Math 1452 Final Exam Spring 2018

1. Consider the region bounded by $y = x^2$, $y = 3x$, and the y-axis. **Set up** (but do not solve) integrals to find

    (a) The volume of the solid generated by rotating this region about the y-axis using washers.

    (b) The volume of the solid generated by rotating this region about the vertical line $x = -2$ using shells.

2. Graph $r = 3\sin(\theta)$ and $r = 3\cos(\theta)$. **Set up** an integral to find the area enclosed by the two graphs.

3. A rectangular gate 2 ft wide and 3 ft high is set into the side of a dam. The top of the dam is 4 ft below the surface of the water. **Set up** an integral for the force of the water against the gate. (Recall the density of water is 62.4 lb/ft³.)

4. Evaluate the following integrals.

    (a) $\displaystyle\int \frac{x+1}{(x-2)(x-1)}\,dx$

    (b) $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

    (c) $\displaystyle\int x^2 \sin(3x)\,dx$

    (d) $\displaystyle\int_2^4 \frac{1}{\sqrt{x-2}}\,dx$

5. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.

    (a) $\displaystyle\sum_{k=2}^{\infty} \frac{3}{k\ln(k)}$

    (b) $\displaystyle\sum_{k=3}^{\infty} \frac{1}{k^2-4}$

    (c) $\displaystyle\sum_{k=0}^{\infty} \frac{3^{k+2}}{2^{k-1}}$

    (d) $\displaystyle\sum_{k=2}^{\infty} \frac{5^k}{(k+1)!}$

6. Does $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k+1}}$ converge absolutely, converge conditionally, or diverge? Explain completely. You must identify all the tests you use and show all the work needed to apply them.

7. Find the first 3 terms of the MacLaurin series for $f(x) = \ln(x+2)$.

8. Let $\mathbf{u} = <3, 0, -1>$ and $\mathbf{v} = <-2, 4, 0>$.

    (a) Find $\mathbf{u} - 3\mathbf{v}$.

    (b) Are $\mathbf{u}$ and $\mathbf{v}$ orthogonal? Why?

    (c) Find the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.