Chapter 14

The Ideal Gas Law and Its Applications

Gases Revisited

Properties of Gases (Section 4.1)

Gases may be compressed.

Gases expand to fill their containers uniformly.

All gases have low densities compared with those of liquids and solids.

Gases may be mixed in the same volume.

A gas exerts constant, uniform pressure in

Gases Revisited

The **Ideal Gas Model** describes the particulate behavior of gases as (Section 4.2):

Gases consist of molecular particles moving at any given instant in straight lines.

Molecules collide with each other and with the container walls without loss of energy.

Gas molecules are very widely spaced.

The actual volume of molecules is negligible compared to the space they occupy.

Gas molecules behave as independent particles; attractive forces between them are negligible.

Gases Revisited

Gas measurements and the units in which they are usually expressed include (Section 4.3):

Pressure, P, expressed in torr or atmospheres (atm)

Volume, V, expressed in liters (L)

Temperature, T, expressed in degrees Celsius (°C) or kelvins (K)

Gases Revisited

Combined Gas Law

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Standard Temperature and Pressure

0°C (273 K) and 1 atm (bar)

Avogadro’s Law

Goal 1

If pressure and temperature are constant, state how volume and amount of gas are related and explain phenomena or make predictions based on that relationship.

Avogadro's Law

Law of Combining Volumes

When gases react with each other, the reacting volumes are always in the ratio of small whole numbers if the volumes are measured at the same temperature and pressure. It extends to gaseous products, too. Could be explained by:

Avogadro's Law

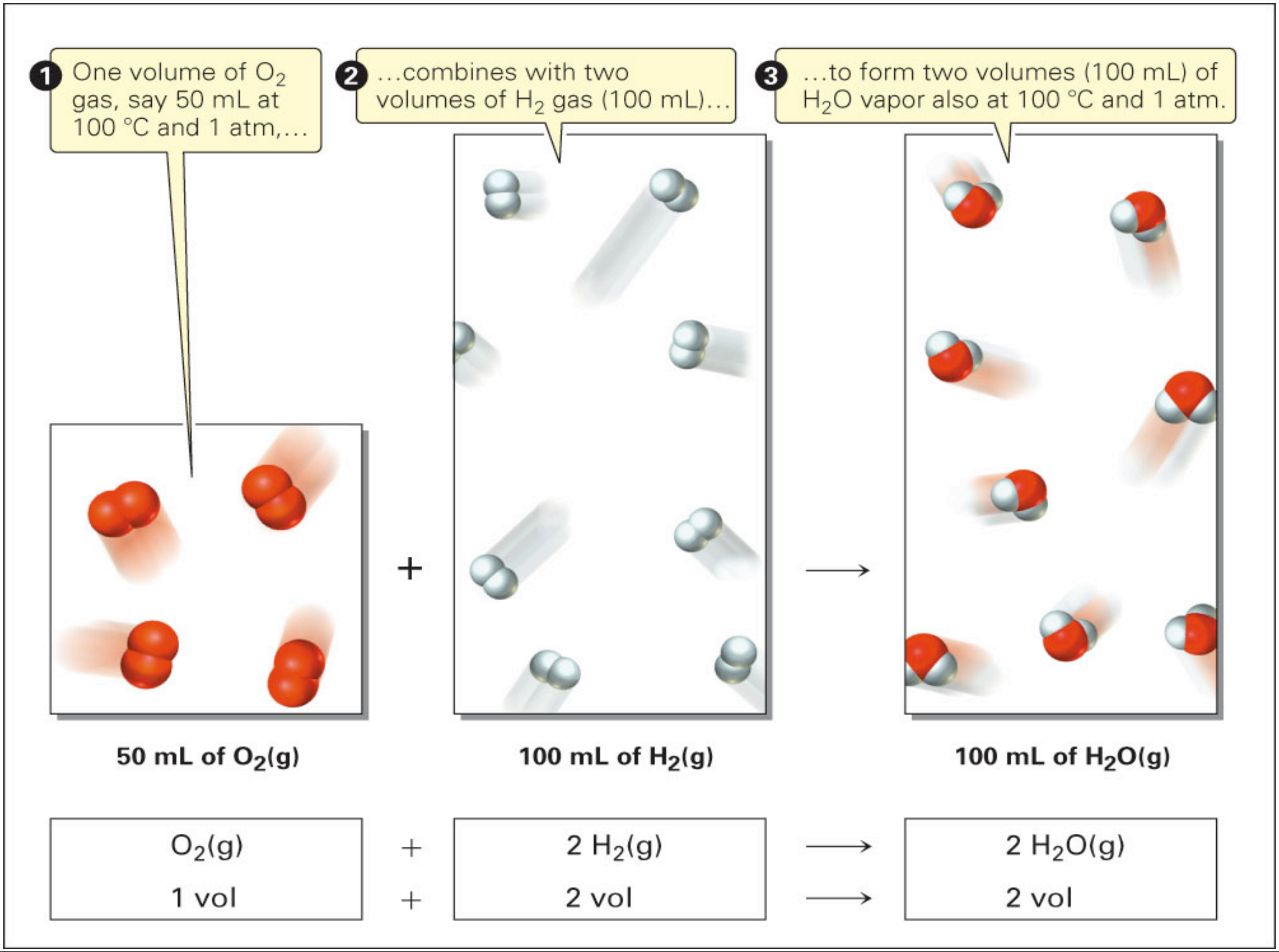
Equal volumes of gases at the same temperature and pressure contain the same number of molecules.

