

## INSTRUCTIONS

Your answers must be entered in your Examination Blue Book; answers on the exam will not be graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, but phones, PDAs, music players, Apple watches, and other electronic devices are not.

Solve problem 1, and solve any 13 of the problems 2–16; they are weighted equally. If you solve more than 13 of the problems 2–16, then mark clearly which ones you want graded, otherwise the first 13 answers to problems 2–16 in your Examination Blue Book will be graded.

## Part I

Solve problem 1 and make sure to explain your reasoning.

- Find the pattern and then find the fifth term of each of the following sequences.
  - 6, 10, 14, 18, ...
  - 1, 4, 9, 16, ...
  - 1, 2, 4, 8, ...

## Part II

Solve 13 of the problems 2–16 below. If you solve more than 13 problems, then mark clearly which ones you want graded, otherwise the first 13 answers in your Examination Blue Book will be graded.

- Use Euler diagrams to decide whether the following syllogisms are valid or invalid.
  - All fish can swim*  
*All dolphins can swim*

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*Therefore: Some dolphins are fish.*
  - Some flowers are blue*  
*Roses are not blue*

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*Therefore: Roses are not flowers.*
- Texas has a population of 28.3 million people and holds 36 seats in the House of Representatives; New Mexico has a population 2.088 million and 3 representatives.
  - Find the average constituency of either state.
  - Which state is more poorly represented?
  - Find the absolute unfairness of the apportionment.
  - Find the relative unfairness of the apportionment.
- The members of a club have ages as listed below
$$23, 45, 19, 25, 23, 21, 25, 28, 26, 25, 31, 32, 33 .$$
Determine the mean and the median age of the members.
- Assume that the interest rate on your credit card is 8%. Last month's balance was \$690. Since then, you made a payment of \$300, bought Post Malone tickets for \$215, returned a swimsuit for \$28, and bought flowers for \$60 for a girl who rejected you.
  - Using the unpaid balance method, what is your credit card balance for this month?
  - What is the finance charge for next month?

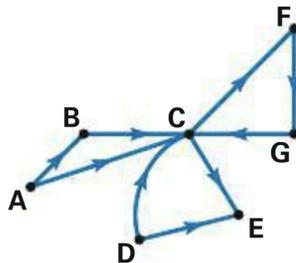
6. You want to save up \$3,000,000 by the day you turn 65 in order to retire happily. If you're turning 28 today, and your retirement account yields an annual interest of 7% compounded monthly, how much money do you need to deposit each month? (Use an ordinary annuity.)
7. The Avengers are in desperate need of another member, and are going to use the Borda count method to determine who the new addition to the team will be. The table below summarizes the preferences of the current members and support staff. Who will be the next hero to join the avengers? Captain Marvel (C), Antman (A), Daredevil (D), or Black Panther (B)?

|            | Number of ballots |    |   |    |
|------------|-------------------|----|---|----|
| Preference | 9                 | 14 | 6 | 13 |
| 1st        | C                 | D  | C | A  |
| 2nd        | A                 | A  | B | D  |
| 3rd        | B                 | B  | D | B  |
| 4th        | D                 | C  | A | C  |

8. Consider a normal distribution with a mean of 36 and a standard deviation of 9.
- (a) What  $z$ -score corresponds to a raw score of 54?
- (b) Using the 68-95-99.7 Rule, what percentage of values would be below 54?
9. The Vernon Law Firm just received 60 new printers for its 5 offices. Below is a table with the number of employees in each office. Use Hamilton's apportionment method to distribute the 60 printers among the Vernon Law Firm offices.

| Office    | A   | B   | C   | D   | E   | Total |
|-----------|-----|-----|-----|-----|-----|-------|
| Employees | 246 | 202 | 196 | 211 | 225 | 1080  |

10. If possible, find the requested paths in the graph below. If it is not possible to find a requested path, explain why not.
- (a) A directed path from B to E.
- (b) A directed path of length 6 from A to E.



11. Determine if the following argument is valid.

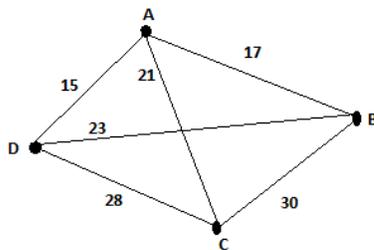
*If I go to the bar, then I will not study*

*I do not go to the bar*

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*Therefore: I study.*

12. A salesman is traveling between four cities: A, B, C, and D. The distances between the cities are on the map. Use the Nearest Neighbor Algorithm to find the shortest route that starts at A, visits all cities, and returns to A.



