New Definition of Epistemic Specifications

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Twenty years ago in a long technical report I expanded the language of ASP by modal operators:

\[ Kp \] — \( p \) is known to be true

\[ Mp \] — \( p \) may be believed to be true.

Theories in the new language were called *epistemic specifications*.

A typical epistemic specification, e.g.

\[ q(X) \leftarrow Kp(X) \]

\[ p(a) \]

\[ p(b) \text{ or } p(c) \]

defined a collection of *belief sets*:

\[ W = \{\{p(a), p(b), q(a)\}, \{p(a), p(c), q(a)\}\} \]
I illustrated the need for a new language by a simple example: Scholarship Eligibility is expressed by rules:

1. \( \text{eligible}(X) \leftarrow \text{highGPA}(X) \)
2. \( \text{eligible}(X) \leftarrow \text{minority}(X), \text{fairGPA}(X) \)
3. \( \neg \text{eligible}(X) \leftarrow \neg \text{fairGPA}(X), \neg \text{highGPA}(X) \)

and the statement: *The students whose eligibility is not determined by the college rules should be interviewed by the scholarship committee.*

Problem: how to represent this rule?
First (Unsuccessful) Attempt

1. $\text{eligible}(X) \leftarrow \text{highGPA}(X)$.  
2. $\text{eligible}(X) \leftarrow \text{minority}(X), \text{fairGPA}(X)$.  
3. $\neg\text{eligible}(X) \leftarrow \neg\text{fairGPA}(X), \neg\text{highGPA}(X)$.  
4. $\text{interview}(X) \leftarrow \neg\text{eligible}(X), \neg\neg\text{eligible}(X)$.

$\text{fairGPA}(\text{ann})$.  
$\neg\text{highGPA}(\text{ann})$.  
$\text{fairGPA}(\text{mike})$ or $\text{highGPA}(\text{mike})$.

The program answers yes to  
$\text{interview}(\text{ann})$  
but unknown to  
$\text{interview}(\text{mike})$.
Eligibility Rules in the New Language

1. \text{eligible}(X) \leftarrow \text{highGPA}(X).

2. \text{eligible}(X) \leftarrow \text{minority}(X), \text{fairGPA}(X).

3. \neg \text{eligible}(X) \leftarrow \neg \text{fairGPA}(X), \neg \text{highGPA}(X).

4. \text{interview}(X) \leftarrow \neg K \text{ eligible}(X),
   \neg K \neg \text{ eligible}(X)

used together with

\text{fairGPA}(ann).

\neg \text{highGPA}(ann).

\text{fairGPA}(mike) \text{ or } \text{highGPA}(mike).

entails that both, Mike and Ann, should be interviewed.
A collection $S$ of sets of ground literals entails $Kp$ if $p$ belongs to every set from $S$.

**Definition**

Let $T$ be an epistemic specification and $S$ be a collection of sets of ground literals in the language of $T$. By $T^S$ we will denote the disjunctive logic program obtained from $T$ by:

1. removing all rules containing subjective literals not entailed by $S$.
2. removing all other occurrences of subjective literals.

A set $S$ is called a *world view* of $T$ if $S$ is the collection of all answer sets of $T^S$. 
Example

A specification:

\[ p(X) \leftarrow K \ q(X). \]
\[ q(a) \text{ or } q(b). \]
\[ q(c). \]

has the worldview

\[ W = \{\{q(c), q(a), p(c)\}, \{q(c), q(b), p(c)\}\} \]

The reduct is:

\[ p(c). \]
\[ q(a) \text{ or } q(b). \]
\[ q(c). \]
Unfortunately recursion through operator K leads to unintended world views. Consider a specification $\Pi$

$$p \leftarrow K \ p$$

and two sets:

$$W_1 = \{\{} \}$$
$$W_2 = \{\{p\}\}.$$  

The reducts are

$$\Pi^{W_1} = \emptyset$$
$$\Pi^{W_2} = \{p.\}$$

Both, $W_1$ and $W_2$ are world views of $\Pi$. But belief in $p$ in $W_2$ is not supported — $W_2$ is unintended world view!
The New Definition: Syntax

Notation:

- $p, \neg p$ — objective literals.
- $K l, M l, \neg K l, \neg M l$ where $l$ is an objective literal possibly preceded by default negation $not$ — subjective literals.
- $M l =_{\text{def}} \neg K \ not \ l$

**Definition**

*Epistemic specification* is a collection of rules of the form:

$$l_1 \ or \ ... \ or \ l_k \leftarrow g_{k+1}, ..., g_m, \ not \ l_{m+1}, ..., \ not \ l_n \quad (1)$$

where the $l$’s are objective literals and the $g$’s are subjective or objective literals.

Programs with variables are shorthands for their ground instantiations.
Definition

Let $T$ be an epistemic specification and $S$ be a collection of sets of ground literals in the language of $T$. By $T^S$ we will denote the disjunctive logic program obtained from $T$ by:

1. removing all rules containing subjective literals not entailed by $S$.
2. removing all other occurrences of subjective literals of the form $\neg K \bot$,
3. replacing remaining occurrences of literals of the form $K \bot$ by $\bot$.

A set $S$ is called a world view of $T$ if $S$ is the collection of all answer sets of $T^S$. 
Consider again specification $\Pi$:

$$p \leftarrow K p$$

and two sets:

$$W_1 = \{\}$$
$$W_2 = \{\{p\}\}.$$ 

As before

$$\Pi^{W_1} = \emptyset$$

But

$$\Pi^{W_2} = \{p \leftarrow p\}$$

$W_1$ is the only world view of $\Pi$!
The new logic may allow a more adequate formalization of the closed world assumption: *if p is not known to be true it is false*. Consider a statement

\[ p \lor q \]

together with CWA for p. Clearly, p is not known to be true, hence it is false, and the agent beliefs are

\[ S = \{\neg p, q\} \]

An ASP representation of CWA does not produce this answer. However if we write CWA as

\[ \neg p \leftarrow \neg K p \]

The resulting program defines the expected world view, S.
Another version of CWA can be formulated as: *if p may not be believed then it is false.*

This can be written as

\[ \neg p \leftarrow \neg M p \]

or equivalently as

\[ \neg p \leftarrow K \neg \text{not } p \]

This, together with

\[ p \text{ or } q \]

has the world view,

\[ A = \{ \{ p \}, \{ q \} \} \]

The old definition gives another, unintended world view

\[ B = \{ \{ q, \neg p \} \}. \]
A specification

\[ p \leftarrow \neg M q \]
\[ q \leftarrow \neg M p \]

is equivalent to

\[ p \leftarrow K \text{not} q \]
\[ q \leftarrow K \text{not} p \]

World views:

\[ S_1 = \{\{p\}\} \]
\[ S_2 = \{\{q\}\} \]
\[ S_3 = \{\{p\}, \{q\}\} \]
A specification

\[ p \leftarrow \neg Kp \]

as expected, has no world views.

A specification

\[ p \leftarrow Mp \]

has two world views:

\[ S_1 = \{\} \]
\[ S_2 = \{p\} \]
The new definition seems to better model our introspective reasoning than the old one. Further study is needed to see if this model is fully adequate.

Questions:

• Conditions for existence and uniqueness of world view, and other mathematical properties.

• Algorithms.

• Good definition of supportedness.

• Applications, e.g. to conformant planning, probabilistic reasoning, etc.