Quantitatively,

Volume is proportional to number of molecules:

$$V \propto n$$

Avogadro’s Law



Avogadro’s Law

The Ideal Gas Law

Goal 2

Explain how the ideal gas equation can be constructed by combining Charles's, Boyle's, and Avogadro's Laws, and explain how the ideal gas equation can be used to derive each of the three two-variable laws.

The Ideal Gas Law

The **Ideal Gas Law** is a mathematical combination of the individual gas laws:

Charles's Law: $V \propto T$

Boyle's Law: $V \propto \frac{1}{P}$

Avogadro's Law: $V \propto n$

The Ideal Gas Law

$$V \propto T \times \frac{1}{P} \times n$$

Inserting a proportionality constant yields an equation:

$$V = R \times T \times \frac{1}{P} \times n$$

Rearranging gives the ideal gas equation
in its most common form:

$$PV = nRT$$

R is called the **universal (or ideal) gas constant.**

The Ideal Gas Law

Example:
A 0.1000-mole sample of helium is placed in a piston and heated to 25.00°C (298.15 K). The volume is adjusted to 4.600 L, and the resulting pressure is measured as 0.5319 atm. Use these data to determine the value of the universal gas constant.

Solution:
Use the ideal gas equation and algebra.

$$PV = nRT$$

$$\frac{PV}{nT}$$

$$\frac{1}{0.1000\ \text{mol}} \quad \frac{1}{298.15\ \text{K}}$$

$$\frac{0.08206\ \text{atm} \times \text{L}}{\text{mol} \times \text{K}}$$

The Ideal Gas Law

A useful variation of the ideal gas equation replaces n, the number of moles, by the mass of a sample, m, divided by molar mass, MM:

$$n = \frac{\text{mass}}{\text{molar mass}} = \frac{m}{MM} = g \times \frac{mol}{g} = mol$$

$$PV = nRT = \frac{m}{MM} RT = \frac{mRT}{MM}$$

The Ideal Gas Equation

Goal 3

Given values for all except one of the variables in the ideal gas equation, calculate the value of the remaining variable.

The Ideal Gas Equation

If you know values for all variables except one in the ideal gas equation, you can use algebra to find the value of the unknown variable.

As with all problems to be calculated algebraically, first solve the equation for the wanted quantity.

The Ideal Gas Equation

Example:

What is the pressure (atm) in a helium-filled 0.77-L balloon if it contains 1.0 g of gas at 22°C?

