cases that involve only two markets. Assume that a given quantity is to be divided between two markets. In the case of this diagram, the quantity to be divided is 50 units. The heavy curve marked I is the demand curve of the first market, again expressed in terms of net prices to the producer. The heavy curve marked II is the demand curve in market II, but note that the quantities have been reversed. For example, if q_1 is sold in market I, the amount sold in market II will be $50-q_1$. So q_2 can be read from right to left.

Under perfect competition, the distribution will be such that the prices are the same in the two markets. As the curves are drawn, the uniform price will be 20. The net returns from the two markets together will be 1,000.

The dashed lines marked I and II show the corresponding curves of marginal net returns from the two markets. Such net returns curves can be obtained graphically from the demand curves by using some fairly simple geometry explained in Robinson's book. Note that at the competitive equilibrium where 35 units are sold in market I and 15 units in market II, the net returns are very unequal. In fact, the net returns from market I are negative, and the net returns from market II are positive. They become equal with the distribution of 23 units to market I and 27 to market II. With that distribution, the net returns from the two markets together are increased to 1,104.

So far, we have assumed that the two heavy lines are ordinary demand curves, except that the curve in market II is reversed. This is all right if the two markets are independent of one another. In case of interdependent markets, we could still draw a diagram like that in figure 9.2. However, curve I would indicate, for example, the various prices in market I corresponding to such combinations as 20 units to market I and 30 to market II, 25 to market I and 25 to market II, and all other combinations adding to 50.

The most general kind of diagram for the case of two markets is that shown in figure 9.3. This diagram is in terms of isoquants, which are similar to contour lines on a topographic map. In the case shown in figure 9.3, each of the curved lines is supposed to connect combinations of q_1 and q_2 that would result in equal net returns.

If this were a topographic map, it would represent a hill. The top of the hill would be at about $q_1=32$ and $q_2=26$. These would be the two quantities which would give the absolute maximum net income to the producer. But suppose he had 70 units to dispose of. Then he would have to choose some point along the straight line indicated on the diagram. The highest point he could reach would be where this straight line is tangent to one of the curves. In this case it is at 300. The diagram indicates that in this imaginary case, the producer could

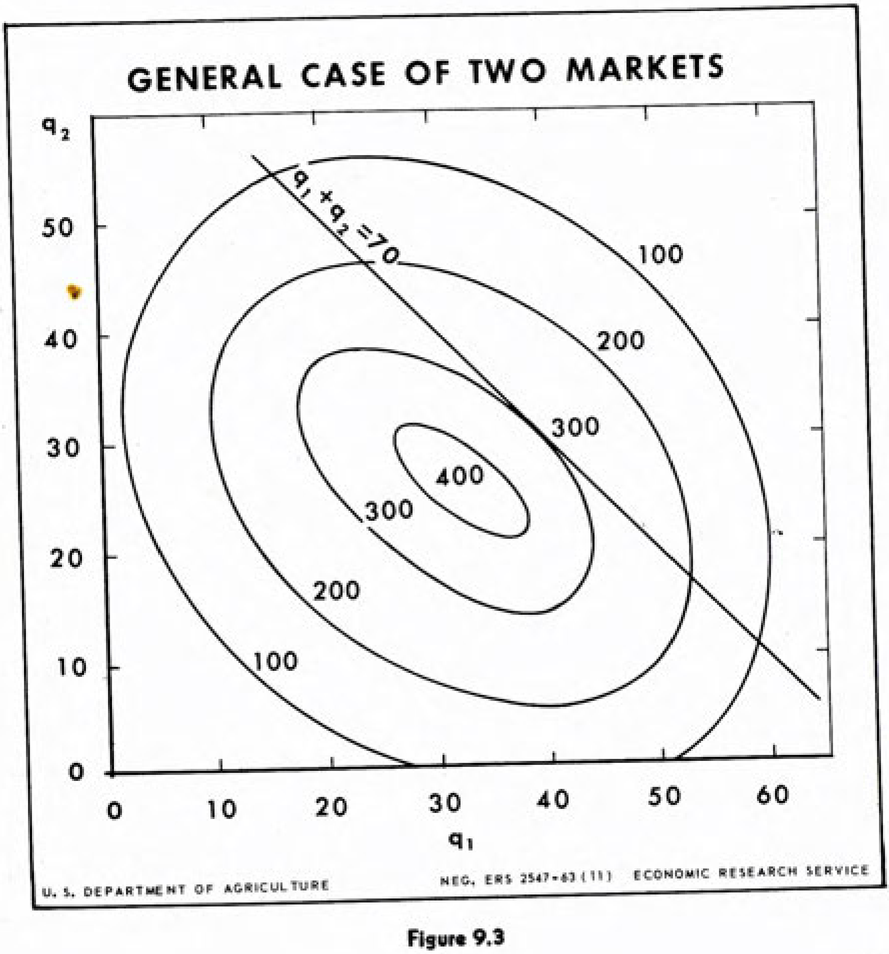

Figure 9.3

get an income of 300 by selling 40 units in the first market and 30 units in the second.

A diagram like figure 9.3 can always be drawn if the basic demand functions are known for each of the two markets. Whether the two markets are independent or interdependent, one can compute the price in each of the markets that would result from any combination of shipments. Then, multiplying the prices by the quantities and adding, he can get the net returns from each combination. These returns can be plotted on the diagram and the proper isoquants can be drawn.

Cases of More Than Two Markets

Distribution among any number of independent markets can be analyzed by diagrams like figure 9.1. When the markets are not independent, however, there is no practicable graphic analysis for cases of more than two markets. The mathematical principles involved in maximizing net returns from a number of markets have been explained by Simkin.[5] A somewhat different mathematical statement of this is in Appendix 6.

10. Distribution Among Markets: Two Examples

> The question of distributing the lemon crop in a given year, among the fresh and processed outlets . . . is of great significance to the industry.—Hoos and Seltzer in *Lemons and Lemon Products, Changing Economic Relationships.*

The theoretical analysis reviewed in Chapter 9 could be applied to a wide variety of practical problems of intermarket distribution. For this purpose we need a set of reliable demand functions.

Unfortunately, most demand analyses deal only with national aggregates. Such studies give a single demand function, showing the prices in a given market corresponding to a range of quantities sold in the market. To analyze distribution among markets, we need a breakdown of total demand to provide a demand function in each market.

But some statistical studies in recent years have analyzed the demand for a commodity in different markets. Two good examples are a study by Meinken[1] and a study by Hoos and Seltzer.[2] In this chapter, I shall use simplified versions of the demand functions found in these two studies, and shall discuss their implications for distribution. The same sort of analysis could be applied to a wide variety of other marketing problems. We need much more quantitative work on demand functions in different markets—by place, by time, by form, and by person.

Distribution of Wheat in 1953

Meinken's wheat study used limited information methods to derive six simultaneous equations for U.S. wheat. Here, we shall be concerned with three of Meinken's equations dealing with: (1) the demand

[5] Simkin, C. G. F. SOME ASPECTS AND GENERALIZATIONS OF THE THEORY OF DISCRIMINATION. Review Econ. Studies, Vol. 1, No. 37, p. 1, 1947–48.

[1] Meinken, Kenneth W. THE DEMAND AND PRICE STRUCTURE FOR WHEAT. U.S. Dept. Agr. Tech. Bul. 1136, 1955.

[2] Hoos, Sidney, and Seltzer, R. E. LEMONS AND LEMON PRODUCTS, CHANGING ECONOMIC RELATIONSHIPS, 1951-52. Bul. 729, Calif. Agr. Expt. Sta. Berkeley, p. 6, 1952.

for wheat used for domestic food, (2) the demand for wheat used for domestic feed for animals, and (3) the demand for wheat for exports. Letting q_1, q_2, and q_3 represent quantities (million bushels) demanded for domestic food, for domestic feed, and for exports, and letting p_1, p_2, p_3 be prices in dollars per bushel, the three equations are as follows:

$$
\begin{aligned}
& q_1 = 531 - 24\ p_1 \\
(10.1) \qquad & q_2 = 631 - 250\ p_2 \\
& q_3 = 2{,}007 - 780\ p_3.
\end{aligned}
$$

My colleague, Forrest Walters, helped derive these equations from 1953 data in Meinken's study (pages 47 and 84, his study). We adjusted the constants in each equation so that, with a uniform price of \$2.30, the estimated quantities were the actual quantities consumed in 1953. These quantities were 476, 56, and 213 million bushels.