13. A game at a carnival involves spinning a wheel to potentially win a prize. There are four colors on the wheel. The probability and payout for each color are listed below. Find the expected value for the game.

| Space  | Probability | Payout |
|--------|-------------|--------|
| Yellow | $1/3$       | -1     |
| Green  | $1/4$       | 1      |
| Blue   | $1/4$       | 2      |
| Purple | $1/6$       | 5      |

14. You roll two six sided fair dice.
- What is the probability of rolling a total of 3?
  - What is the probability of rolling a total of 4?
  - Which of these two events has higher probability?
15. Suppose \$12,000 is invested at an annual interest rate of 8% for 5 years. Find the future value if the interest is compounded as follows.
- Monthly
  - Annually

16. Consider the weighted voting system

$$[22 : 12, 8, 5, 20]$$

made up of voters A, B, C, and D.

- What is the quota of this weighted voting system? Are there any dictators?
- List all the winning coalitions.
- Find the critical voters in each winning coalition.
- Compute the Banzhaf Power Index for each voter.

**DEFINITION** In a weighted voting system, a voter's **Banzhaf power index\*** is defined as

$$\frac{\text{the number of times the voter is critical in winning coalitions}}{\text{the total number of times voters are critical in winning coalitions}}.$$

| Method                     | How the Winning Candidate Is Determined  |
|----------------------------|--|
| Plurality                  | The candidate receiving the most votes wins.   |
| Borda count                | Voters rank all candidates by assigning a set number of points to first choice, second choice, third choice, and so on; the candidate with the most points wins.   |
| Plurality-with-elimination | Successive rounds of elections are held, with the candidate receiving the fewest votes being dropped from the ballot each time, until one candidate receives a majority of votes.  |
| Pairwise comparison        | Candidates are compared in pairs, with a point being assigned the voters' preference in each pair. (In the case of a tie, each candidate gets a half point.) After all pairs of candidates have been compared, the candidate receiving the most points wins. |

**FORMULA FOR FINDING THE FUTURE VALUE OF AN ORDINARY ANNUITY** Assume that we are making  $n$  regular payments,  $R$ , into an ordinary annuity. The interest is being compounded  $m$  times a year and deposits are made at the end of each compounding period. The future value (or amount),  $A$ , of this annuity at the end of the  $n$  periods is given by the equation

$$A = R \frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}.$$

**THE COMPOUND INTEREST FORMULA** Assume that an account with principal  $P$  is paying an annual interest rate  $r$  and compounding is being done  $m$  times per year. If the money remains in the account for  $n$  time periods, then the future value,  $A$ , of the account is given by the formula

$$A = P \left(1 + \frac{r}{m}\right)^n.$$

Notice that in this formula, we have replaced  $r$  by  $\frac{r}{m}$ , which is the annual rate divided by the number of compounding periods per year, and  $t$  by  $n$ , which is the number of compounding periods.

### THE UNPAID BALANCE METHOD FOR COMPUTING THE FINANCE CHARGE ON A CREDIT CARD LOAN

This method also uses the simple interest formula  $I = Prt$ ; however,

$P$  = previous month's balance + finance charge + purchases made – returns – payments.

The variable  $r$  is the annual interest rate, and  $t = \frac{1}{12}$ .

**FORMULA FOR CONVERTING RAW SCORES TO z-SCORES** Assume a normal distribution has a mean of  $\mu$  and a standard deviation of  $\sigma$ . We use the equation

$$z = \frac{x - \mu}{\sigma}$$

to convert a value  $x$  in the nonstandard distribution to a z-score.

**DEFINITION** Assume that an experiment has outcomes numbered 1 to  $n$  with probabilities  $P_1, P_2, P_3, \dots, P_n$ . Assume that each outcome has a numerical value associated with it and these are labeled  $V_1, V_2, V_3, \dots, V_n$ . The **expected value** of the experiment is

$$(P_1 \cdot V_1) + (P_2 \cdot V_2) + (P_3 \cdot V_3) + \dots + (P_n \cdot V_n).$$

**DEFINITIONS** Suppose that representatives are apportioned between two states A and B. We define the **absolute unfairness** of this apportionment as the difference between the larger average constituency and the smaller one. If state A has the larger average constituency, then the absolute unfairness is

$$(\text{average constituency of state A}) - (\text{average constituency of state B}).$$

If the two states have the same average constituencies, then we say that the two states are *equally well represented*.

**DEFINITION** When apportioning the representatives for two states, we define the **relative unfairness** of the apportionment as

$$\frac{\text{the absolute unfairness of the apportionment}}{\text{the smaller average constituency of the two states}}.$$

### HAMILTON'S APPORTIONMENT METHOD

- Find the standard divisor for the apportionment (total population/total number of representatives).
- Find the standard quota (state's population/standard divisor) for each state and round it down to its lower quota. Assign that number of representatives to each state.
- If there are any representatives left over, assign them to states in order according to the size of the fractional parts of the states' standard quotas.