A New Numerical Approach to the Solution of Partial Differential Equations with Optimal Accuracy on Irregular Domains and Cartesian Meshes. Application to the Wave and Heat Equations.

A. Idesman

Texas Tech University, Box 41021, Lubbock, TX, USA, E-mail: alexander.idesman@ttu.edu

A new numerical approach based on the minimization of the local truncation error is suggested for the solution of partial differential equations. Uniform Cartesian meshes are used for the space discretization. Similar to the finite difference method, the form and the width of the discrete (stencil) equations are assumed in advance. A discrete system of equations includes regular uniform stencils for internal points and non-uniform stencils for the grid points close to the boundary. The procedure for the calculation of the coefficients of stencil equations is based on the minimization of the order of the local truncation error and provides the optimal accuracy at the given width of stencil equations. Independent of weak formulations (Continuous Galerkin, Discontinuous Galerkin and other approaches) used with known techniques, the new technique will exceed the accuracy of the known techniques at the same widths of stencil equations. In contrast to the finite elements, there is no necessity to calculate by integration the elemental mass and stiffness matrices that is time consuming for high-order elements. As a mesh, the grid points of a uniform rectangular (square) mesh as well as the points of the intersection of the boundary of a complex domain with the horizontal, vertical and diagonal lines of the Cartesian mesh are used; i.e., in contrast to the finite element meshes, a trivial mesh is used with the new approach. Changing the width of the stencil equations, different high-order numerical techniques can be developed. The main advantages of the new approach are a high accuracy, trivial meshes and the simplicity of the formation of a discrete (semi-discrete) system for irregular domains. Currently the new technique is applied to the solution of the wave, heat and Laplace equations. At the same number of degrees of freedom, the new approach yields much more accurate results than known numerical techniques (e.g., the finite element method, finite volume method, finite difference method, isogeometric elements and others) and significantly (by a factor of 1000 and more) reduces the computation time at a given accuracy.

Speaker Bio: Dr. Idesman is a Professor in the Department of Mechanical Engineering at Texas Tech University. His research interests include the development of high-order accurate numerical techniques for PDE including wave and heat equations, elastodynamics, the application of these techniques to engineering problems, the development of numerical algorithms for nonlinear problems with inelastic materials at small and large strains. Dr Idesman has authored and co-authored 89 journal and conference papers.