Instructions. Solve 14 of the problems 1–16 and write your solutions in a blue book. If you solve more than 14 problems, you must *clearly mark which* 14 *you want to have graded.* For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and PDAs are not.

1. If you start at the corner of 19th St. and Boston Av. in Lubbock, in how many ways can you walk to the corner of Tech Terrace Park at the corner of 23rd St. and Flint Av. if you may only walk West or South (down or left on the map) at each intersection?

2. An island's only inhabitants are penguins and bears. If there are 16 heads and 36 feet on the island, how many penguins and how many bears live on the island?

3. Consider the following preference lists for thirteen voters

$$ABCD : 4 \quad BACD : 4 \quad CDAB : 3 \quad DCAB : 1 \quad DCBA : 1$$

Determine the winner (A, B, C, or D) using three of the following methods:

   (a) Plurality voting

   (b) Borda count

   (c) The Hare method

   (d) Pairwise comparison

4. By using direct, indirect, or transitive reasoning, write down a valid argument. Use at least two hypotheses and state which type of reasoning you are using.

5. Set

$$U = \{1, 2, 3, \ldots, 10\},$$
$$A = \{1, 3, 5, 7, 9\},$$
$$B = \{2, 4, 6, 8, 10\},$$
$$C = \{1, 2, 4, 8, 9\}, \text{ and}$$
$$D = \{1, 2, 3\} \, .$$

(a) Find the complementary set $\overline{A}$ of A with respect to U

(b) List all the subsets of D

(c) Find the cardinality of A

(d) Find the intersection $C \cap D$

(e) Find the complementary set $\overline{A \cap C}$ with respect to U

(f) Find the set $B \times D$

(g) Decide if A and B are disjoint

6. Recall that the definition of *neither p nor q* is $\sim(p \vee q)$. Show that the definition could also be $\sim p \wedge \sim q$.

7. Consider the following graph:
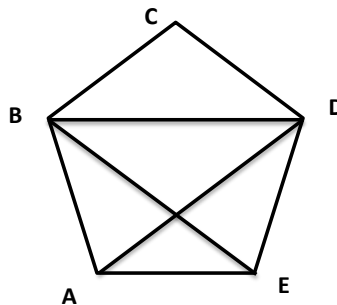
(a) Is this graph traversable? If so, write a path order in which it may be traversed.

(b) Is this graph an Euler Circuit? Explain why or why not.

(c) Does the graph have a Hamiltonian Cycle? If so, give an example.

8. A survey was conducted among 1200 people on which sodas they like. 540 people said they like Dr. Spice. 481 people said they like Mountain Mist. 465 people said they like PC Cola. 180 people like Dr. Spice and Mountain Mist. 210 people like Dr. Spice and PC Cola. 200 people like Mountain Mist and PC Cola. 85 people like all 3 sodas.

(a) Draw the Venn diagram that represents this data

(b) How many people polled do not like any of the choices?

(c) How many people polled like exactly 2 of the 3 choices?

(d) How many people polled only like PC Cola?

9. Batman lists the Batmobile on Craigslist and offers financing in the form of a Batmobile loan through Bruce Wayne Enterprises. The loan is for 15 years and the interest is 5% per year compounded monthly. Given that the monthly payment is $750, how much is Batman asking for the Batmobile, i.e. what is the loan amount?

10. A city is considering linking the irrigation systems of four small farms by building water pipelines to connect them to the city's aquifer. The proposed pipelines, accompanied by their costs in thousands of dollars, are shown below.
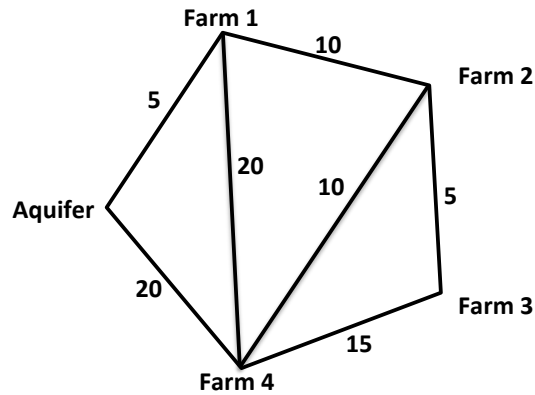

Find the cheapest way that the city can build these pipelines so that each farm receives water. (*Hint:* The pipeline has to have the water flow from the aquifer, so there cannot be any loops in the resulting pipeline system.)

11. The state of Texas has a population of 25 million. The population density of the state of Virginia is approximately two times the population density of Texas, but the area of Texas is circa six times the area of Virginia. What is the approximate population of Virginia?

12. A department of English assigns its TAs to junior level classes based on enrollment using Hamilton's plan. Here are the enrollment data for a semester when 750 students enrolled in the five junior level classes.

Class	Enrollment
Engl-1	64
Engl-2	138
Engl-3	218
Engl-4	188
Engl-5	142
Total	750

(a) There are 30 TAs to be assigned, how many do each class get?

(b) If instead there were 31 TAs to be assigned, how many do each class get? Does a paradox occur?

13. Suppose the half life of a certain radioactive isotope is 59 days. After how many days will only 32% of the original isotope be left.

14. In the Ninja world, the Grand Fireball Technique begins as a small fireball of 50 cm in diameter. It grows exponentially, and after 10 seconds the fireball will have grown to 1000 cm in diameter.

   (a) What is the rate of growth of the fireball in %/second?

   (b) What will the diameter of the fireball be after 20 seconds?

15. In basketball, the area know as "the paint" is a rectangular region with dimensions of 6 feet at the baseline and 12 feet. On top of this is a half circle that extends toward the other team's goal; it has diameter of 6 feet (to coincide with the baseline). The paint plus this half circle are together know as the key.

12 ft.

6 ft.
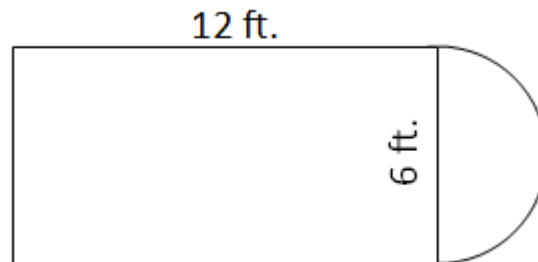
   (a) What is the perimeter of the key?

   (b) What is the area of the key?

16. Bill wishes to save up $350,000 to buy a new house for Ted's birthday in 10 years. So, Bill goes to the bank and opens an account that yields 5% interest compounded monthly. How much money should Bill deposit into the account at the end of every month in order to save up enough money to buy the house for Ted in 10 years?