TOPOLOGY DOCTORAL PRELIMINARY EXAMINATION
MAY 2002

WORK ALL PROBLEMS. ASSUME THAT ALL SPACES UNDER CON-
SIDERATION ARE HAUSDORFF (T_2). GIVE A PRECISE STATEMENT
OF ANY MAJOR THEOREM REFERENCED IN ANY ARGUMENT.

1.) Show that if each of X and Y is regular, then the product $X \times Y$ is
regular. Give an example to show that the product of two normal spaces
need not be normal. Clearly describe the topology of the factor spaces in
your example and indicate why each is normal but the product is not.

2.) Let $f : X \longrightarrow Y$ be a closed, continuous surjection with X locally
connected. Show that Y is also locally connected.

3.) Let X be a compact metric space and $\{U_\alpha\}_{\alpha \in A}$ be an open cover of
X. Show that there exists $\epsilon > 0$ such that if H is any subset of X with
$\operatorname{diam}(H) \leq \epsilon$ then $H \subset U_\alpha$ for some $\alpha \in A$.

4.) State and prove the Baire Category Theorem for complete metric spaces.

5.) Recall that a space X is *second countable* if there exists a countable
collection of open sets which form a basis for the topology of X. Show that
if X is second countable then **every** collection of open sets forming a basis
for the topology of X has a countable subcollection which forms a basis for
the topology on X.

6.) Let $(\tilde{X}, p)$ be a covering space for the space X. Let $f : [0,1] \to X$ be a
path and let b be a point of $\tilde{X}$ such that $p(b) = f(0)$. Show that there exists
a unique path $g : [0,1] \to \tilde{X}$ such that $g(0) = b$ and $f = p \circ g$.

7.) Let X be an arcwise connected space which is the union of the subsets
X_1 and X_2. Show that if each of X_1 and X_2 is an open, simply connected
subset of X and if $X_1 \cap X_2$ is arcwise connected, then X is simply connected.

8.) Prove the Borsuk-Ulam Theorem for $n = 2$: There is no continuous
antipode-preserving function $f : S^2 \to S^1$ from the 2−dimensional sphere to
the unit circle. (A map f between spheres is said to be *antipode-preserving*
if $f(-x) = -f(x)$ for each x.)