The equations (10.1) assume that the three markets are independent of one another; that is, they assume that the quantity consumed in a given market depends only upon the price in that market. It is not affected by prices in any other market. The domestic and export markets are practically independent of one another, at least over a wide range of prices. This is because of various governmental actions on such things as import quotas and import duties. The market for wheat used as animal feed is, of course, affected by prices of corn and other feed grains.

The feed equation in (10.1) assumes a given price of corn. But also, one might well question whether the demand for wheat feed is independent of prices for wheat used as food. At present, the two markets are doubtless not independent of one another, but they might conceivably be separated for practical purposes by such means as coloring the wheat used as animal feed. For the analysis, we shall assume that the equations (10.1) are accurate, and that the markets are independent of one another. This enables us to make a simple analysis.

First, we transpose equations (10.1) to estimate the prices associated with various quantities. This can be done legitimately because equations (10.1) are estimates of the true structural equations. When these equations are reversed, we get the following equations for estimating prices.

$$
\begin{aligned}
& \hat{p}_1 = 22.13 - 0.04167\ q_1 \\
(10.2) \qquad & \hat{p}_2 = 2.52 - 0.00400\ q_2 \\
& \hat{p}_3 = 2.57 - 0.00127\ q_3.
\end{aligned}
$$

Multiplying prices by quantities, we get the following equations for net returns from each of the three markets.

$$
\begin{aligned}
& \hat{r}_1 = 22.13\ q_1 - 0.04167\ q_1^2 \\
(10.3) \qquad & \hat{r}_2 = 2.52\ q_2 - 0.00400\ q_2^2 \\
& \hat{r}_3 = 2.57\ q_3. - 0.00127\ q_3^2
\end{aligned}
$$

Differentiating each of these equations with respect to quantities marketed, we get the following equations for marginal net returns:

$$\hat{r}_{11}=22.13-0.08334\ q_1$$
$$(10.4)\qquad \hat{r}_{22}=2.52-0.00800\ q_2$$
$$\hat{r}_{33}=2.57-0.00256\ q_3,$$

where $\hat{r}_{ij}$ means the derivative of returns in market i with respect to the quantities sold in market j. In the special case of independent markets, all $\hat{r}_{ij}$'s are zero except those shown, since the returns from any market are affected only by the quantity sold in that market.

Figure 10.1 shows a graphic analysis of this problem. The heavy curves show the returns for domestic food, for domestic feed, and for exports as functions of the quantities sold in these three markets. The heavy straight line shows the uniform price of $2.30 in Kansas City. This was the actual price in 1953. The actual cost of export wheat to foreigners was the U.S. price minus a subsidy of 37.5 cents plus shipping costs; but the wheat farmer got the same price, whether or not the wheat was exported. This is one of several current examples of farm programs that are essentially two-price arrangements to the

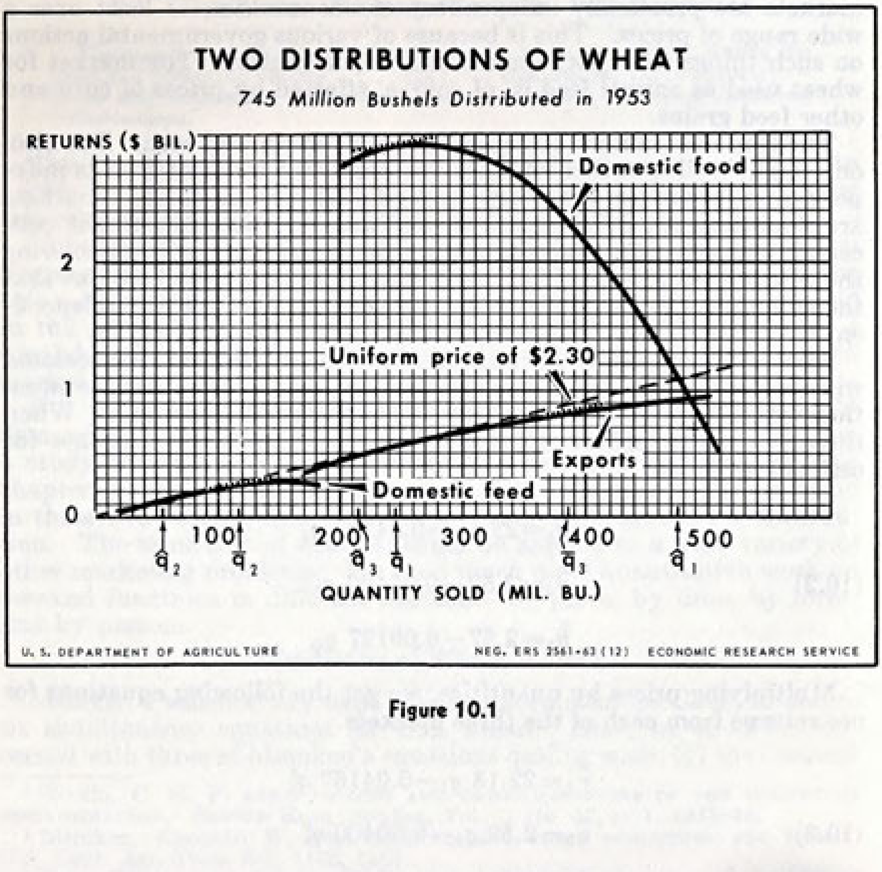

Figure 10.1

buyer but single-price deals to the farmer. In the following analysis, we assume that the subsidy would be the same under any distribution, and are concerned with the market value of wheat prices at Kansas City.

The distribution with a uniform price of $2.30 would be indicated by the intersection of the heavy straight line and the heavy curves. This would give the actual distribution of 476 million bushels for domestic food, 56 million bushels for domestic feed, and 213 million bushels for exports. (It has to come out this way because the constants were adjusted to make it do so.) Total sales in these three markets were 745 million bushels. Priced at $2.30 a bushel, the wheat was worth $1.72 billion.

But note that marginal returns at these three points are very unequal. The returns curve for domestic food slopes sharply downward at the point $q_1=476$ (i.e., marginal returns in that market are decidedly negative). The returns curves for domestic feed and for exports slope slightly upward at $q_2=56$, and at $q_3=213$ (i.e., marginal returns in those markets are slightly positive). This means that the market value of the wheat crop could be substantially increased by selling less of it for domestic food and more for domestic feed and for exports. A graphic solution would be to move to the left on the domestic food curve and to the right on the curves for domestic feed and for exports until three points were found where the tangents of the three curves were parallel to one another, and where the total quantity was 745 (which was the actual quantity distributed in 1953).

More precisely, we can do the same thing by the mathematics explained in Appendix 6. The analysis indicates that maximum returns would be obtained by the distribution shown in the second column of table 10.1.

However, this is a hypothetical maximum. It assumes that the linear demand equations are valid when extrapolated far beyond the observed values. Note, for example, that domestic food sales would be reduced from 476 million bushels to 246 million, and that the price would be increased from $2.30 to $11.88 a bushel. Even if the extrapolation would work (which is very doubtful), it would be politically impossible to consider such a drastic reduction in wheat for domestic food and such an enormous increase in prices.

If the Congress were to approve a three-price program for wheat, it would doubtless set an upper limit upon the price for wheat used for food in the domestic market. An administrator of such a program would also have to take account of many other practical problems. For example, if he set the price of feed wheat too low, there would be great opposition from corn farmers. Likewise, if he set the price of export wheat too low, there would be opposition from wheat farmers in other countries. The problem would be to find some distribution of 745 million bushels that would be politically feasible and that would increase the returns to wheat farmers.