Solution:

V, m, and T are directly given in the problem statement. You also need MM, and that can be found based on the name of the gas, helium. From the periodic table, MM for He is 4.003 g/mol. You now have all variables but one.

We will use the 0.0821 L • atm/mol • K value for R because the problem statement asks for P in atm.

The Ideal Gas Equation

$$PV = \frac{m}{MM} RT$$

$$P = \frac{m\,R\,T}{(MM)V} = \frac{1.0\ \text{g}}{4.003\ \text{g/mol}} \times 0.0821\ \frac{\text{L} \times \text{atm}}{\text{mol} \times \text{K}} \times \frac{(22 + 273)\ \text{K}}{0.77\ \text{L}}$$

$$= 7.9\ \text{atm}$$

Gas Density

Goal 4

Calculate the density of a known gas at any specified temperature and pressure.

Goal 5

Given the density of a pure gas at specified temperature and pressure, or information from which it can be found, calculate the

Gas Density

$$\text{Density} \equiv \frac{\text{mass}}{\text{volume}}$$

$$\text{In symbols, } D \equiv \frac{m}{V}$$

$$PV = \frac{m}{MM} RT$$

$$P = \frac{m\,R\,T}{(MM)\,V} = \frac{m}{V} \times \frac{RT}{MM} = D \times \frac{RT}{MM}$$

$$P = D \times \frac{RT}{MM} \qquad D = \frac{(MM)\,P}{R\,T}$$

So density is proportional to molar mass of a gas

Gas Density

Example:

What is the density of helium at 22°C and 744 torr?

Solution:

Solve with algebra.

$$D = \frac{(MM)\,P}{R\,T} = \frac{4.003\text{ g}}{\text{mol}} \times \frac{744\text{ torr}}{760\text{ torr/atm}} \times \frac{\text{mol} \times \text{K}}{.0821\text{ L} \times \text{atm}} \times \frac{1}{(22+273)\text{ K}} = 0.162\text{ g/L}$$

USING GAS DENSITY

The density of air at 15 ^{o}C and 1.00 atm is 1.23 g/L. What is the molar mass of air?

1. Calc. moles of air.

   V = 1.00 L   P = 1.00 atm   T = 288 K

   n = PV/RT = 0.0423 mol

2. Calc. molar mass

   mass/mol = 1.23 g/0.0423 mol = 29.1 g/

Molar Volume

Goal 6

Calculate the molar volume of any gas at any given temperature and pressure.

Goal 7

Given the molar volume of a gas at any specified temperature or pressure, or information from which molar volume may be determined, and eitehr the number of

Molar Volume

Molar Volume of a Gas

The volume occupied by one mole of gas molecules.

In symbols, $MV \equiv \dfrac{V}{n}$

$$PV = nRT$$

$$V = \frac{nRT}{P}$$

$$\frac{V}{n} = \frac{RT}{P} = MV(\text{molar volume})$$

Molar Volume

At the common reference conditions of standard temperature (0°C, 273 K) and pressure (1 atm),

$$MV = \frac{V}{n} = \frac{RT}{P} = \frac{0.0821\ L \times atm}{mol \times K} \times \frac{273\ K}{1\ atm} = 22.4\ L/mol$$

