The Beta Anomaly and Mutual Fund Performance

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Abstract

We contend that mutual fund performance cannot be measure using the alpha from standard asset pricing models if passive portfolios have nonzero alphas. We show how controlling for passive alpha produces an alternative measure of fund manager skill that we call active alpha. Active alpha is persistent and associated with higher returns and improved portfolio performance. Therefore, it makes sense for investors to allocate funds towards high active alpha managers. We find that while many investors do allocate their cash flows to funds with standard alphas, a subset of investors also allocate funds to managers that exhibit high active alpha performance as well.

1 Introduction

The empirical asset pricing literature supplies convincing evidence that high-beta assets often deliver lower expected returns than the CAPM model predicts, and that lower beta assets deliver returns above CAPM expectations (Black, Jensen, and Scholes (1972), Gibbons, Ross, and Shanken (1989), Baker, Bradley, and Wurgler (2011)). Recently, Frazzini and Pedersen (2014) reinvigorate this debate with a compelling theoretical argument for what is broadly termed the beta anomaly. They propose an additional betting against beta (BAB) factor that captures the return spread from this CAPM anomaly.

Given the evidence for the beta anomaly, it has long been suspected that mutual fund managers can capture significant alpha by investing in low-beta stocks. The standard academic response to measuring mutual fund behavior is currently to use the 4-factor model suggested by Carhart (1997). According to this model, in the absence of active management, the expected excess return for a fund is the sum of the products of the betas with four factor risk-premia. The expected difference between the portfolio return and its benchmark return is the Carhart measure of abnormal performance, or the alpha. The Carhart approach in effect assumes that a matching passive portfolio alpha is zero. However, given the current uncertainty regarding the correct multi-factor model to apply to equity returns, and the recent introduction of the BAB factor, whether any asset pricing model effectively controls for the beta anomaly is unclear.

This paper examines whether accounting for the beta anomaly can systematically affect inferences about mutual fund performance. According to the capital asset pricing model, higher mutual fund alpha indicates skill. However, it could also reflect a lower beta exposure to the market. That is, if fund A tends to hold high-beta assets relative to fund B, we ought to expect that, given equal skill, A has a lower alpha than B. In the standard attribution framework, however, we might spuriously attribute this result to differences in skill between A and B. It is not clear a priori how to account for the beta anomaly in mutual fund performance evaluation. More generally, it is not clear how to estimate the value-added of a fund when factor sensitivities are associated with a consistent pattern of alphas. We address the accounting issue by introducing a new performance measure that we call active alpha. Active alpha measure subtracts the passive alpha component from the funds' standard alpha. Passive alpha is measured as the value-weighted alpha of those individual stocks whose betas are similar to estimated fund beta. If the active alpha is positive, investors seeking that particular level of risk would benefit from holding such mutual fund.

In our sample of actively managed U.S. domestic equity funds, we find that alphas are almost monotonically declining in beta for mutual funds, as they are in general for equities. In contrast, we find that active alpha is almost monotonically increasing in beta. It seems apparent that the relation we observe between mutual fund standard alpha and fund beta is a consequence of the beta anomaly. Inference based on our active alpha measure, which accounts for cross-sectional return differences due to the beta anomaly, differs dramatically from that based on standard alpha measures. More specifically, we find that high-beta mutual funds tend to have positive and significant active alpha measures, but low-beta mutual funds tend to have positive and significant standard alpha measures. Moreover, higher active alpha is positively associated with several desirable portfolio characteristics including market-adjusted return and the portfolio Sharpe ratio.

There are other benefits of using active alpha to measure managerial skill. By controlling for passive beta outperformance or underperformance, active alpha controls for any time-variation in average mutual fund beta documented by Boguth and Simutin (2018). Further, adding the BAB factor to excess return models does not appear to completely control for the low-beta anomaly in mutual funds. Although, the magnitude of the alpha-beta relation is smaller in a six-factor model that adds the Pastor and Stambaugh (2003) liquidity factor and the Frazzini and Pedersen (2014) BAB factor to the commonly-used Carhart (1997) four factor model, we find that standard alphas are still significantly negatively related to fund beta.