Table 10.1 summarizes three distributions. The first column shows the actual distribution in 1953 which gave farmers an income of $1.72 billion. The second column shows the hypothetical maximum which theoretically would return $3.96 billion. The last column shows a distribution that might possibly be feasible. Feasibility in this case is a matter of judgment. Many wheat experts, and many political scientists, might want to make changes in the numbers in the last column. But it should serve as an illustration. The distribution

TABLE 10.1.—*Three distributions of U.S. wheat crop in 1953*

Use	Actual			Hypothetical maximum			Possibly feasible		
	Volume	Price per bu.	Farm income	Volume	Price per bu.	Farm income	Volume	Price per bu.	Farm income
	Mil. bu.	*Dol.*	*Bil. dol.*	*Mil. bu.*	*Dol.*	*Bil. dol.*	*Mil. bu.*	*Dol.*	*Bil. dol.*
Domestic food	476	@ 2.30 =	1.09	246	@11.88 =	2.92	435	@ 4.00 =	1.74
Domestic feed	56	@ 2.30 =	0.13	116	@ 2.06 =	0.24	70	@ 2.24 =	0.16
Exports	213	@ 2.30 =	0.50	383	@ 2.08 =	0.80	240	@ 2.27 =	0.54
Totals	745		1.72	745		3.96	745		2.44

shown in the last column would raise the price of wheat used for domestic food from $2.30 to $4.00 a bushel. This is, of course, a substantial increase in price. But it probably would not raise the price of bread very much, since the cost of wheat is a small fraction of the retail price of bread. This distribution would increase the quantities sold for domestic feed and for exports. It would lower the prices for those categories slightly below the actual prices that existed in 1953. The estimated return from this distribution is $2.44 billion. This is an increase of almost three-quarters of a billion dollars over the actual returns in 1953. In this particular example, returns would be increased from each of the three markets.

The estimates summarized in table 10.1 assume that the same quality and grade of wheat would be used in all three markets. Actually, the lower grades of wheat are used for feed. In 1953, for example, the wheat used for feed doubtless was priced at less than $2.30 a bushel. In actual practice, an administrator would take into account the normal discounts for the quality of wheat used in feed. But despite this, the rough estimates of increased returns from a three-price arrangement are probably accurate enough for practical purposes.

This is not the place to discuss in detail the mechanics of operating a three-price program. Conceivably, such a program might be carried out by a national cooperative that had control over the entire crop. Or some government agency, such as the Commodity Credit Corporation, might be in control of the crop. Or, possibly, it might be put under some kind of Federal marketing order. In any case, the quantity that any wheat grower could sell for domestic food would have to be limited. Various kinds of "certificate plans" have been suggested as a means of maintaining the necessary price differentials.

Fresh and Processed Lemons

When lemon crops have been large, a common practice in recent years has been to divert some of the surplus into various kinds of processed lemon products. Many of these lemon products return very low net prices to lemon producers (sometimes even negative returns). Still, some degree of diversion of this kind may well be profitable.

Hoos and Seltzer have studied the economics of this situation.[3] I shall use two demand functions that they developed. Some of the

[3] See footnote 2, page 70.

early studies overestimated the increased returns that could be obtained by diverting surplus lemons to processing use at lower net prices. This is because the researchers assumed that the market for fresh lemons was independent of the market for processed lemon products. Hoos and Seltzer properly questioned this assumption, since fresh and processed lemons are used for many of the same purposes. When Hoos and Seltzer estimated the demand function for fresh lemons, they measured not only how the price of fresh lemons was affected by marketings of fresh lemons, but also how it was affected by the quantity of lemons sold for processing. Similarly, when they measured the demand for processed lemons, they estimated not only how the price of processed lemons was affected by the quantity sold for processing, but also how it was affected by the quantity sold in the fresh market. Thus, Hoos and Seltzer were considering the general case for two related markets.

I took Hoos and Seltzer's price-quantity coefficients.[4] Then I computed constants which would adjust the equations to the prices and quantities existing in the season of 1934–35. The net prices to the lemon producer, and quantities in that year, were

(10.5) $p_1=\$1.71$ a box (on tree) $q_1=7.18$ million boxes

$p_2=\ 0.07$ a box (on tree) $q_2=3.55$ million boxes.

Using these numbers, and the price-quantity coefficients from the Hoos-Seltzer equations, I got estimating equations for 1934–35 as follows:

$$\begin{aligned} p_1&=5.638-0.465\,q_1-0.166\,q_2 \\ p_2&=0.897-0.043\,q_1-0.146\,q_2. \end{aligned} \tag{10.6}$$

From equations (10.6) it is simple to figure the net returns to the grower as a function of the two quantities. They are

$$R=\text{Returns}=5.638\,q_1-0.465\,q_1^2-0.209\,q_1q_2+0.897\,q_2-0.146\,q_2^2. \tag{10.7}$$

Differentiating R with respect to q_1 and with respect to q_2, we get

$$\begin{aligned} \text{Marginal returns: } R_1&=5.638-0.930\,q_1-0.209\,q_2 \\ R_2&=0.897-0.209\,q_1-0.292\,q_2. \end{aligned} \tag{10.8}$$

Inserting into (10.8) the quantities that were actually sold in each of the markets in 1934–35, the indicated marginal returns in the fresh market were -1.78, and the indicated marginal returns in the processed market were -1.64. The fact that both marginal returns were negative shows clearly that total sales were larger than the amount that would have brought the greatest income. This is commonly the case in agriculture.

But the fact that the marginal returns from the two markets were almost equal indicates that lemon growers in that year came close to maximizing the income they could get from their total output of 10.73 million boxes. Using the mathematical methods described in Appendix 6, we could find the absolute maximum returns for any size crop

[4] See footnote 2, page 70. Equations are adapted from Hoos and Seltzer's equations 1 and 2, table 9, p. 77.

$Q = q_1 + q_2$. We would do this by solving equations (10.9).

$$(10.9) \qquad \begin{aligned} 0.930q_1+0.209q_2+1.000r&=5.638\\ 0.209q_1+0.292q_2+1.000r&=0.897\\ 1.000q_1+1.000q_2&=Q, \end{aligned}$$

where r is the marginal net return in each market, and where Q is the fixed quantity to be distributed.

The general solution is

$$(10.10) \qquad \begin{aligned} q_2&=-5.897+0.897\,Q\\ q_1&=5.897+0.103\,Q. \end{aligned}$$

When Q is 10.73, as it was in 1934–35, equations (10.10) indicate that the maximum net returns to the grower would be obtained by selling 7.00 million boxes in the fresh market and 3.73 million for processing. This is very close to the actual sales in that year of 7.18 million in the fresh market and 3.55 million for processing.

Assuming that the equations held exactly, the allocation of 7.00 million boxes to the fresh market and 3.73 to the processing market would have given the lemon growers net returns of \$12.54 million. Actually, they got \$12.53 million.

Naturally, an organized group of lemon growers would want a general sort of analysis which would indicate the best allocation for any size of crop. A handy chart for such a purpose would be one like figure 10.2. The heavy curved lines in figure 10.2 are isoquants, like those in figure 9.2. For example, all the points on the line marked 12 indicate combinations of q_1 and q_2 that would give the growers net returns of \$12 million. Similarly, the points on the curve marked 10 would give the growers \$10 million. The point where the two dashed lines cross indicates the combination of q_1 and q_2 which would maximize the net return to growers when they had a total of 10.73 million boxes to sell. The dashed line indicates all the combinations of q_1 and q_2 which add to 10.73 million boxes. It is simply a straight line, with a slope of -1.

You could find the maximum point on the dashed line by interpolating an isoquant which was tangent to the dashed line. It would be tangent at the point where the two dashed lines cross. At that point the slope of the isoquant would be -1. The dotted line marked "expansion path" cuts all the isoquants at the points where their slopes are -1. It shows the increases in each market that would maximize returns as output expanded. If the crop had been greater or smaller than 10.73, it would have been a simple matter to draw the dashed line at a higher or lower level. In any such case, the maximum income would be obtained by the combination of quantities for fresh use and for processing indicated by the intersection of the dashed line with the dotted line.