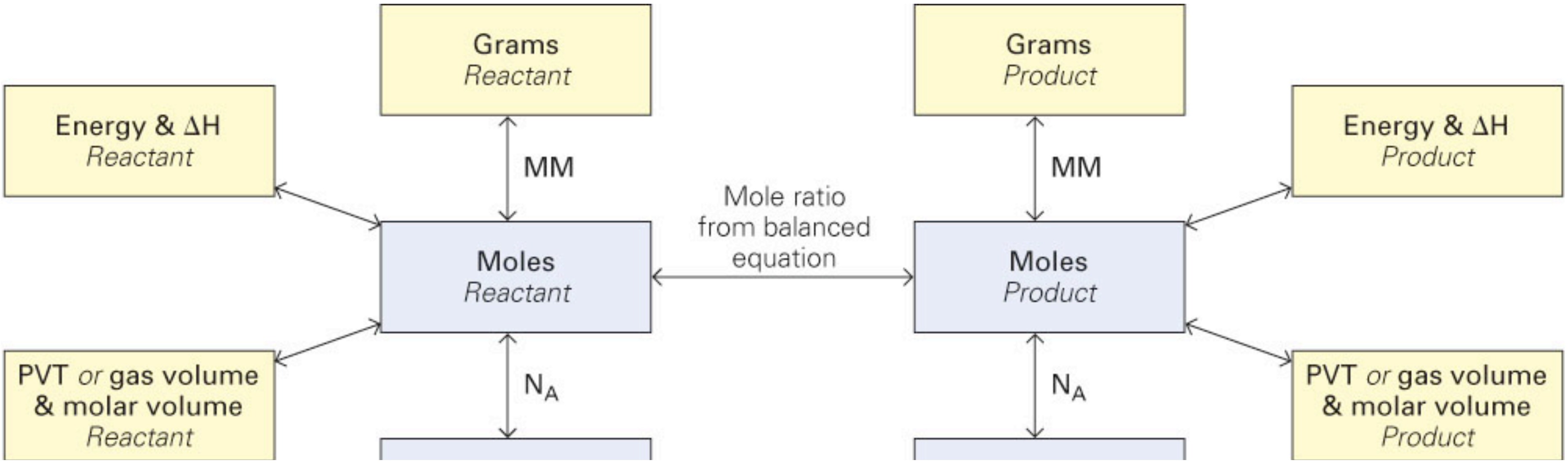
The STP molar volume of a gas is 22.4 L/

Gas Stoichiometry at STP

Goal 8

Given a chemical equation, or a reaction for which the equation can be written, and the mass or number of moles of one species in the reaction, or the STP volume of a gaseous species, find the mass or number of moles of another species, or the STP volume of another

Gas Stoichiometry at STP



Gas Stoichiometry at STP

STP gas stoichiometry is based on the fact that the

molar volume of an ideal gas at STP is 22.4 L/

Gas Stoichiometry at STP

Molar volume (L/mol) and molar mass (g/mol) are similar.

However, they differ in two important respects:

The molar mass of a substance is constant, independent of temperature and pressure. By contrast, the molar volume of a gas is variable, depending on temperature and pressure.

Each substance has its own unique molar mass.

Gas Stoichiometry at STP

Example:

What volume of hydrogen, measured at STP, is released when a 1.44 g chip of solid calcium is added to a hydrochloric acid solution?

Solution:

GIVEN: 1.44 g Ca   WANTED: volume H_2 (assume L)

$Ca + 2\ HCl \rightarrow H_2 + CaCl_2$

$$1.44\text{ g Ca} \times \frac{1\text{ mol Ca}}{40.08\text{ g Ca}} \times \frac{1\text{ mol }H_2}{1\text{ mol Ca}} \times \frac{22.4\text{ L }H_2}{\text{mol }H_2} = 0.805\text{ L }H_2$$

Stoich: Ideal Gas Eqn Method

Goal 9

Given a chemical equation, or a reaction for which the equation can be written, and the mass or number of moles of one species in the reaction, or the volume of any gaseous species at a given temperature and pressure, find the mass or number of moles of any other species, or the volume of any other gaseous species at a given temperature and pressure.

Stoich: Ideal Gas Eqn Method

Procedure

Solving a Gas Stoichiometry Problem

Ideal Gas Equation Method

Volume Given

(want g or mol)

1. Use the ideal gas equation to change given volume to moles: $n = PV/RT$.
2. Use the result in Step 1 to calculate the wanted quantity (such as grams) using Steps 2 and 3 of the stoichiometry path.

Stoich: Ideal Gas Eqn Method

Procedure

Solving a Gas Stoichiometry Problem

Ideal Gas Equation Method

Volume Wanted

(start with g or mol)

1. Calculate moles of wanted substance using Steps 1 and 2 of the stoichiometry path.
2. Use the ideal gas equation to change moles calculated above to volume: $V = nRT/P$.

