# MATH 1300-FINAL EXAM 

## DECEMBER 2018

## Instructions:

Your answers must be entered in your Examination Blue Book; answers written on this exam will not be graded. For full credit, you must show complete, correct and legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, but phones, PDAs, music players, Apple watches, and other electronic devices are not.

Solve problem 1 below, and solve any 13 of problems $2-16$ on the following pages. The problems are weighted equally during grading. If you solve more than 13 of problems $2-16$, you must indicate clearly which 13 you want graded. Otherwise, the first 13 will be selected by your grader.

## Part I

Problem 1: In the graph below, determine the number of even and odd vertices, respectively. Does the graph have a bridge?

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## Part II

Problem 2: Sam deposited $\$ 2,000$ in an account with an annual interest rate of $2.4 \%$, compounded quarterly. What is the value of the account after 8 years? Round your answer to nearest cent.

Problem 3: A snail climbs four feet up a tree during the day and slides three feet down at night. The tree is ten feet tall. How many days will it take the snail to reach the top of the tree for the first time?

Problem 4: Consider the following weighted voting system:

$$
[12: 4,7,8,3]
$$

(a) Are there any winning coalitions? If so, list them.
(b) Is there a dictator? Explain your answer.

Problem 5: There are 300 girls and 120 boys in the student athletics program at Monterey High School. The program supervisor has already chosen 3 girls and 2 boys to serve on a six-person committee to evaluate proposed athletics guidelines.
(a) Find the Huntington-Hill number for each group.
(b) Use the Huntington-Hill apportionment principle to decide if the sixth member on the committee should be a girl or a boy.

Problem 6: Use a truth table to determine if the statements $(p \vee \sim q) \wedge \sim(p \wedge q)$ and $p \wedge(p \wedge q)$ are logically equivalent.

Problem 7: 84 people are voting on which restaurant is to cater their event. There are three options: A, B and C. The ballots were submitted as follows:

|  | 21 | 19 | 29 | 15 |
| :---: | :---: | :---: | :---: | :---: |
| 1st | A | B | C | B |
| 2nd | B | A | B | C |
| 3rd | C | C | A | A |

(a) Which option wins when using the Borda count method?
(b) Which option wins when using plurality?

Problem 8: In (a)-(c), write the statement in symbolic form:
(a) The Democrats will control the Senate and environmental programs will not be cut. (Use $p$ for "The Democrats will control the Senate" and $q$ for "Environmental programs will be cut".)
(b) If you exercise, then you will feel great or you will sleep better. (Use $p$ for "You exercise", $q$ for "You will feel great" and $r$ for "You will sleep better".)
(c) Press the Enter key if the screen is black and the Tab key if it is not. (Use $p$ for "The screen is black", $q$ for "Press the Enter key" and $r$ for "Press the Tab key".)

Problem 9: A traveling salesman lives in Buffalo (B) and wants to travel to Atlanta (A), Chicago (C), and Denver (D) to sell his saxophones. Using the Nearest Neighbor method on the graph below, what is the cheapest flight path to visit all three cities and return home? How much will this trip cost?


Problem 10: A pair of ordinary dice is rolled, and the numbers showing on the top sides are added together.
(a) What is the probability of getting a sum strictly less than 4 ?
(b) Using your understanding of the complement of an event, find the probability of getting a sum greater than or equal to 4 .

Keep the answers as fractions.
Problem 11: Leo wants to save up for retirement, so he sets up an ordinary annuity. If he makes monthly payments of $\$ 600$ and the annuity has an annual interest rate of $6 \%$, compounded monthly, how much money will be in his account after 25 years? Round your answer to nearest cent.

Problem 12: Find the mean, median, mode, range and standard deviation of the following distribution:

$$
45,12,50,18,30,28,12,40,33,21,37
$$

In your answers and calculations, you may round numbers to two decimal places.

Problem 13: Below are the probabilities and values associated with five outcomes of an experiment. Calculate the expected value for the experiment.

| Outcome | Probability | Value |
| :---: | :---: | :---: |
| A | 0.2 | 4 |
| B | 0.2 | 8 |
| C | 0.1 | -1 |
| D | 0.3 | 2 |
| E | 0.2 | -3 |

Problem 14: A city consists of four districts with the following populations:

- North: 4,180
- South: 5,320
- East: 1,500
- West: 7,050

Use Hamilton's apportionment method to assign 20 city council seats to the four districts.