In case the crop were small enough, these two lines would not cross at any point on the diagram. Technically, they would cross at a point indicating negative sales for processing, which would be a nonfeasible solution, since the grower cannot sell less than nothing in any market. Actually, if the crop were less than about 6.6 million boxes, the grower would apparently maximize his income by selling it all in the fresh market. But when the crop got larger than about 6.6 million boxes, it would be profitable for the grower to divert a substantial portion of

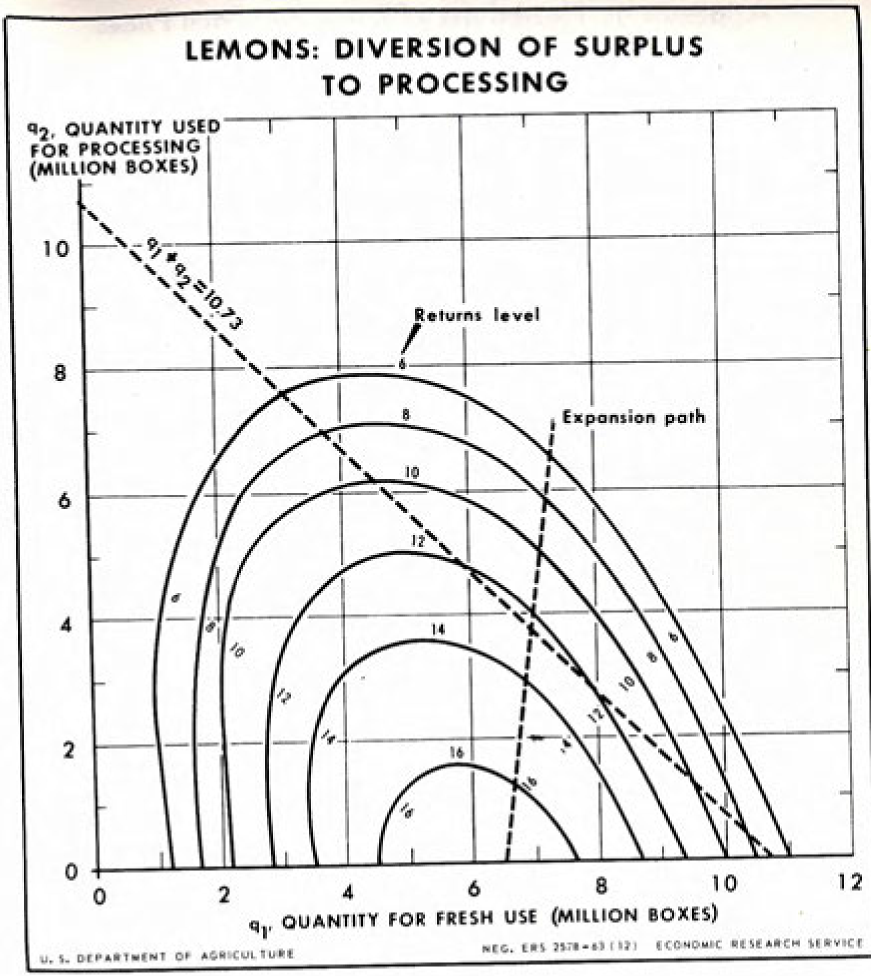

Figure 10.2

the surplus into processing. These results apply, of course, to marketings in years near 1934–35. To analyze current diversion problems, we would need a postwar analysis of demand.

The above analysis of lemon distribution is entirely static. It indicates the distribution that would have maximized returns in a single year. A more important question for the lemon industry may well be how to maximize net returns over a long period of years. To analyze this, we would need an analysis of long-term demand. It is quite possible that the distribution of the lemon crop in a given year may affect the returns in following years. The diversion of surplus lemons into such products as canned and frozen lemon juice very likely is an important factor in market development for these products. Market development is naturally a dynamic process. Also, of course, the development of markets for processed lemon products might reduce the demand for fresh lemons in the long run. To analyze the flow of net returns over a long period of years would require a much more elaborate statistical analysis of demand and a more difficult analysis than the one discussed above.

Appendix 1. Flexibilities of Retail and Farm Prices

Let $\hat{r}$ be the expected retail price,
$\hat{f}$ the expected farm price,
q the quantity to be sold and consumed, and
y consumer income.

(A.1.1) Assume that $\hat{r}=f_1\ (q, y)$
and $\hat{f}=f_2\ (q, y)$.

The elasticity of expected retail price with respect to quantity, with income held constant (i.e., "price flexibility"), will be written $F_{\hat{r}q.y}$. Similarly, $F_{\hat{f}q.y}$ will be the elasticity of farm price, with income held constant.

Moreover, $F_{\hat{r}y.q}$ and $F_{\hat{f}y.q}$ will be the elasticities of retail and of farm prices with respect to income, with quantity held constant.

By definition,

(A.1.2)
$$F_{\hat{r}q.y}=\frac{\partial r}{\partial q}\cdot\frac{q}{r} \qquad F_{\hat{f}q.y}=\frac{\partial f}{q}\cdot\frac{q}{f}$$
$$F_{\hat{r}y.q}=\frac{\partial r}{\partial y}\cdot\frac{y}{r} \qquad F_{\hat{f}y.q}=\frac{\partial f}{y}\cdot\frac{y}{f}$$

We ask how these elasticities compare at retail and at farm levels—depending upon the nature of the price spreads.

A. With percentage price spreads—

$$f=(1-k)\ r.$$

$$\text{Then } F_{\hat{f}q.y}=(1-k)\,\frac{\partial r}{\partial q}\,\frac{q}{(1-k)r}=\frac{\partial r}{\partial q}\cdot\frac{q}{r}=F_{\hat{r}q.y}$$

$$\text{and } F_{\hat{f}y.q}=(1-k)\,\frac{\partial r}{\partial y}\,\frac{y}{(1-k)r}=\frac{\partial r}{\partial y}\cdot\frac{y}{r}=F_{\hat{r}y.q}.$$

So, with percentage spreads, both price flexibilities are the same at retail and at farm levels.

B. With constant absolute spreads—

$$f=r-c.$$

Then

$$F_{\hat{f}q.y}=\frac{\partial\hat{r}}{\partial q}\cdot\frac{q}{r-c}, \text{ and } \frac{F_{\hat{f}q.y}}{F_{\hat{r}q.y}}=\frac{r}{r-c}.$$

Also

$$F_{\hat{f}y.q}=\frac{\partial\hat{r}}{\partial y}\cdot\frac{y}{r-c}, \text{ and } \frac{F_{\hat{f}y.q}}{F_{\hat{r}y.q}}=\frac{r}{r-c}.$$

So, prices are more elastic (more flexible) at the farm than at retail—both with respect to quantity and with respect to income—if the price spread is a constant number of dollars and cents.

Appendix 2. An Orthogonal Regression

For illustration, I have used the data from table 3.1, time period, 1948–62.

Variables, x_1=log of index of food prices; x_2=log of per capita of disposable income; and x_3=log of per capita food consumption.

The correlation matrix was

$$R=\begin{bmatrix}1.00000 & 0.92219 & 0.66911\\ 0.92219 & 1.00000 & 0.86791\\ 0.66911 & 0.86791 & 1.00000\end{bmatrix}.$$

We assume that $\lambda=\frac{\sum e_1^2}{\sum x_1^2}=\frac{\sum e_2^2}{\sum x_2^2}=\frac{\sum e_3^2}{\sum x_3^2}$, where e_k is the error of measurement, or specification in x_k.

If, aside from such errors, the correlation were perfect—and if the errors are uncorrelated with one another and with the variables themselves—

$$\begin{vmatrix}1-\lambda & 0.92219 & 0.66911\\ 0.92219 & 1-\lambda & 0.86791\\ 0.66911 & 0.86791 & 1-\lambda\end{vmatrix}=(1-\lambda)^3-2.05141(1-\lambda)+1.07108=0.$$

We want the smallest positive root. First solve the cubic for the value of $(1-\lambda)$ which is nearest 1, but less than 1. It is approximately 0.97770, corresponding to $\lambda_0=0.02230$.

