Nonmonotonic causal theories

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Outline

• Introduction
• Syntax and Semantics
• Knowledge Representation
• C+ Language
• Theorems
Introduction

This paper
• Defines nonmonotonic causal theories (NCT)
• Investigate “definite” NCT
• Shows how they can be used for knowledge representation
• Introduces action language C+ based on NCT
• Compares NCT with related works
Syntax of Causal theory

• Multi-valued propositional signature: set of constants, \( c \), and their finite domains \( \text{Dom}(c) \)

Note: \( i : c \), \( i \) is nonnegative integer and \( c \) is constant.

• Atom: \( c = v \) where \( v \) is in \( \text{Dom}(c) \)

• Formula: a propositional combination of atoms. e.g. \( \text{Loc}_a = \text{table} \land \text{Loc}_b = \text{table} \)
Causal Rules

• **Causal rule:**
  
  \[ F \leftarrow G, \text{ where } F \text{ and } G \text{ are formulas.} \]

  **Causal theory:** a set of causal rules.
Semantics
(Informal)

• $F \leftarrow G$: if $G$ is true then there is a cause for $F$ to be true.

Note that $F \leftarrow \top$ means that $F$ is caused to be true while $\bot \leftarrow \neg F$ says that $F$ is true.

• $F$ is true in causal theory $T$ if and only if $F$ is caused by $T$
Examples

• Let c, d be boolean constants

• $T_1 = \{ c = t \iff \top. \ d = t \iff c = t. \}$

  *Does $T_1$ entail $d = t$? Yes.*

• $T_2 = \{ \bot \iff \neg c = t. \ d = t \iff c = t. \}$

  *Does $T_2$ entail $d = t$? No.*
Semantics

• An *interpretation* is a function that maps constants to elements of their domains.

• An interpretation $I$ *satisfies* an atom $c = v$ (symbolically, $I \models c=v$) if $I(c) = v$.

• A *model* of a set of formulas is an interpretation that satisfies all formulas.
Semantics

- $T^I$: The reduct $T^I$ of $T$ relative to $I$ is the set of the heads of all rules in $T$ whose bodies are satisfied by $I$.
- $I$ is a model of $T$ if $I$ is the unique model of $T^I$. 
Examples

1. \( \sigma = \{ c \}, \text{Dom}(c) = \{ 1, 2, 3 \} \)
\( T_1: c = 1 \iff c = 1 \)
\( I(c) = 1; T^l = \{ c = 1 \}. \)

2. \( \sigma = \{ c, d \}, \text{Dom}(c) = \{ 1, 2, 3 \}; \text{Dom}(d) = \{ t, f \} \)
\( T_2: c = 1 \iff c = 1 \)
\( I(c) = 1; I(d) = t; T^l = \{ c = 1 \} \) note: \( I \) is not the unique model of \( T^l \)

3. \( \sigma = \{ c \}, \text{Dom}(c) = \{ 1, 2, 3 \} \)
\( T_3: c = 1 \iff c = 1, c = 2 \iff t. \)
\( I(c) = 2; T^l = \{ c = 2 \}. \)

4. \( \sigma = \{ c \}, \text{Dom}(c) = \{ 1, 2, 3 \} \)
\( T_4: c = 1 \iff c = 1, c = 2 \iff c = 2. \)
\( a. \quad I(c) = 1; T^l = \{ c = 1 \}. \)
\( b. \quad I(c) = 2; T^l = \{ c = 2 \}. \)
Definite theories

- Definition:
  A causal theory $T$ is definite if
  - The head of every rule of $T$ is an atom or $\bot$
  - No atom is the head of infinitely many rules of $T$
Completion Process

• There is a completion process that reduces the problem of finding a model of a definite causal theory to the problem of finding a model of a set of formulas.
Completion Process

• For each atom A whose domain is not a singleton, add formula
  \[ A \equiv G_1 \lor \ldots \lor G_n \] where \( G_1, \ldots, G_n \) are the bodies of the rules of T with head A.

  (Do nothing for singleton formula)

  if \( n=0 \), add formula \( A \equiv \bot \).

• For each constraint \( \bot \leftarrow F \), add formula \( \neg F \).
Example of Completion

• Causal Theory T:

\[ \sigma = \{ c, d \}, \ \text{Dom}(c) = \{ 1, \ldots, n \} \]

\[ \text{Dom}(d) = \{ t, f \} \]

\[ c=1 \iff c=1 \]
\[ c=2 \iff c=2 \]
\[ c=1 \iff d=t \]
\[ c=3 \iff \top \]
\[ d=t \iff d=t \]

\[ c=1 \equiv c=1 \lor d=t \]
\[ c=2 \equiv c=2 \]
\[ c=3 \equiv \top \]
\[ c=v \equiv \bot \ (v \in \text{Dom}(c) \backslash \{1,2,3\}) \]
\[ d=t \equiv d=t \]
\[ d=f \equiv \bot \]
Theorem

- The models of a definite causal theory are precisely the models of its completion.
Knowledge Representation

• Default reasoning
• Actions and changes
• Example: monkey and bananas
Default

- \( c=v \iff c=v \) can be used to express the default knowledge. Example:

\[ \sigma = \{ c \}, \text{Dom}(c) = \{1, \ldots, n\} \]

1. \( T_1: c = v_1 \iff c = v_1 \)

\[ T_1 \vdash c = v_1 \]

2. \( T_2: c = v_1 \iff c = v_1. \ c = v_2 \iff \top. \)

\[ T_2 \vdash c = v_2; T_2 \vdash c \neq v_1 \]
Action and changes

1: \( p \leftarrow 0 : a \)

0: \( p \leftarrow 0 : p \).