In our main analysis we show that active alpha is persistent, indicating that it captures some skill over and above allocating assets to low-beta stocks. This finding raises the question of whether investors recognize and respond to active alpha when allocating their cash flows across funds. Related to this question is the fascinating question of what excess return model investors use to allocate their fund flows. Using a Bayesian framework that allows for alternative degrees of belief in different asset pricing models, Busse and Irvine (2006) show that fund flow activity varies by investor beliefs and by the time period under consideration. They report that a 3-year return history has a stronger correlation with fund flows than a single year's performance. Berk and van Binsbergen (2016) use mutual fund flows to test which asset pricing model best fits investor behavior. They test a large number of asset pricing models and time horizons and find that over most, but not all, horizons the CAPM best reflects investor behavior. Barber, Huang, and Odean (2016) find heterogeneous investor responses to fund performance. They report that investors respond most actively to beta risk and treat other factors such as size, value and momentum (the factors in the Carhart (1997) model), as fund alpha. However, they find that more sophisticated investors tend to use more sophisticated benchmarks when evaluating fund performance. Agarwal, Green, and Ren (2017) examine hedge fund flows and find that investors place relatively greater emphasis on exotic risk exposures that can only be obtained from hedge funds. Yet they find little performance persistence in these exotic risks.

Since active alpha is persistent, we investigate how investors allocate their mutual fund flows between standard alpha and active alpha. We find that consistent with the literature, standard alpha generates future fund flows. However, we also find that a subset of investors allocate their fund flows based on our active alpha measure of mutual fund performance. This finding suggests that some mutual fund investors are sophisticated enough to control for the beta anomaly since they invest based on active alpha, a skill measure that controls for portfolio beta. Conversely, we also find investors allocate fund flows based on the passive alpha, or the outperformance that can be obtained by simply generating a low beta portfolio.

To provide an economic explanation for the empirical sensitivity of mutual fund flows to active alpha over and beyond standard alpha, we develop a simple model of mutual fund flows with the presence of both sophisticated and naive investors. In our model, some investors are sophisticated and are able to invest in a passive benchmark with the same risk as the fund. Other investors are naive and only make risky investments via the fund. Both types of investors update the fund's managerial skill as Bayesians. It turns out that sophisticated investors' demand for the fund is positively related to posterior expectations of the active alpha, whereas naive investors' demand for the fund is positively related to posterior expectations of the standard alpha. Intuitively, sophisticated investors consider only active alpha, since they can identify (and short) the passive benchmark portfolio, in turn extracting only the performance truly attributable to managerial ability. On the other hand, naive investors care equally about all sources of alpha, since they are comfortable making risky investments only with the manager.

The model predicts that the flow sensitivities to active alpha and to standard alphas can be either positive or negative, depending on the relative presence of sophisticated investors (and on the persistence of active and passive alphas). Importantly, the empirical fact that flows respond positively to both active alpha and standard alpha measure can be consistent with our rational learning model only given the coexistence of sophisticated and naive investors. The empirical magnitudes of the capital response also suggest that sophisticated investors are relatively rare. We provide supporting evidence for this investor heterogeneity using mutual fund flows from institutional versus retail share classes. As we would expect, it is the flows from institutional share classes that significantly respond to active alpha.

Our paper contributes to the literature on mutual fund performance accounting for return anomalies from the empirical asset pricing literature. Ours is the first to account for the beta anomaly and to produce an estimate of managerial skill that does not attribute skill to a low-beta portfolio tilt. However, the factor-model regression approach is not the only popular mutual fund performance attribution method. The characteristic-based benchmark approach of Daniel, Grinblatt, Titman, and Wermers (DGTW, 1997) is also prominent. Since then, the literature has recognized the importance of accounting for the stock characteristics such as size, value and momentum effects in fund returns. Busse et al. (2017) propose to marry the factor-model regression approach and DGTW approach via a double-adjusted mutual fund performance. Back, Crane, and Crotty (2017) show that fewer funds have significant positive performance than one would expect by chance after alphas are adjusted for coskewness.

As we propose fund beta as a predictor of fund's value, others have proposed fund characteristics that predict performance, including industry concentration in Kacperczyk et al. (2005), the return gap in Kacperczyk et al. (2008), and peer benchmarking in Hunter et al. (2014).