Replacing the 1's in the diagonal of R, and computing the adjugate

$$[R-\lambda_0]=\begin{bmatrix}0.97770 & 0.92219 & 0.66911\\ 0.92219 & 0.97770 & 0.86791\\ 0.66911 & 0.86791 & 0.97770\end{bmatrix}$$

$$\text{Adj. }[R-\lambda_0]=\begin{bmatrix}0.20263 & -0.32090 & 0.14619\\ -0.32090 & 0.50819 & -0.23151\\ 0.14619 & -0.23151 & 0.10546\end{bmatrix}.$$

In units of standard deviations, the orthogonal regression is proportional to any row of the adjugate. For example, $(0.20263x_1/s_1)-(0.32090x_2/s_2)+(0.14619x_3/s_3)=0$.

The standard deviations were $s_1=0.026422$, $s_2=0.064628$, and $s_3=0.007136$.

Thus, in terms of original units, the orthogonal regression is

$$7.669x_1-4.965x_2+20.486x_3=0.$$

This can be written in three different ways: dividing successively by -7.669, by 4.965, and by -20.486, the equation can be written in the following forms:

$$-1.000x_1+0.647x_2-2.671x_3=0$$

$$1.545x_1-1.000x_2+4.125x_3=0$$

$$-0.374x_1+0.242x_2-1.000x_3=0.$$

The observed statistics are compatible with the hypothesis that the correlation between the true values of the three variables is perfect, and that the observed scatter around the regression plane is due to relative errors of a little over 2 percent in each variable. (Here, relative error means the variance of the error divided by the variance of the variable itself.)

Note that $b_{ij}=1/b_{ji}$. For example, a unit increase in consumption reduces food price by 2.67120 units. And a unit increase in food price reduces consumption by 1/2.67120=0.37436 units. There is no inconsistency due to changing the dependent variable. In effect, all three variables are treated as dependent upon one another.

Appendix 3. Relationships Among Demand Coefficients

Ever since the days of Walras and Pareto, many mathematical economists have considered relationships that should exist between demand coefficients. In recent years, Slutsky, Hotelling, Henry Schultz, Hicks, Wold and Juréen, Frisch, and Brandow have made especially important contributions.[1]

But, so far as I know, the first to apply these results to actual statistical analysis was Brandow. His pioneering bulletin opens the way to broader statistical studies of interrelated demands than ever before possible. This is much more than an interesting academic exercise; it can be an extremely valuable tool in program analysis and development, as Brandow shows. We have been using Brandow's demand matrix (with some modifications) in the U.S. Department of Agriculture to estimate the expected effects of changes in such things as price-support levels and acreage allotments.

Brandow does not discuss in any detail the basis for the several relationships that are supposed to hold among the demand coefficients. Rather, he refers to Wold and Juréen and to Frisch. Also, following almost all mathematical economists, he gives the relationships in terms of "elasticities of demand," meaning the elasticity of consumption with respect to prices and with respect to income.

Elasticities are always abstract and difficult. Also, the relationships between the elasticities of demand are based upon reasoning about how a rational consumer would adjust his purchases, q_1, q_2, . . ., q_n, assuming given total expenditures m, and assuming fixed prices, p_1, p_2, . . ., p_n. Thus, the quantities are treated as dependent variables, while prices and total expenditures are assumed given.

I find it easier, and often more useful, to think in terms of $\partial p_k/\partial q_i$ and $\partial p_k/\partial m$. (In the rest of this appendix, these will be called p_{ki}

---

[1] Slutsky, Eugenio. SULLA TEORIA DEL BILANCIO DEL CONSUMATORE. Giornale degli Economisti. Milan, LI–1, 1915.

Schultz, Henry. THE THEORY AND MEASUREMENT OF DEMAND. Chapter 1, Univ. Chicago Press, Chicago, 1938.

Hicks, J. R. VALUE AND CAPITAL. 2nd ed. Clarendon Press, Oxford, 1946.

Wold, Herman and Juréen, Lars. DEMAND ANALYSIS. Chapters 6 and 7. Wiley & Sons, New York, 1953.

Frisch, Ragnar. A COMPLETE SYSTEM FOR COMPUTING ALL DIRECT AND CROSS ELASTICITIES IN A MODEL OF MANY SECTORS. Econometrica 27–2, April 1959.

Brandow, G. E. INTERRELATIONS AMONG DEMANDS FOR FARM PRODUCTS AND IMPLICATIONS FOR CONTROL OF MARKET SUPPLY. Penn. State Univ. Bul. 680, pp. 13–15, University Park, 1961.

Hotelling, Harold. EDGEWORTH'S TAXATION PARADOX AND THE NATURE OF DEMAND AND SUPPLY FUNCTIONS. Jour. Political Econ. XL–5, October 1932.

and p_{km}.) These are the estimates of price changes corresponding to given changes in quantities marketed and in total expenditures in a market. In competitive markets, changes in prices are generally determined by changes in quantities marketed and changes in income—not the other way around. In this appendix I shall show the principal relationships which should hold among such coefficients. Then, I shall translate these relationships into terms of price flexibilities—meaning the elasticity of price with respect to the several quantities and with respect to expenditures. These relationships are exactly parallel to those found by such writers as Wold and Juréen and Frisch, except that ours will be expressed as relationships between *price* derivatives, whereas theirs were expressed as relationships between quantity derivatives.

Demand Equations

Consumers in a given market, during a given year, buy n different goods and services. Let $q_1, q_2, \ldots, q_n$ represent the per capita quantities purchased, and let m be the per capita expenditures for all the n goods and services together.

Suppose we want a set of equations to estimate the expected price of each good as a function of the q's and m. The two most commonly used sets of equations are

$$
\text{(A.3.1)}\quad
\begin{aligned}
p_1 &= a_{11}q_1 + a_{12}q_2 + \ldots + a_{1n}q_n + a_{1m}m \\
p_2 &= a_{21}q_1 + a_{22}q_2 + \ldots + a_{2n}q_n + a_{2m}m \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
p_n &= a_{n1}q_1 + a_{n2}q_2 + \ldots a_{nn}q_n + a_{nm}m,
\end{aligned}
$$

and

$$
\text{(A.3.2)}\quad
\begin{aligned}
\log p_1 &= b_{11}\log q_1 + b_{12}\log q_2 + \ldots + b_{1n}\log q_n + b_{1m}\log m \\
\log p_2 &= b_{21}\log q_1 + b_{22}\log q_2 + \ldots + b_{2n}\log q_n + b_{2m}\log m \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\log p_n &= b_{n1}\log q_1 + b_{n2}\log q_2 + \ldots + b_{nn}\log q_n + b_{nm}\log m.
\end{aligned}
$$

Of course, neither set of equations is likely to hold accurately throughout any wide ranges of the q's and of m. But either should provide reasonably accurate measures of expected changes in prices with given small (or assumed) changes in the q's and in m; that is, a_{kj} in (A.3.1) should be a reasonably accurate measure ot the flexibility of p_k with respect to q_j. (In the rest of this appendix, such flexibilities will be called f_{kj}.)

We want to establish certain relationships that must hold among the a's in (A.3.1), or among the b's in (A.3.2). The relationships among the a's can be established directly, remembering that $a_{kj} = p_{kj}$ and $a_{km} = p_{km}$. If someone prefers to work with relationships among the b's, the relationships among the a's can be translated to flexibilities, remembering that $b_{kj} = f_{kj}$. We shall show the relationships in both forms.

The Expenditure Equation

We can establish many of the principal relationships by analyzing the basic equation of expenditures,

$$
\text{(A.3.3)}\qquad p_1q_1 + p_2q_2 + \ldots + p_nq_n = m.
$$

The first two relationships discussed below can be established from (A.3.3) alone—without using such concepts as utility and indifference.

