Reasoning about Reasoning
An overview of my research on Epistemic Specifications

Patrick Kahl
Texas Tech University

March 5, 2013
What do I mean by “Reasoning about Reasoning”? Let’s start with a definition for reasoning:

So what I mean is finding an answer to a problem by considering various possible solutions where the problem itself includes some introspective reasoning about judgements formed by logic process.
Introspective reasoning implies some sort of self-examination, or auto-*epistemic* reasoning w.r.t. knowledge representation.
1988 — “The Stable Model Semantics for Logic Programming” (Michael Gelfond/Vladimir Lifschitz)
1991 — “Classical Negation in Logic Programs and Disjunctive Databases” (Michael Gelfond/Vladimir Lifschitz)
Brief History of ASP: Solvers

1997 — Smodels/Lparse (Ilkka Niemelä/Patrik Simons/Tommi Syrjänen, Helsinki University of Technology)
1997 — DLV (Wolfgang Faber/Georg Gottlob/Nicola Leone/Gerald Pfeifer, Technical University of Vienna; Thomas Eiter, University of Giessen; Simona Perri/Francesco Scarcello, University of Calabria)
2003 — NoMoRe (Thomas Linke, Potsdam University)
2003 — Cmodels (Yuliya Lierler/Vladimir Lifschitz, UT Austin)
2004 — ASSAT (Fangzhen Lin/Yuting Zhao, Hong Kong University of Science and Technology)
2007 — Potassco [clasp, etc.] (Martin Gebser/Roland Kaminski/Benjamin Kaufmann/André Neumann/Torsten Schaub, Potsdam University)
2009 — ASPeRiX (Claire Lefèvre/Pascal Nicolas/Stéphane Ngoma, University of Angers)
In 1991, Michael Gelfond proposed an extension to ASP called *Epistemic Specifications*, adding modal operators K and M. Used before a literal, K and M form *subjective literals*.

- \( K \ell \) — “\( \ell \) is known to be true”
- \( M \ell \) — “\( \ell \) is possibly true”

A program in this language is called an *epistemic specification*. 
An epistemic specification $\Pi$ defines collections of *belief sets* called *world views* of $\Pi$. If $W$ is a world view of $\Pi$, then:

- $K \ell$ is true in $W$ if $\ell$ belongs to every belief set of $W$; and
- $M \ell$ is true in $W$ if $\ell$ belongs to at least one belief set of $W$.

This program defines world view $W = \{\{p, q, s, t\}, \{p, r, s, t\}\}$. 

\[
\begin{align*}
\text{p.} & \quad \text{q or r.} \\
\text{s $\leftarrow K p.$} & \quad \text{t $\leftarrow M q.$}
\end{align*}
\]
The need for extending ASP is illustrated by a simple example.

Suppose *Scholarship Eligibility* is expressed by the ASP rules:

\[
\begin{align*}
\text{eligible}(X) & \leftarrow \text{highGPA}(X). \\
\text{eligible}(X) & \leftarrow \text{minority}(X), \text{fairGPA}(X). \\
\neg\text{eligible}(X) & \leftarrow \neg\text{fairGPA}(X), \neg\text{highGPA}(X).
\end{align*}
\]

along with the following special rule:

*Students whose eligibility is not determined by the above rules should be interviewed by the scholarship committee.*

Problem: How do we represent this last rule?
Let’s try the following ASP program:

\% eligibility rules
eligible(X) ← highGPA(X).
eligible(X) ← minority(X), fairGPA(X).
¬eligible(X) ← ¬fairGPA(X), ¬highGPA(X).

\% interview rule (first attempt)
interview(X) ← not eligible(X), not ¬eligible(X).

Now suppose the data includes the following:

fairGPA(mike) or highGPA(mike).

Unfortunately, this program does not entail interview(mike).
Let’s try again using Epistemic Specifications:

% eligibility rules
eligible(X) ← highGPA(X).
eligible(X) ← minority(X), fairGPA(X).
¬eligible(X) ← ¬fairGPA(X), ¬highGPA(X).

% interview rule (using subjective literals)
interview(X) ← ¬K eligible(X), ¬K ¬eligible(X).

Now add the data:

fairGPA(mike) or highGPA(mike).

This new program does entail interview(mike).
If $\Pi$ is a ground epistemic specification and $W$ is a non-empty collection of sets of ground literals in the language of $\Pi$, then:

- $K \, \ell$ is satisfied with respect to $W$ iff $\forall S \in W : \ell \in S$;
- $\neg K \, \ell$ is satisfied with respect to $W$ iff $\exists S \in W : \ell \notin S$;
- $M \, \ell$ is satisfied with respect to $W$ iff $\exists S \in W : \ell \in S$; and
- $\neg M \, \ell$ is satisfied with respect to $W$ iff $\forall S \in W : \ell \notin S$. 
**Definition**

Let $\Pi$ be an epistemic specification and $W$ be a collection of sets of ground literals in the language of $\Pi$. By $\Pi^W$ — called the *epistemic reduct* of $\Pi$ with respect to $W$ — we denote the ASP program obtained from $\Pi$ by:

1. removing all rules containing a subjective literal that is not satisfied by $W$; and then
2. removing all remaining occurrences of subjective literals.