2 Data and Methods

2.1 Mutual fund sample

The Morningstar and CRSP merged dataset provides information about mutual fund names, returns, total assets under management (AUM), inception dates, expense ratios, turnover ratios, investment strategies classified into Morningstar Categories, and other fund characteristics. From this data set we collect monthly return and flow data on over 2,838 U.S. diversified equity mutual funds actively managed for the period 1983-2014. Panel A of Table 1 presents summary information about the sample. There are 298,055 fund-month observations. Mean fund size of $1.28 billion is each fund's total assets under management (AUM), aggregated across share classes, divided by the total stock market capitalization in the same month. To account for the growth over time in the mutual fund industry, we scale this ratio by multiplying it by the total stock market capitalization at the end of 2011 as in Pastor, Stambaugh and Taylor (2015). We compute the fund age from the fund's inception date and find the typical fund has a life of 199 months. Funds earn an average gross return of 0.78% per month and collect fees of 9.9 basis points per month. Monthly firm volatility is 4.64% and average fund beta is 0.99. This beta average suggests that in the fund beta sort results presented below, one can consider the middle decile portfolios to roughly bracket the market beta.

2.1.1 Estimating mutual fund alphas

We estimate the abnormal return (alpha) for each mutual fund using each of the five performance evaluation models: i) the CAPM, ii) the Fama-French (1993) three factor model (FF3), iii) the Carhart (1997) four factor model, iv) the factor model we call PS5 is a five factor model augmenting the Carhart (1997) four-factor model with the Pastor and Stambaugh (2003) liquidity factor as in Boguth and Simutin (2018), and v) the Carhart (1997) four factor model augmented with the Pastor and Stambaugh (2003) liquidity factor and the Frazzini and Pedersen (2014) betting against beta factor (FP6). Alpha estimates are updated monthly based on a rolling estimation window for each model. For example, in the case of the four-factor model for each fund in month t, we estimate the following time-series regression using thirty-six months of returns data from months $\tau = t-1, \ldots t-36$:

$$(R_{p\tau} - R_{f\tau}) = \alpha_{pt} + \beta_{pt} (R_{m\tau} - R_{f\tau}) + s_{pt} SMB_{\tau} + h_{pt} HML_{\tau} + m_{pt} UMD_{\tau} + e_{p\tau}, \tag{1}$$

where $R_{p\tau}$ is the mutual fund return in month τ, $R_{f\tau}$ is the return on the risk-free rate, $R_{m\tau}$ is the return on a value-weighted market index, SMB_{τ} is the return on a size factor (small minus big stocks), HML_τ is the return on a value factor (high minus low book-to-market stocks), and UMD_τ is the return on a momentum factor (up minus down stocks). The parameters β_{pt}, s_{pt}, h_{pt}, and m_{pt} represent the market, size, value, and momentum tilts (respectively) of fund p; α_{pt} is the mean return unrelated to the factor tilts; and $e_{p\tau}$ is a mean zero error term. (The subscript t denotes the parameter estimates used in month t, which are estimated over the thirty-six months prior to month t.) We then calculate the alpha for the fund in month t as its realized return less returns related to the fund's market, size, value, and momentum exposures in month t:

$$\widehat{\alpha}_{pt} = (R_{pt} - R_{ft}) - \left[\widehat{\beta}_{pt}\,(R_{mt} - R_{ft}) + \widehat{s}_{pt}SMB_t + \widehat{h}_{pt}HML_t + \widehat{m}_{pt}UMD_t\right]. \tag{2}$$

We repeat this procedure for all months (t) and all funds (p) to obtain a time series of monthly alphas and factor-related returns for each fund in our sample.

There is an analogous calculation of alphas for other factor models that we evaluate. For example, we estimate a fund's FP6 alpha using the regression of Equation (1), but add the Pastor and Stambaugh (2003) liquidity factor and Frazzini and Pedersen (2014) betting against beta factor as independent variables. To estimate the CAPM alpha, we retain only the market excess return as an independent variable.

2.1.2 Estimating stock alphas

We build the beta-matched passive portfolio from the return characteristics of individual stocks. We estimate abnormal performance for individual stocks in an analogous manner to that of mutual fund alphas described above. First, we estimate the abnormal return (alpha) for each stock using each of the five performance evaluation models. Alpha estimates are updated monthly based on a rolling estimation window. Consider the four-factor model, which includes factors related to market, size, value, and momentum in the estimation of a stock's return. In this case, for each stock in month t, we estimate the following time-series regression using thirty-six months of returns data from months $\tau = t-1, \dots t-36$where $R_{q\tau}$ is the stock return in month τ, $R_{f\tau}$ is the return on the risk-free rate, $R_{m\tau}$ is the return on a value-weighted market index, SMB_{τ} is the return on a size factor (small minus big stocks), HML_{τ} is the return on a value factor (high minus low book-to-market stocks), and UMD_{τ} is the return on a momentum factor (up minus down stocks). The parameters β_{qt}, s_{qt}, h_{qt}, and m_{qt} represent the market, size, value, and momentum tilts (respectively) of stock q; α_{qt} is the mean return unrelated to the factor tilts; and $e_{q\tau}$ is a mean zero error term. (The subscript t denotes the parameter estimates used in month t, which are estimated over the thirty-six months prior to month t.) We then calculate the alpha for the stock in month t as its realized return less returns related to the stock's market, size, value, and momentum exposures in month t:

$$\widehat{\alpha}_{qt} = (R_{qt} - R_{ft}) - \left[\widehat{\beta}_{qt} (R_{mt} - R_{ft}) + \widehat{s}_{qt} SMB_t + \widehat{h}_{qt} HML_t + \widehat{m}_{qt} UMD_t\right]. \tag{3}$$

We repeat this procedure for all months (t) and all stocks (q) to obtain a time series of monthly alphas and factor-related returns for each stock in our sample.

2.1.3 Estimating mutual fund passive alphas

We calculate the passive alpha for each fund in month t using the alphas and market betas from individual stocks as in Equation (2). The passive alpha for each fund is the value-weighted alpha of those individual stocks whose beta are in a 10 percent range around estimated fund beta, such that:

$$\widehat{\beta}_{qt} > 95\% \times \widehat{\beta}_{pt}, \widehat{\beta}_{qt} < 105\% \times \widehat{\beta}_{pt}. \tag{4}$$

Let $\widehat{\theta}_{pt}$ denote the esimate of passive alpha for the fund in month t.

2.1.4 Estimating mutual fund active alphas

The fund's passive alpha allows us to calculate the active alpha for the fund in month t as the standard alpha for the fund in month t less the passive alpha in month t :

$$\widehat{\delta}_{pt} = \widehat{\alpha}_{pt} - \widehat{\theta}_{pt}, \tag{5}$$

where $\widehat{\delta}_{pt}$ is our active alpha estimate for fund p in month t.

2.2 Horizon for performance evaluation

To estimate longer horizon alphas, we cumulate monthly alphas by fund-month. For example, to estimate annual standard alpha:

$$A_{pt} = \prod_{s=0}^{11} (1 + \widehat{\alpha}_{p,t-s}) - 1, \tag{6}$$

where the monthly alpha estimates are calculated from a particular asset pricing model.

Analogously, we calculate the fund's annual active alpha as follows:

$$\Delta_{pt} = \prod_{s=0}^{11} \left(1 + \widehat{\delta}_{p,t-s}\right) - 1, \tag{7}$$

where monthly active alpha estimates can also vary depending on the asset pricing model used as to generate expected returns.

3 Results

3.1 Mutual fund alphas

In Table 1, panel B presents summary information on mutual fund standard alphas, estimated as usual, without controlling for any heterogeneity across funds in their betas. Standard alphas are measured against four different asset pricing models that researchers have used to estimate fund performance: the CAPM, the Fama-French 3-factor model (FF3), the Carhart 4-factor model (Carhart4), and a five-factor model (that we designate as PS5) using Carhart's (1997) four factors plus the liquidity factor in Pastor and Stambaugh (2003). Average mutual fund standard alphas based on these models are generally less than 1 basis point per month, with the exception of the CAPM, which produces a slightly more positive average outperformance of 6 basis points per month. These alphas all represent risk-adjusted returns before fees, so that if we subtract the average monthly expense ratio of 9.8 basis points, we would see that the average fund underperforms across all the benchmark models.

Panel C of Table 1 presents the same statistics for active alpha. On average, active alphas are lower than standard alphas for each asset pricing model. After removing the passive alpha component of fund performance, mutual fund managers do not show any degree of stock-picking ability, at least on average. Average active alpha ranges from 1 basis point for the CAPM to -5 basis points for the PS5 benchmark model. Perhaps interestingly, active alphas also suggest considerably larger heterogeneity across managerial skills than standard alphas do.

3.2 Mutual fund beta anomaly

Table 2 examines the degree to which mutual fund alphas are exposed to the beta anomaly. Again, alphas are measured against the four asset pricing models that we have used in section 3.1. In each month, we sort funds into 10 portfolios by their betas and compute the time-series average alphas for each beta-sorted portfolio. We note that mutual fund alphas are all based on gross returns and so do not represent the net alphas earned by investors.