Weighted Sum of Income Coefficients

Suppose m varies, while all the q's are fixed. Then

(A.3.4) $$q_1p_{1m}+q_2p_{2m}\ldots+q_np_{nm}=1.$$

The meaning of this is simple. The change in expenditure for good k is q_kp_{km}. If total expenditure is increased, say by a dollar, the sum of expenditures for each separate good must increase by the same amount.

In terms of the a's in (A.3.1), equation (A.3.4) is

(A.3.5) $$q_1a_{1m}+q_2a_{2m}+\ldots+q_na_{nm}=1.$$

In terms of price flexibilities with respect to m, (A.3.4) becomes

(A.3.6) $$r_1f_{1m}+r_2f_{2m}+\ldots+r_nf_{nm}=1,$$

where r_k is the proportion of total expenditures allocated to the k^{th} good.

In terms of the b's in (A.3.2), equation (A.3.6) is

(A.3.7) $$r_1b_{1b}+r_2b_{2m}\ldots+r_nb_{nm}=1.$$

Weighted Column Sums

Suppose now that q_k varies, while all other q's and m are fixed. Then

(A.3.8) $$q_1p_{1k}+q_2p_{2k}+\ \ldots\ +q_np_{nk}=-p_k,$$

or, in terms of the a's,

(A.3.9) $$q_1a_{1k}+q_2a_{2k}+\ \ldots\ +q_na_{nk}=-p_k.$$

These equations, too, can be explained easily. When q_k changes, the change in the expenditure for the i^{th} good is q_ip_{ik} if $i\neq k$. But the change in expenditure for the k^{th} good is $p_k+q_kp_{kk}$. Equation (A.3.8) results from summing all changes in expenditure—which sum is zero because m is fixed.

In terms of price flexibilities with respect to q_k, (A.3.8) is equivalent to

(A.3.10) $$r_1f_{1k}+r_2f_{2k}+\ \ldots\ +r_nf_{nk}=-r_k,$$

or

(A.3.11) $$r_1b_{1k}+r_2b_{2k}+\ \ldots\ +r_nb_{nk}=-r_k.$$

Weighted Row Sums

Here we assume homogeneity. That is, if incomes were raised by x percent, and if all quantities were fixed, each price would be raised x percent. This is logically correct if *each consumer's expenditure is increased by x percent.* Otherwise, it is not exactly correct but may serve as an approximation.

Using the Euler theorem,

$$q_1p_{k1}+q_2p_{k2}+\ \ldots\ +q_np_{kn}=p_{km}, \tag{A.3.12}$$

or

$$q_1a_{k1}+q_2a_{k2}+\ \ldots\ +q_na_{kn}=a_{km}. \tag{A.3.13}$$

In terms of price flexibilities, (A.3.12) becomes

$$f_{k1}+f_{k2}+\ \ldots\ +f_{kn}=f_{km}, \tag{A.3.14}$$

$$b_{k1}+b_{k2}+\ \ldots\ +b_{kn}=b_{km}. \tag{A.3.15}$$

Symmetry

In addition to the above conditions based upon the expenditure equation, we can establish certain logical relationships between p_{ik} and p_{ki} on the assumption that the typical consumer is rational.

Let U_k be the marginal utility of the k^{th} good, p_k its price, and U_m the marginal utility of expenditures. The rational consumer spends his money in such a way that

$$U_k=p_kU_m \text{ and } U_i=p_iU_m. \tag{A.3.16}$$

Since the order of differentiation is immaterial, the second derivatives $U_{ki}=U_{ik}$.

So

$$p_kU_{mi}+U_mp_{ki}=p_iU_{mk}+U_mp_{ik}. \tag{A.3.17}$$

Also,

$$U_{im}=p_iU_{mm}+U_mp_{im}=U_{mi}, \tag{A.3.18}$$

and

$$U_{km}=p_kU_{mm}+U_mp_{km}=U_{mk}.$$

Substituting (A.3.12) into (A.3.11), and simplifying,

$$p_kp_{im}+p_{ki}=p_ip_{km}+p_{ik}, \tag{A.3.19}$$

or

$$p_{ki}=p_{ik}+p_ip_{km}-p_kp_{im}, \tag{A.3.20}$$

or

$$a_{ki}=a_{ik}+p_ia_{km}-p_ka_{im}. \tag{A.3.21}$$

In terms of price flexibilities, (A.3.19) becomes

$$r_u(r_i f_{im}+f_{ki})=r_i(r_k f_{km}+f_{ik}), \tag{A.3.22}$$

or

$$f_{ki}=\frac{r_i}{r_k}f_{ik}-r_i(f_{km}-f_{im}), \tag{A.3.23}$$

or

$$b_{ki}=\frac{r_i}{r_k}b_{ik}-r_i(b_{km}-b_{im}). \tag{A.3.24}$$

Note that when the proportion of income spent for good i is small, f_{ki} is approximately equal to $\frac{r_i}{r_k}f_{ik}$. In such cases,

$$b_{ki}\sim\frac{r_i}{r_k}b_{ik}; \tag{A.3.25}$$

or, more simply

$$p_{ki}\sim p_{ik}, \tag{A.3.26}$$

or

$$a_{ki}\sim a_{ik}. \tag{A.3.27}$$

Appendix 4. Symmetric Regressions

Chapter 6 included a symmetric set of equations for estimating retail prices of beef, pork, and chicken. A general linear set of symmetric equations is

$$\begin{aligned} p_5&=a_{51}q_1+a_{52}q_2+a_{53}q_3+a_{54}y_4\\ p_6&=a_{52}q_1+a_{62}q_2+a_{63}q_3+a_{64}y_4\\ p_7&=a_{53}q_1+a_{63}q_2+a_{73}q_3+a_{74}y_4, \end{aligned} \tag{A.4.1}$$

where q_1, q_2, q_3 are per capita quantities of the three goods consumed, where p_5, p_6, p_7 are the retail prices of the same goods (for example, p_5 is the price of the first good, etc.), and where y is per capita consumer income.

Note that the first three columns of a_{ij}'s to the right of (A.4.1) are assumed to be symmetric. The logic of this has been covered in Appendix 3. If we should fit three separate least squares regressions to the three equations in (A.4.1), we would get two different estimates of a_{52}, of a_{53}, and of a_{63}. One possible way around this difficulty is to determine the three equations simultaneously, and in such a way as to minimize the sum of sums of squared errors.[1] Thus, if e_1 is the error of estimate in the first equation, e_2 in the second, e_3 in the third, we may minimize $E=(e_1^2+e_2^2+e_3^2)$. To do this, we must compute E from (A.4.1), differentiate it with respect to each of the a_{ij}'s, and set

[1] Brandow followed a similar procedure in Pennsylvania State University Bulletin 680, in equation i. 2, table 5, pp. 29–30. University Park. Aug. 1961.

each derivative equal to zero. This is a long and tedious job, but it involves no difficulties. It results in the following matrix equation, where $m_{ij}=\sum x_i x_j$.

$$
(\text{A.4.2})\quad
\begin{bmatrix}
m_{11} & m_{12} & m_{13} & m_{14} & & & & & \\
m_{12} & (^1) & m_{23} & m_{24} & m_{12} & m_{13} & m_{14} & & \\
m_{13} & m_{23} & (^2) & m_{34} & & m_{12} & & m_{13} & m_{14} \\
m_{14} & m_{24} & m_{34} & m_{44} & & & & & \\
 & m_{12} & & & m_{22} & m_{23} & m_{24} & & \\
 & m_{13} & m_{12} & & m_{23} & (^3) & m_{34} & m_{23} & m_{24} \\
 & m_{14} & & & m_{24} & m_{34} & m_{44} & & \\
 & & m_{13} & & & m_{23} & & m_{33} & m_{34} \\
 & & m_{14} & & & m_{24} & & m_{34} & m_{44}
\end{bmatrix}
\begin{bmatrix} a_{51} \\ a_{52} \\ a_{53} \\ a_{54} \\ a_{62} \\ a_{63} \\ a_{64} \\ a_{73} \\ a_{74} \end{bmatrix}
=
\begin{bmatrix} m_{15} \\ (^4) \\ (^5) \\ m_{45} \\ m_{26} \\ (^6) \\ m_{46} \\ m_{37} \\ m_{47} \end{bmatrix}.
$$

[1] $m_{11}+m_{22}$.