Problem 15: Consider a normal distribution with a mean of 30 and a standard deviation of 6 , and answer the following questions:
(a) What $z$-score corresponds to the raw score of 60 ?
(b) What is the raw score corresponding to a $z$-score of 1.5 ?

Problem 16: Bruce finally decided to buy a new house. While browsing online, he found a unique beach condo for sale in Galveston for only $\$ 229,000$. The International Bank of Statistics offers him a 30year home loan at $7.5 \%$ interest rate, compounded monthly since he will be making monthly payments. Bruce impulsively purchases the home. How much will his monthly payments be? Round your answer to nearest cent.

| Method | How the Winning Candidate Is Determined |
| :--- | :--- |
| Plurality | The candidate receiving the most votes wins. |
| Borda count | Voters rank all candidates by assigning a set number of points to first <br> choice, second choice, third choice, and so on; the candidate with <br> the most points wins. |
| Plurality-with- <br> elimination | Successive rounds of elections are held, with the candidate receiving <br> the fewest votes being dropped from the ballot each time, until one <br> candidate receives a majority of votes. |
| Pairwise <br> comparison | Candidates are compared in pairs, with a point being assigned the <br> voters' preference in each pair. (In the case of a tie, each candidate <br> gets a half point.) After all pairs of candidates have been compared, <br> the candidate receiving the most points wins. |

THE HUNTINGTON-HILL APPORTIONMENT PRINCIPLE If states $X$ and $Y$ have already been allotted $x$ and $y$ representatives, respectively, then state $X$ should be given an additional representative in preference to state $Y$ provided that

$$
\frac{(\text { population of } Y)^{2}}{y \cdot(y+1)}<\frac{(\text { population of } \mathrm{X})^{2}}{x \cdot(x+1)}
$$

Otherwise, state $Y$ should be given the additional representative. We will often refer to a number of the form $\frac{(\text { population of } \mathrm{X})^{2}}{x \cdot(x+1)}$ as a Huntington-Hill number.

## FORMULA FOR FINDING THE FUTURE VALUE OF AN ORDINARY

ANNUITY Assume that we are making $n$ regular payments, $R$, into an ordinary annuity. The interest is being compounded $m$ times a year and deposits are made at the end of each compounding period. The future value (or amount), $A$, of this annuity at the end of the $n$ periods is given by the equation

$$
A=R \frac{\left(1+\frac{r}{m}\right)^{n}-1}{\frac{r}{m}}
$$

THE COMPOUND INTEREST FORMULA Assume that an account with principal $P$ is paying an annual interest rate $r$ and compounding is being done $m$ times per year. If the money remains in the account for $n$ time periods, then the future value, $A$, of the account is given by the formula

$$
A=P\left(1+\frac{r}{m}\right)^{n} .
$$

Notice that in this formula, we have replaced $r$ by $\frac{r}{m}$, which is the annual rate divided by the number of compounding periods per year, and $t$ by $n$, which is the number of compounding periods.

## FORMULA FOR FINDING PAYMENTS ON AN AMORTIZED LOAN Assume

 that you borrow an amount $P$, which you will repay by taking out an amortized loan. You will make $m$ periodic payments per year for $n$ total payments and the annual interest rate is $r$. Then, you can find your payment by solving for $R$ in the equation$$
P\left(1+\frac{r}{m}\right)^{n}=R\left(\frac{\left(1+\frac{r}{m}\right)^{n}-1}{\frac{r}{m}}\right)^{*}
$$

FORMULA FOR CONVERTING RAW SCORES TO z-SCORES Assume a normal distribution has a mean of $\mu$ and a standard deviation of $\sigma$. We use the equation

$$
z=\frac{x-\mu}{\sigma}
$$

to convert a value $x$ in the nonstandard distribution to a $z$-score.

DEFINITION Assume that an experiment has outcomes numbered 1 to $n$ with probabilities $P_{1}, P_{2}, P_{3}, \ldots, P_{n}$. Assume that each outcome has a numerical value associated with it and these are labeled $V_{1}, V_{2}, V_{3}, \ldots, V_{n}$. The expected value of the experiment is

$$
\left(P_{1} \cdot V_{1}\right)+\left(P_{2} \cdot V_{2}\right)+\left(P_{3} \cdot V_{3}\right)+\cdots+\left(P_{n} \cdot V_{n}\right) .
$$