$W$ is a *world view* if $W$ is the collection of answer sets of $\Pi^W$. 
Under the old definition, recursion through operators $K$ and $M$ could sometimes lead to unintended world views.

Consider the following epistemic specification:

$$\Pi = \{ p \leftarrow K \ p. \}$$

"believe $p$ if $p$ is known to be true"

and two collections of sets of ground literals:

$$W_1 = \{ \{ \} \}$$

$$W_2 = \{ \{ p \} \}$$

The associated epistemic reducts for $\Pi$ are:

$$\Pi^{W_1} = \{ \}$$

(the empty program)

$$\Pi^{W_2} = \{ p. \}$$

$W_1$ and $W_2$ are both world views of $\Pi$, but intuitive support for belief in $p$ is missing from $\Pi - W_2$ is an unintended world view!
Under the old definition, recursion through operators $K$ and $M$ could sometimes lead to unintended world views.

Consider the following epistemic specification:

$$\Pi = \{ p \leftarrow M p. \}$$ ("believe $p$ if $p$ is possibly true")

and two collections of sets of ground literals:

$$W_1 = \{ \{ \} \}$$

$$W_2 = \{ \{ p \} \}$$

The associated epistemic reducts for $\Pi$ are:

$$\Pi^{W_1} = \{ \}$$ (the empty program)

$$\Pi^{W_2} = \{ p. \}$$

$W_1$ and $W_2$ are both world views of $\Pi$, but intuitive support for belief in $p$ is missing from $\Pi - W_2$ is an unintended world view!
Definition

Let $\Pi$ be an epistemic specification and $W$ be a non-empty collection of sets of ground literals in the language of $\Pi$. By $\Pi^W$ we denote the ASP program obtained from $\Pi$ by:

1. removing rules with a subjective literal not satisfied by $W$;
2. removing subjective literals of the form $\neg K \ell$ or $\neg M \ell$;
3. replacing subjective literals of the form $K \ell$ with $\ell$; and
4. replacing each rule $r$ containing a subjective literal of the form $M \ell$ with two rules, $r_1$ and $r_2$ (based on $r$), where:
   a. $M \ell$ is replaced with $\ell$ in $r_1$; and
   b. $M \ell$ is replaced with $\text{not } \ell$ in $r_2$.

$W$ is a world view if $W$ is the collection of answer sets of $\Pi^W$. 
Consider again the following epistemic specification:

\[ \Pi = \{ p \leftarrow K \ p. \} \]

and two collections of sets of ground literals:

\[ W_1 = \{ \} \]
\[ W_2 = \{ \{ p \} \} \]

As before:

\[ \Pi^{W_1} = \{ \} \text{ (the empty program)} \]

but now:

\[ \Pi^{W_2} = \{ p \leftarrow p. \} \]

\( W_1 \) is the only world view for \( \Pi \) — the problem is fixed for \( K \).
Now consider the “M” version of the program:

\[ \Pi = \{ p \leftarrow M \ p. \} \]

and two collections of sets of ground literals:

\[ W_1 = \{ \{ \} \} \]
\[ W_2 = \{ \{ p \} \} \]

As before:

\[ \Pi^{W_1} = \{ \} \text{ (the empty program)} \]

but now:

\[ \Pi^{W_2} = \{ p \leftarrow p., p \leftarrow \text{not } p. \} \]

\( W_1 \) is the only world view for \( \Pi \) — the problem is fixed for M.
Epistemic Specification Solvers

1994 — ELMO (Richard Watson, UTEP)
- Implemented using Prolog
- No functional terms
- Limited to stratified programs

2007 — Wviews (Yan Zhang, University of Western Sydney)
- Guesses subjective literal values to create ASP programs
- Uses DLV to verify world views
- Fails to generate correct world views in some cases

2012 — ESparser & ESmodels (Zhizheng Zhang/Rongcun Cui/Kaikai Zhao, Southeast University, Nanjing, China)
- Uses modified version of claspD to verify world views
- Tests indicate grounder has problem
Outline of New Algorithm to Solve Epistemic Specification

- **Input:** epistemic specification
  - **Conversion:** ASP program created from input
    - **Computation:** ASP solver used to find answer sets of conversion
      - **Analysis:** answer sets grouped into potential world views
        - **Verification:** ASP solver used to verify potential world views
          - **Output:** world views
The new definition models our introspective reasoning better than the old one. It is clear that certain unintended world views are now avoided; however, further study is needed to see if the new definition is adequate and able to model our intuition.

Future Work:

- Good definition of support.
- Conditions for existence and uniqueness of world view, proof of relationship to old definitions.
- Proof of soundness & completeness of algorithm, complexity analysis of algorithm, implementation proof-of-concept.
- Applications, relationship to probabilistic reasoning, etc.
Questions? Comments?