[2] $m_{11}+m_{33}$.

[3] $m_{22}+m_{33}$.

[4] $m_{25}+m_{16}$.

[5] $m_{35}+m_{17}$.

[6] $m_{36}+m_{27}$.

In our beef-pork-chicken problem, the m_{ij}'s were

$$
(\text{A.4.3})\quad
\begin{array}{c}
\begin{array}{rrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
1,725 & -440 & 474 & 33,897 & -445 & 86 & -1,092 \\
-440 & 215 & -106 & -6,965 & 183 & -127 & 245 \\
474 & -106 & 224 & 13,473 & 110 & 44 & -444 \\
33,897 & -6,965 & 13,473 & 884,052 & 2,790 & 2,200 & -27,439
\end{bmatrix}.
\end{array}
$$

The results given in (6.2) of chapter 6 were obtained by inserting these m_{ij}'s into (A.4.2), solving for the a_{ij}'s, and inserting these a_{ij}'s into (A.4.1).

Appendix 5. An Indifference Surface

Chapter 7 presented an indifference surface for beef and pork. It was derived from data in table 7.1. First, we got the regression equation

$$
(\text{A.5.1})\quad \log r = -4.788588 - 0.855546 \log q_1 + 0.955203 \log q_2 + 1.452289 \log q_3,
$$

where r is the ratio of retail beef price to retail pork price, and q_1, q_2, q_3 are per capita consumption of beef, pork, and all other goods.

If log q_2 had been held constant at the mean of the series, (3.274239), the equation would have been

$$\log r = -0.033447 - 0.855546 \log q_1 + 0.955203 \log q_2. \tag{A.5.2}$$

or

$$r = 0.92590 q_1^{-0.855546} q_2^{0.955203}. \tag{A.5.3}$$

Consider the equation [1]

$$z = \frac{a(1-c)}{1+b} q_1^{1+b} + q_2^{1-c}. \tag{A.5.4}$$

Its first derivatives are

$$z_1 = a(1-c) q_1^b \text{ and } z_2 = (1-c) q_2 - c. \tag{A.5.5}$$

So

$$z_1/z_2 = a q_1^b q_2^c. \tag{A.5.6}$$

Now, if z were any monotonic increasing function of utility,

$$z_1/z_2 = u_1/u_2 = p_1/p_2 = r. \tag{A.5.7}$$

In our case, we would have

$$z = 0.28713 q_1^{0.14445} + q_2^{0.04480}. \tag{A.5.8}$$

If price ratios were estimated from (A.5.7) and (A.5.6), the estimates would be the same as those from (A.5.2).

To compute any indifference curve from (A.5.7), take any arbitrary value, $z = k_1$. Then

$$q_2 = (k_i - 0.28713 q_1^{0.14445})^{22.321}. \tag{A.5.9}$$

Using this equation, I computed several points on the curves corresponding to $k_i = 1.72, 1.73, 1.74, 1.75, 1.76$. These k_i's are arbitrary values of z, chosen to fill the range of the observed data.

Price ratios (or ratios of marginal utilities) could be estimated from figure 7.2 by estimating the slope of an indifference line passing through any given point (q_1, q_2).

---

[1] My former colleague, Glenn L. Burrows, suggested this equation to me. I used it in a paper published in 1956.

Appendix 6. Distribution To Maximize Returns

Assume that a commodity may be distributed among n markets. Let the quantities distributed to the n markets be $q_1, q_2, \ldots, q_n$. Let $p_1, p_2, \ldots, p_n$ be the net prices received by farmer or distributor (that is, p_k is the delivered price in market k minus any such costs as freight, storage, or processing that would be incurred by the farmer or distributor in order to sell in the k^{th} market). First consider the general case, where the markets may or may not be independent of one another.

Let the demand function in market k be

$$p_k = f_k(q_1, q_2, \ldots, q_n). \tag{A.6.1}$$

Returns from the k^{th} market are

$$r_k = q_k p_k, \tag{A.6.2}$$

and returns from all n markets together are

$$R = q_1p_1 + q_2p_2 + \ldots + q_np_n. \tag{A.6.3}$$

Assume that the demand functions (A.6.1) are known for each of the n markets. Then it is a simple matter to compute the returns function (A.6.3). This function can be maximized by the ordinary rules found in any good text on the calculus.

Let R_k stand for $\partial R/\partial q_k$, and let R_{kj} stand for $\partial R^2/\partial q_k q_j$. If there are no restrictions upon the total amount to be distributed in the n markets, the necessary conditions for maximum returns are

$$R_1 = R_2 = \ldots = R_n = 0. \tag{A.6.4}$$

There are n equations in (A.6.4), with the n unknowns, $q_1, q_2 \ldots q_n$.

If the total amount to be sold is fixed at Q (a common situation in agricultural marketing), the conditional (or constrained) maximum is found by differentiating

$$R - \lambda(q_1 + q_2 + \ldots + q_n - Q), \tag{A.6.5}$$

where λ is a Lagrangean multiplier, and setting each derivative equal to zero. Thus, necessary conditions for maximum returns in this case are

$$\begin{aligned} R_1 = R_2 = \ldots = R_n &= \lambda \\ q_1 + q_2 + \ldots + q_n &= Q. \end{aligned} \tag{A.6.6}$$

There are $n+1$ equations in (A.6.6) with the $n+1$ unknowns,

$$q_1, q_2, \ldots q_n, \lambda.$$

The Lagrangean multiplier λ turns out to be the marginal returns, which are the same in each of the n markets.

Sufficient conditions for maximizing returns are that the Hessian matrix,

$$H=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix}, \tag{A.6.7}$$

be negative definite. This means that each diagonal element of H must be negative, each 2-rowed principal minor must be positive, each 3-rowed principal minor must be negative, . . ., and in general each even-rowed minor must be positive and each odd-rowed minor must be negative.

Marginal Returns in Independent and Dependent Markets

In the special case where the n markets are completely independent of one another, (1) becomes

$$p_k=f(q_k), \tag{A.6.8}$$

and the derivative of $R=p_kq_k$ with respect to q_k is

$$R_k=p_k+q_kp_{kk}, \tag{A.6.9}$$

where p_{kk} is the derivative of p_k with respect to q_k (from the formula for the derivative of a product).

Equation (A.6.9) can be written

$$R_k=p_k(1+f_{kk}) \tag{A.6.10}$$

where f_{kk} is the flexibility of p_k with respect to q_k. When using the true, structural demand function, $f_{kk}=1/e_{kk}$, where e_{kk} is the elasticity of q_k with respect to p_k. For that reason, (A.6.10) is often written as

$$R_k=p_k(1+1/e_{kk}). \tag{A.6.11}$$

In the general case (whether or not the markets are dependent), marginal returns are obtained by differentiating equation (A.6.3). This gives

$$\begin{aligned} R_1&=p_1+q_1p_{11}+q_2p_{21}+\ \cdots\ +q_np_n, \\ R_2&=p_2+q_1p_{12}+q_2p_{22}+\ \cdots\ +q_np_{n2} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ R_n&=p_n+q_1p_{1n}+q_2p_{2n}+\ \cdots\ +q_np_{nn}, \end{aligned} \tag{A.6.12}$$

where p_{ij} is the derivative of p_i with respect to q_j.

In the special case of independent markets, $p_{ij}=0$ whenever $i\neq j$. So in this case, (A.6.12) is equivalent to (A.6.9).