Stoich: Ideal Gas Eqn Method

Example:

What volume of CO_2, measured at 22°C and 755 torr, is produced when 10.0 g CH_4 is burned completely?

Solution:

Step 1 is to calculate the moles of the wanted substance using the first two steps in the stoichiometry path.

$CH_4 + 2\ O_2 \rightarrow CO_2 + 2\ H_2O$

GIVEN: 10.0 g CH_4 WANTED: mol CO_2

$$10.0\ \text{g}\ CH_4 \times \frac{1\ \text{mol}\ CH_4}{16.04\ \text{g}\ CH_4} \times \frac{1\ \text{mol}\ CO_2}{1\ \text{mol}\ CH_4} = 0.623\ \text{mol}\ CO_2$$

Stoich: Ideal Gas Eqn Method

The second step is to use the ideal gas equation to change moles from the first step to volume.

$PV = nRT$

$$V = \frac{nRT}{P} = \frac{(0.623\ \text{mol}\ CO_2)\,(.0821\ \text{L} \times \text{atm/mol-K})(295\ \text{K})}{\dfrac{755\ \text{torr}}{760\ \text{torr/atm}}}$$

$$= 15.2\ \text{L}\ CO_2$$

Vol–Vol Gas Stoichiometry

Goal 10

Given a chemical equation, or a reaction for which the equation can be written, and the volume of any gaseous species at a given temperature and pressure, find the volume of any other gaseous species at a given temperature and pressure.

Vol-Vol Gas Stoichiometry

Volume-Volume Gas Stoichiometry

$V \propto n$ (at constant T and P)

The ratio of volumes of gases in a reaction is the same as the ratio of moles, provided that the gas volumes are measured at the same temperature

Vol-Vol Gas Stoichiometry

Example:

Hydrogen and nitrogen gases react to form gaseous ammonia. How many liters of hydrogen are required to react with 5.5 L of nitrogen? Both gases are measured at STP.

Solution:

Since both gases are at the same temperature and pressure, the mole ratio in the balanced equation is also a volume ratio.

$3\ H_2 + N_2 \rightarrow 2\ NH_3$

$$5.5\ \text{L}\ N_2 \times \frac{3\ \text{L}\ H_2}{1\ \text{L}\ N_2} = 17\ \text{L}\ H_2$$

Vol-Vol Gas Stoichiometry

Example:

Hydrogen and nitrogen gases react to form gaseous ammonia. How many liters of hydrogen, measured at 26°C and 0.977 atm, are required to react with 5.5 L of nitrogen, measured at -11°C and 2.49 atm?

Solution:

Change the given temperature and pressure to the wanted temperature and pressure for the gas given, and then solve by using the volume

Vol–Vol Gas Stoichiometry

	Volume	Temperature	Pressure
Initial Value (1)	5.5 L	–11°C; 262 K	2.49 atm
Final Value (2)	V_2	26°C; 299 K	0.977 atm

Vol-Vol Gas Stoichiometry

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{P_1V_1T_2}{P_2T_1} = V_1 \times \frac{P_1}{P_2} \times \frac{T_2}{T_1} =$$

$$5.5\ \mathrm{L} \times \frac{2.49\ \mathrm{atm}}{0.977\ \mathrm{atm}} \times \frac{299\ \mathrm{K}}{262\ \mathrm{K}} = 16\ \mathrm{L}\ N_2$$

Vol-Vol Gas Stoichiometry

GIVEN: 16 L N_2 (at 26°C and 0.977 atm)

WANTED: L H_2 (at 26°C and 0.977 atm)

$3\ H_2 + N_2 \rightarrow 2\ NH_3$

3 L H_2/1 L N_2

$$16\ \mathrm{L}\ N_2 \times \frac{3\ \mathrm{L}\ H_2}{1\ \mathrm{L}\ N_2} = 48\ \mathrm{L}\ H_2$$