0: \( \neg p \leftarrow 0 : \neg p \)

0: \( a \leftarrow 0 : a \).

0: \( \neg a \leftarrow 0 : \neg a \)

1: \( p \leftarrow (0 : p) \land (1 : p) \)

1: \( \neg p \leftarrow (0 : \neg p) \land (1 : \neg p) \)
The Models of the example

• \( M_1 = \{ 0:p=t, 0:a=t, 1:p=t \} \)
• \( M_2 = \{ 0:p=t, 0:a=f, 1:p=t \} \)
• \( M_3 = \{ 0:p=f, 0:a=t, 1:p=t \} \)
• \( M_4 = \{ 0:p=f, 0:a=f, 1:p=f \} \)

If we use \( i \) and \( i+1 \) instead of 0 and 1, for the length \( m \) (\( i < m \)), we will have \( 2^{m+1} \) models.
Monkey and Banana (MB)

- **Signature:**
  \[ i : \text{Loc}(x), \text{where } x \in \{ \text{Monkey, Bananas, Box} \} \]
  \[ i : \text{HasBananas}, i : \text{OnBox} \]
  \[ i : \text{walk}, i : \text{climbOn} \]
  \[ i : \text{PushBox}(l), \text{where } l \in \{ L1, L2, L3 \} \]
  \[ i : \text{climbOff}, i : \text{GraspBananas} \]
• $i + 1 : \text{Loc(Monkey)} = l \leftarrow i : \text{Walk}(l)$.
• $i + 1 : \text{HasBananas} \leftarrow i : \text{GraspBananas}$.
• $\perp \leftarrow i : (\text{GraspBananas} \land \neg \text{OnBox})$.
• $\perp \leftarrow i : (\text{GraspBananas} \land \text{Loc(Monkey)} \neq \text{Loc(Banans)})$.
• $i + 1 : \text{Loc(Box)} = l \leftarrow i : \text{PushBox}(l)$.
• $i + 1 : \text{Loc(Monkey)} = l \leftarrow i : \text{PushBox}(l)$.
• ........
Reasoning and planning

• Prediction:

Given the complete MB theory, we know, initially the monkey is at L₁, the bananas are at L₂, and the box is at L₃. The monkey walks to L₃ then pushes the box to L₂. Are the bananas and the box at the same location?

\[ ((0:\text{Loc}(\text{monkey})=L_1 \land (0: \text{Loc}(\text{Bananas})=L_2) \land (0: \text{Loc}(\text{Box})=L_3) \land (0: \text{Walk}(L_3)) \land (1: \text{PushBox}(L_2)) \rightarrow 2: (\text{Loc}(\text{Monkey})=\text{Loc}(\text{Bananas}) \land \text{Loc}(\text{Bananas})=\text{Loc}(\text{Box})). \]
Reasoning and planning

• Postdiction

The monkey walked to location $L_3$ and pushed the box. Does it follow that the box was initially at $L_3$

$$[(0: \text{Walk}(L_3) \land (1: \forall \text{PushBox}(l))] \rightarrow 0: \text{Loc}(\text{Box})=L_3$$
Reasoning and planning

• Planning

Initial states:
0:Loc(Monkey) = L₁, 0:Loc(Bananas) = L₂, 0:Loc(Box) = L₃

• Goal:

m:HasBananas

• The shortest plan:

0:Walk(L₃), 1:PushBox(L₂), 2:ClimbOn, 3:GraspBananas.
Definitions of C+

Static determined fluent: only appears at the head of static causal laws
Fluent formula: all constants occurring in it are fluent constants.
Action formula: at least one action constant and no fluent constants.
Syntax in C+

- Static laws: *caused F if G*
- Action dynamic law: *caused A if H*
- Fluent dynamic law: *caused F if G after H*
- *exogenous c*
- *inertial p*

where *F and G are fluent formulas, A is an action formula, H is a formula, c is an action constant, p is a simple fluent constant*
Translation

• caused $F$ if $G$  $\rightarrow$  $i:F \leftarrow i:G$

• caused $F$ if $G$ after $H$  $\rightarrow$  $i+1:F \leftarrow i+1:H \wedge i:G$

• exogenous $c$  $\rightarrow$  $i:c=v \leftarrow i:c =v$

• inertial $p$  $\rightarrow$  $i+1:p=v \leftarrow i+1:p=v \wedge i:p=v$

• For every simple fluent constant $c$ and every $v$ in $Dom(c)$. Add  $0:c=v \leftarrow 0 :c=v$
Example of C+

\( a \) causes \( p \)

exogenous \( a \)

inertial \( p \)
Theorems

• An interpretation $I$ is a model of a causal Theory $T$ if and only if, for every formula $F$,

\[ I \models F \iff T^I \models F \]

• If a causal theory $T$ contains a causal rule $F \leftarrow G$ then $T$ entails $G \Rightarrow F$. 
Theorems

• Let $T_1$ and $T_2$ be causal theories of a signature $\sigma$ such that every rule in $T_2$ is a constraint. An interpretation of $\sigma$ is a model of $T_1 \cup T_2$ iff it is a model of $T_1$ and does not violate any of the constraints in $T_2$

• The models of a definite causal theory are precisely the models of its completion.