Instructions

Your answers must be entered in your Examination Blue Book; answers on the exam will not be graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, but phones, PDAs, music players, Apple watches, and other electronic devices are not.

Solve problem 1, and solve any 13 of the problems 2–16; they are weighted equally. If you solve more than 13 of the problems 2–16, then mark clearly which ones you want graded, otherwise the first 13 answers to problems 2–16 in your Examination Blue Book will be graded.

Part I

Solve problem 1 and make sure to explain your reasoning.

1. Andrew, Tonya, Katherine, and Zach each have the power to control one of the four elements, air, water, earth, and fire. Using the following clues, determine who can control which element.

   (a) Andrew has a crush on the person who can control water.

   (b) Tonya does not like the person controlling air or the person controlling fire.

   (c) Katherine, who is the youngest, is liked by everyone.

   (d) Zach, who is best friends with the person controlling air, is younger than Tonya and older than the person controlling water.

Part II

Solve 13 of the problems 2–16 below. If you solve more than 13 problems, then mark clearly which ones you want graded, otherwise the first 13 answers in your Examination Blue Book will be graded.

2. Every month, you have a "Treat Yo Self" day, where you spend \$350 on yourself. If, instead, you put this money into an ordinary annuity that yields 7% interest, how much money would you accumulate over the course of 15 years?

3. Consider the weighted voting system

   $$[14\text{: } 4, 6, 13, 5]\,.$$

   (a) List all possible coalitions and determine their weights.

   (b) Identify the winning coalitions, and for each one determine the critical members.

   (c) Compute the Banzhaf Power Index for each voter.

4. Assume that you are rolling two fair dice. What is the probability that the total showing is greater than 10? Leave your answer as a reduced fraction.

5. The Society for Literature, Science, and the Arts is electing a new president; the candidates are Lalith, Malith, and Nalith. The voter preferences are as follows:

	Number of ballots					
Preference	15	18	16	6	4	12
1st	L	L	M	M	N	N
2nd	M	N	N	L	L	M
3rd	N	M	L	N	M	L

   (a) Determine the winner of the election under the *Borda Count Method*.

   (b) Determine the winner of the election under the *Plurality Method*.

6. Find the pattern and then determine the seventh term of each of the following sequences.

   (a) 4, 11, 18, 25, ...

   (b) 6, 12, 24, 48, ...

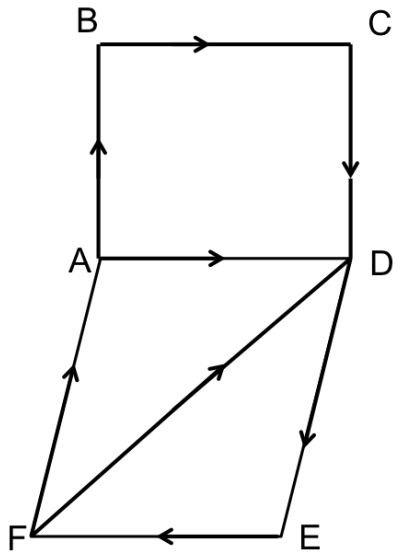
   (c) 2, 9, 28, 65, ...

7. Consider a normal distribution with a mean of 140 and a standard deviation of 14.

   (a) What z-score corresponds to the raw score of 112?

   (b) Using the 68–95–99.7 Rule, what percentage of observations would be below 112?

8. Consider the directed graph:



   (a) Find a directed path from A to E

   (b) Find a directed path of length 6 from A to A

9. Sophie wants to figure out the finance charge on her credit card. Assume that her annual interest rate is 15%, and the unpaid balance at the beginning of last month, May, was $1,000. Use this information and the table below to answer the following questions:

Date	**Transaction**
June 1	Previous balance $1,000
June 5	Payment of $50
June 13	Charged $650 for a TV

(June has 30 days)

   (a) What is the finance charge using the unpaid balance method? Round your answer to the nearest cent.

   (b) What is the finance charge using the average daily balance method? Round your answer to the nearest cent.

10. A district in a small country has three polling divisions. Polling division A has 17,000 people and 8 representatives, polling division B has 31,000 people and 13 representatives, and polling division C has 11,500 people and 5 representatives.

    (a) Calculate the Huntington–Hill number for each polling division.

    (b) Use Huntington–Hill apportionment method to decide which polling division is more deserving of an additional representative.

11. In your Math 1300 class, the ages of the students are as listed:

    $$18, 17, 19, 19, 20, 25, 18, 23, 26, 18, 18, 19, 27$$

    Answer the following questions.

    (a) Find the lower half

    (b) Find the upper half

    (c) Find the first quartile

    (d) Find the third quartile

    (e) Find the five-number summary

    (f) Construct a box-and-whisker plot using parts (a) through (e)

12. Construct a truth table for the following statement

    $$(p \vee \sim q) \iff (\sim q \wedge \sim p)$$

13. You have $\$34,000$ remaining on your student loan and the annual interest rate is 12%. You wish to pay off the loan in 20 years. Assume that you can refinance your loan at 6%. Use the table below to answer the questions below. Round your answers to the nearest cent.

Monthly payments on a $1,000 loan.

