Math 1300 ©
Final Exam Department of Math and Stat
Fall 2014 Texas Tech University

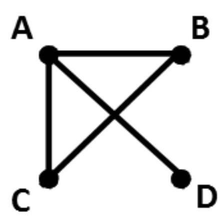
Instructions: Solve 13 of the problems 1–15. If you solve more than 13 problems, you must clearly mark which 13 you want to have graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and PDAs are not.

1. Construct a truth table for the given compound statement.

$$(p \to q) \vee (\sim p \wedge \sim q)$$

2. James has set up an ordinary annuity to save for his retirement. If he wants to have $70,000 in his account after 15 years and the annuity has an annual interest rate of 7.5% compounded monthly, how much should his monthly payments be?

3. Consider the following graph:



   (a) Can the graph be traced? Explain your answer using Euler's theorem.

   (b) If the given graph is Eulerian, find an Euler circuit in it. If it is not Eulerian, first Eulerize it and then find an Euler circuit. Begin the Euler circuit at vertex A.

4. An apartment complex has three buildings. Building A has 120 units, building B has 141 units, and building C has 39 units. An 11-person committee will set rules to govern the complex.

   (a) Apportion this committee based on the number of units per building using Hamilton's method.

   (b) Now increase the number of members in the committee by one and reapportion the committee (again using Hamilton's method).

   (c) Does an apportionment paradox occur? Explain your answer.

5. If you randomly select a single card from a standard 52-card deck, what is the probability that you draw a heart or a face card?

6. Find the mean, median, and mode of the following distribution.

$$12, 11, 7, 9, 8, 6, 6, 4, 5, 10, 1, 2$$

7. Consider the following voting preference table:

	10	5	3	2	1
1^{st}	C	B	A	B	C
2^{nd}	B	A	B	C	A
3^{rd}	A	C	C	A	B

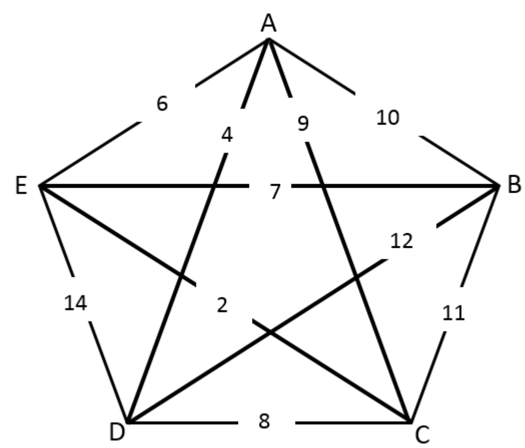
(a) Determine who wins the election according to the plurality method.

(b) Determine who wins the election according to the Borda Count method.

(c) Using the Borda count method, was the Majority Criterion violated? Explain your answer.

8. Alex, Becca, and Carly play on the same soccer team. Last season, they scored a total of 35 goals. If Becca scored three fewer goals than Alex, and Carly scored two more goals than Alex, how many goals did each soccer player score?

9. Use the weighted graph given below to answer the following questions:



(a) How many Hamilton circuits would you have to consider if you planned to use the brute force algorithm to find a path that minimizes the traveling cost?

(b) Use the nearest neighbor algorithm to find a Hamilton circuit beginning at vertex A.

(c) What is the weight of the path found in part b?

10. Determine whether the following syllogism is valid or invalid using Euler diagrams.

Some animals are dangerous.
All lions are animals.

Some lions are dangerous.

11. Use the unpaid balance method to find the finance charge on the credit card account. Last month's balance, the payment, the annual interest rate, and any other transactions are stated below.

    - Last month's balance: \$509
    - Payment: \$208
    - Annual interest rate: 19%
    - Bought shoes: \$89
    - Bought jacket: \$127

12. Determine which state is more poorly represented: State A has a population of 488,895 people and 11 representatives; State B has a population of 325,098 people and 9 representatives.

13. Consider the weighted voting system

$$[\,11 : 3, 4, 5, 10\,]$$

    where the weights represent voters A, B, C, and D respectively.

    (a) List all coalitions, state their weights, and identify the winning coalitions. For each winning coalition, determine the critical voters.

    (b) Compute the Banzhaf power index for each voter.

14. You are playing a game in which a single die is rolled. If you roll a 2 or a 5, you win \$36, otherwise you lose \$36. Is the game fair? Explain your answer.

15. Consider a normal distribution with a mean of 23 and a standard deviation of 3.

    (a) What z-score corresponds to the raw score 26?

    (b) Use the 68-95-99.7 Rule to determine what percentage of values would be below 26.

THE COMPOUND INTEREST FORMULA Assume that an account with principal P is paying an annual interest rate r and compounding is being done m times per year. If the money remains in the account for n time periods, then the future value, A, of the account is given by the formula

$$A = P\left(1 + \frac{r}{m}\right)^n.$$

Notice that in this formula, we have replaced r by $\frac{r}{m}$, which is the annual rate divided by the number of compounding periods per year, and t by n, which is the number of compounding periods.

THE UNPAID BALANCE METHOD FOR COMPUTING THE FINANCE CHARGE ON A CREDIT CARD LOAN This method also uses the simple interest formula $I = Prt$; however,

P = previous month's balance + finance charge + purchases made − returns − payments.

The variable r is the annual interest rate, and $t = \frac{1}{12}$.

FORMULA FOR FINDING THE FUTURE VALUE OF AN ORDINARY ANNUITY Assume that we are making n regular payments, R, into an ordinary annuity. The interest is being compounded m times a year and deposits are made at the end of each compounding period. The future value (or amount), A, of this annuity at the end of the n periods is given by the equation

$$A = R\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}.$$

FORMULA FOR FINDING PAYMENTS ON AN AMORTIZED LOAN Assume that you borrow an amount P, which you will repay by taking out an amortized loan. You will make m periodic payments per year for n total payments and the annual interest rate is r. Then, you can find your payment by solving for R in the equation

$$P\left(1 + \frac{r}{m}\right)^n = R\left(\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right)^*.$$

Method	How the Winning Candidate Is Determined
Plurality	The candidate receiving the most votes wins.
Borda count	Voters rank all candidates by assigning a set number of points to first choice, second choice, third choice, and so on; the candidate with the most points wins.
Plurality-with-elimination	Successive rounds of elections are held, with the candidate receiving the fewest votes being dropped from the ballot each time, until one candidate receives a majority of votes.
Pairwise comparison	Candidates are compared in pairs, with a point being assigned the voters' preference in each pair. (In the case of a tie, each candidate gets a half point.) After all pairs of candidates have been compared, the candidate receiving the most points wins.

HAMILTON'S APPORTIONMENT METHOD

a) Find the standard divisor for the apportionment (total population/total number of representatives).

b) Find the standard quota (state's population/standard divisor) for each state and round it down to its lower quota. Assign that number of representatives to each state.

c) If there are any representatives left over, assign them to states in order according to the size of the fractional parts of the states' standard quotas.

RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS If E and F are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

GENERAL RULE FOR COMPUTING $P(F|E)$ If E and F are events in a sample space, then $P(F|E) = \dfrac{P(E \cap F)}{P(E)}$.

FORMULA FOR CONVERTING RAW SCORES TO z-SCORES Assume a normal distribution has a mean of μ and a standard deviation of σ. We use the equation

$$z = \frac{x - \mu}{\sigma}$$

to convert a value x in the nonstandard distribution to a z-score.