Special Case of Linear Demand Functions

If equations (1) are linear,

$$
\begin{aligned}
&p_1=b_{10}+b_{11}q_1+b_{12}q_2+\ \dots\ +b_{1n}q_n\\
\text{(A.6.13)}\qquad &p_2=b_{20}+b_{21}q_1+b_{22}q_2+\ \dots\ +b_{2n}q_n\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
&p_n=b_{n0}+b_{n1}q_1+b_{n2}q_2+\ \dots\ +b_{nn}q_n,
\end{aligned}
$$

and returns are

$$
\begin{aligned}
&R=b_{10}q_1+b_{11}q_1^2+(b_{12}+b_{21})q_1q_2+\ \dots\ +(b_{1n}+b_{n1})q_1q_n\\
&\quad+b_{20}q_2+\ \dots\ +b_{22}q_2^2+\ \dots\ +(b_{2n}+b_{n2})q_2q_n\\
\text{(A.6.14)}\qquad &\quad\dots\dots\dots\dots\dots\dots\dots\dots\\
&\qquad\qquad\qquad\qquad\qquad+b_{n0}q_n+\ \dots\ b_{nn}q_n^2.
\end{aligned}
$$

In this case, the necessary conditions for a constrained maximum are

$$
\begin{aligned}
&2b_{11}q_1+(b_{12}+b_{21})q_2+\ \dots\ +(b_{1n}+b_{n1})q_n-\lambda=-b_{10}\\
&(b_{12}+b_{21})q_1+2b_{22}q_2+\ \dots\ +(b_{2n}+b_{n2})q_n-\lambda=-b_{20}\\
\text{(A.6.15)}\qquad &\dots\dots\dots\dots\dots\dots\dots\dots\\
&(b_{1n}+b_{n1})q_1+(b_{2n}+b_{n2})q_2+\ \dots\ +2b_{nn}q_n-\lambda=-b_{n0}\\
&q_1+q_2+\ \dots\ +q_n=Q.
\end{aligned}
$$

Sufficient conditions are (A.6.15) together with a demonstration that

$$
\text{(A.6.16)}\qquad H=\begin{bmatrix}
2b_{11} & (b_{12}+b_{21}) & \dots & (b_{1n}+b_{n1})\\
(b_{12}+b_{21}) & 2b_{22} & \dots & (b_{2n}+b_{n2})\\
\dots & \dots & \dots & \dots\\
(b_{1n}+b_{n1}) & (b_{2n}+b_{n2}) & \dots & 2b_{nn}
\end{bmatrix}
$$

is negative definite.

In general, the diagonal elements of (A.6.16) are negative. Ordinarily, the diagonal elements will be much larger in absolute value than the nondiagonal elements. So, in ordinary cases of linear demand, the solution of equations (15) for the q's and for λ will give a true maximum.

In the extra-special case of linear demands and independent markets, $b_{ij}=0$ for all $i\neq j$. Then equations (A.6.15) are easy to solve, and (A.6.16) shows that the solution is a true maximum—assuming that b_{kk} is negative in all markets. To solve (A.6.15) divide the first row by $2b_{11}$, divide the second row by $2b_{22}$, . . ., and so on; then subtract each of the n revised rows from the last row. This eliminates all the q's and gives a simple equation of the form $c_1\lambda=c_2+Q$, or $\lambda=(c_2+Q)/c_1$. Then each of the n revised rows can be solved easily for one of the q's. A case in point is the wheat model discussed in Chapter 10.

With interdependent markets, some of the b_{ij}'s$\neq 0$ when $i\neq j$. Equations (A.6.15) can be solved by any of the usual methods, such as the Gauss-Doolittle method. But first it can be simplified by subtracting equation 1 from equations 2, 3, . . ., n in order to eliminate λ.

A method that is well adapted to the problem discussed here can be developed from the general formula for inverting a bordered matrix.[1] The matrix of coefficients to the left of (A.10.9) can be written in partitioned form,

$$\begin{bmatrix} b & i' \\ i & 0 \end{bmatrix}$$

where b is the $(n\times n)$ submatrix of coefficients of the q's, where $i=[1 \quad 1 \quad . \; . \; . \quad 1]$, where i' is the transpose of i, and where 0 is a scalar zero. It can be shown that

$$\text{(A.6.17)} \qquad \begin{bmatrix} b & i' \\ i & 0 \end{bmatrix} = \begin{bmatrix} (b^{-1}-s's/S) & s'/S \\ s/S & -1/S \end{bmatrix}$$

where s is the $(1\times n)$ vector of column sums of b^{-1}, where s' is the $(n\times 1)$ vector of row sums of b^{-1}, and S is the sum of all elements of b^{-1}. To demonstrate this, substitute ib^{-1} for s, $b^{-1}i'$ for s', and $ib^{-1}i'$ for S; then multiply the original matrix by its inverse; the result is a unit matrix.

After b has been inverted, the inverse of the bordered matrix can be computed very easily and quickly from (A.6.17).

Special Case of Constant Demand Elasticity

In the case of a single market, economists often assume a demand of constant elasticity,

$$\text{(A.6.18)} \qquad p=aq^{-b}$$

Such curves often fit the data fairly well within the observed range of p and q. But they are not suitable for extrapolation over any great range. Such an assumed demand function implies that returns from a single market,

$$\text{(A.6.19)} \qquad r=pq=aq^{1-b}$$

have no maximum. If $b<1$, returns would rise indefinitely as quantity increased. If $b>1$, returns would approach infinity as quantity approached zero.

---

[1] See Frazer, R. A., Duncan, W. J., and Collar, A. R., ELEMENTARY MATRICES. Pp. 112–115. Cambridge Univ. Press, Cambridge, 1955. Also, Faddeeva, V. N., COMPUTATIONAL METHODS OF LINEAR ALGEBRA, translated from Russian by Curtis D. Benster. Pp. 105–111. Dover Publications, Inc., New York, 1959.

In case of several markets, such demand functions as

$$
\begin{aligned}
p_1 &= a_{10} q_1^{-a_{11}} q_2^{a_{12}} \cdots q_n^{a_{1n}} \\
p_2 &= a_{20} q_1^{a_{21}} q_2^{-a_{22}} \cdots q_n^{a_{2n}} \\
&\cdots\cdots\cdots\cdots\cdots \\
p_n &= a_{n0} q_1^{a_{n1}} q_2^{a_{n2}} \cdots q_n^{-a_{nn}}
\end{aligned} \tag{A.6.20}
$$

are also poorly adapted to the study of intermarket distribution. (The direct price flexibilities $-a_{kk}$ are assumed to be negative; cross flexibilities such as a_{ij} can be negative, zero, or positive.)

With demand equations (A.6.20) and a given quantity to distribute, there would be no real distribution that would maximize returns unless each $a_{kk}<1$ (i.e., unless demand in each market were elastic). Otherwise, almost infinite returns could be obtained by practically starving the most inelastic market and distributing practically the entire supply to the other $n-1$ markets.

Iteration

In actual practice—whatever demand functions are assumed—it is possible to start with some particular distribution, such as the actual distribution of last year. Then marginal returns from each market can be computed by (A.6.12). (These equations can be used even for graphic demand functions.) Unless $R_1=R_2= \ldots\ldots =R_n$, some shifts will be indicated to increase returns. This iterative process can be continued indefinitely as long as the R'_ks are unequal, and as long as the distribution stays within the range where the demand equations (A.6.1) are believed to hold.

Apparently, Hoos and Seltzer had in mind an iterative procedure when they advised California lemon producers not to distribute their crop between the fresh and processed markets in the way that would apparently maximize returns.[2] Rather, they advised lemon growers that "consideration might well be given to gradually decreasing the percentage of the crop allocated to the fresh outlet and correspondingly increasing the percentage of the crop going to the processed outlet."

Such an iterative procedure has the great advantage of making it possible to restudy the demand functions after each step. If the allocation to processed lemons is gradually increased from year to year, the statistician may be able to get dynamic measures of demand changes, and thus may find the key to maximizing the flow of returns over a period of several years. This would take into account the effect of distribution in year t upon returns in years $t+1$, $t+2$, ..., and so on.

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[2] Hoos, Sidney and Seltzer, R. E. LEMONS AND LEMON PRODUCTS. Bul. 729, Calif. Agr. Expt. Sta., pp. 59. Berkeley. 1952.

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SUBJECT INDEX

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