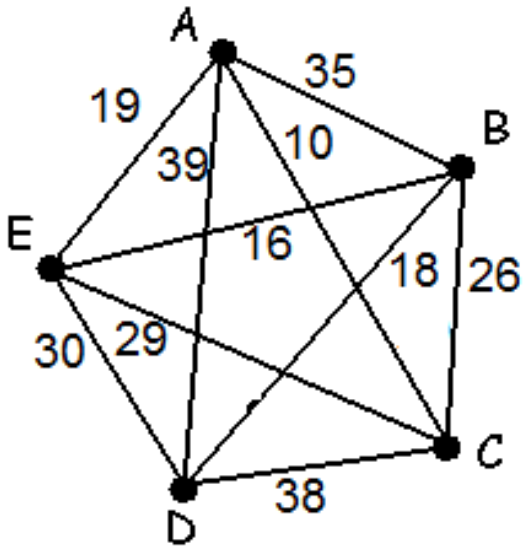
Annual Interest Rate	Number of Years for the Loan				
	3	4	10	20	30
4%	$29.53	$22.58	$10.12	$6.06	$4.77
5%	29.97	23.03	10.61	6.60	5.37
6%	30.42	23.49	11.10	7.16	6.00
8%	31.34	24.41	12.13	8.36	7.34
10%	32.27	25.36	13.22	9.65	8.78
12%	33.21	26.33	14.35	11.01	10.29

    (a) What is the monthly payment on the original loan?

    (b) How much will your new monthly payments be?

    (c) How much interest will you save over the 20 years?

14. Consider the weighted graph



(a) If you were to use the Brute Force Algorithm, how many possible Hamilton circuits would you have to consider?

(b) Use the Nearest Neighbor algorithm to find a Hamilton circuit that begins at vertex E.

(c) What is the weight of the Hamilton circuit found in part (b)?

15. Use an Euler diagram to determine whether the following argument is valid or invalid.

All tall people are clumsy.
All clumsy people own dogs.
Some educators have dogs.

$\therefore$ *Some educators are tall.*

16. If you randomly draw a card from a standard 52-card deck, what is the probability that it is a Spade given that you know it is a black card?

DEFINITION In a weighted voting system, a voter's **Banzhaf power index*** is defined as

$$\frac{\text{the number of times the voter is critical in winning coalitions}}{\text{the total number of times voters are critical in winning coalitions}}.$$

Method	How the Winning Candidate Is Determined
Plurality	The candidate receiving the most votes wins.
Borda count	Voters rank all candidates by assigning a set number of points to first choice, second choice, third choice, and so on; the candidate with the most points wins.
Plurality-with-elimination	Successive rounds of elections are held, with the candidate receiving the fewest votes being dropped from the ballot each time, until one candidate receives a majority of votes.
Pairwise comparison	Candidates are compared in pairs, with a point being assigned the voters' preference in each pair. (In the case of a tie, each candidate gets a half point.) After all pairs of candidates have been compared, the candidate receiving the most points wins.

GENERAL RULE FOR COMPUTING $P(F|E)$ If E and F are events in a sample space, then $P(F|E) = \dfrac{P(E \cap F)}{P(E)}$.

FORMULA FOR FINDING THE FUTURE VALUE OF AN ORDINARY ANNUITY Assume that we are making n regular payments, R, into an ordinary annuity. The interest is being compounded m times a year and deposits are made at the end of each compounding period. The future value (or amount), A, of this annuity at the end of the n periods is given by the equation

$$A = R\frac{\left(1 + \dfrac{r}{m}\right)^n - 1}{\dfrac{r}{m}}.$$

THE UNPAID BALANCE METHOD FOR COMPUTING THE FINANCE CHARGE ON A CREDIT CARD LOAN This method also uses the simple interest formula $I = Prt$; however,

P = previous month's balance + finance charge + purchases made − returns − payments.

The variable r is the annual interest rate, and $t = \frac{1}{12}$.

THE COMPOUND INTEREST FORMULA Assume that an account with principal P is paying an annual interest rate r and compounding is being done m times per year. If the money remains in the account for n time periods, then the future value, A, of the account is given by the formula

$$A = P\left(1 + \frac{r}{m}\right)^n.$$

Notice that in this formula, we have replaced r by $\frac{r}{m}$, which is the annual rate divided by the number of compounding periods per year, and t by n, which is the number of compounding periods.

FORMULA FOR CONVERTING RAW SCORES TO *z*-SCORES Assume a normal distribution has a mean of μ and a standard deviation of σ. We use the equation

$$z = \frac{x - \mu}{\sigma}$$

to convert a value x in the nonstandard distribution to a z-score.

THE AVERAGE DAILY BALANCE METHOD FOR COMPUTING THE FINANCE CHARGE ON A CREDIT CARD LOAN

1. Add the outstanding balance for your account for each day of the month.
2. Divide the total in step 1 by the number of days in the month to find the average daily balance.
3. To find the finance charge, use the formula $I = Prt$, where P is the average daily balance found in step 2, r is the annual interest rate, and t is the number of days in the month divided by 365.

THE HUNTINGTON–HILL APPORTIONMENT PRINCIPLE If states X and Y have already been allotted x and y representatives, respectively, then state X should be given an additional representative in preference to state Y provided that

$$\frac{(\text{population of Y})^2}{y \cdot (y + 1)} < \frac{(\text{population of X})^2}{x \cdot (x + 1)}$$

Otherwise, state Y should be given the additional representative. We will often refer to a number of the form $\frac{(\text{population of X})^2}{x \cdot (x + 1)}$ as a **Huntington–Hill number.**