Panel A reports the standard alpha of each beta decile calculated relative to different asset pricing models. The beta anomaly is clearly evident, with the standard alphas sorted by beta showing a consistently declining pattern. Relative to the CAPM, funds in the lowest beta decile have 250 basis points of average outperformance per year, while funds in the highest beta decile underperform by 99 basis points, which implies an economically large performance spread of 348 basis points. The use of alternative asset pricing models do not lower the magnitude of this spread very much. The often-used Carhart (1997) 4-factor model reduces the spread in alphas between beta-sorted portfolios 1 (P1) and 10 (P10) to 298 basis points. The Fama-French (1993) model and the four-factor model augmented with the Pastor-Stambaugh (2003) liquidity factor do marginally better than the Carhart (1997) model, with the P1-P10 alpha spreads of 253 basis points and 269 basis points, respectively. Clearly, if fund abnormal performance is measured by standard alphas, the low-beta mutual funds would exhibit a great degree of skill, as evidenced by their outperformance relative to the benchmark models. On the other hand, the high-beta mutual funds would predictably underperform and so represent bad investment opportunities. Of course, this pattern could simply suggest that the beta anomaly in assets held by mutual funds is an important source of standard alpha heterogeneity across funds.

Panel B reports the results for active alpha, which controls for the beta anomaly effect by using a passive beta-matched stock portfolio (Equation (5)) in estimating managerial skill. The pattern of active alphas is markedly different than that of standard alphas. Now, skill tends to increase with beta, suggesting that high-beta portfolio managers actually exhibit higher skills on average than low-beta portfolio managers once we control for the beta anomaly. The active alpha spread is quite large based on the CAPM at 331 basis points per year, but the use of multi-factor models do reduce this spread considerably to a minimum of 156 basis points in the case of the PS5 model.

We present the time series of performance spreads in annualized standard alpha and active alpha between the high-beta and low-beta mutual fund portfolios in Figure 1. Each of the three graphs plots the standard alpha and active alpha spreads for the high- vs. low-beta portfolios using three different asset pricing models, respectively, the CAPM, the Carhart4 model, and the FP6 model. Consistent with the results in Table 2, the active alpha spread is generally positive. Moreover, this spread is generally larger than the corresponding spread in standard alpha.

3.2.1 Mutual fund beta anomaly and the BAB factor

Frazzini and Pedersen (2014) contend that the beta anomaly is driven by funding constraints and propose a betting-against-beta factor (BAB) that captures the return affect related to the tightness of this particular constraint. Since the BAB factor is intended to be useful as a control variable for the low-beta anomaly, it is natural to ask whether an asset pricing model agumented with the BAB factor suffices to remove the performance-beta relation in mutual fund returns. To the extent that the Frazzini-Pedersen (2014) explanation for the low-beta anomaly is correct, the BAB factor should be related to the size of the anomaly. In turn, it should explain at least some of the low-beta premium in mutual fund standard alpha.

We proceed to analyze more formally the effect of beta on both standard alpha and active alpha using the Fama-MacBeth regressions in Table 3. Since Table 2 shows that the relation between alpha and beta is similar across all four standard asset pricing models, Table 3 only reports, for the sake of brevity, the regression results using the CAPM, the Carhart4 model, the PS5 model, and the PS5 model augmented with the BAB factor (FP6) as performance benchmarks.

The first four columns in Panel A of Table 3 regress standard alpha for each of the four asset pricing models on only a constant and beta as a single regressor. The objective of estimating these regressions is to determine the size and the significance of the alpha-beta relation documented in Table 2, and to examine whether the addition of the BAB factor to existing multi-factor models suffices to account for this relation in mutual fund returns.

Column (1) reports that the coefficient on beta for the CAPM is -0.05 and is statistically significant. This result indicates that we would expect a fund with a beta of 0.5 to deliver around 5% improvement annually in standard alpha relative to a fund with a beta of 1.5. The results for the Carhart model in column (2) are similar with a slightly larger increase of 6% in annual alpha per unit decrease in market risk.

In column (4), we report the alpha-beta relation using a six-factor model that includes the BAB factor (FP6). As we would expect from Frazzini and Pedersen (2014), the addition of the BAB factor to the benchmark portfolios does reduce the magnitude of this relation between mutual fund alpha and beta. However, the coefficient on beta is 3.1% per year per unit of beta, which continues to be statistically significant. Despite the use of the FP6 model, there still is a significant alpha premium to low-beta mutual funds. This suggests that including the BAB factor to the usual portfolios for fund performance benchmarks does not completely explain away the low-beta anomaly in mutual fund alpha. Columns (5)-(8) present multivariate regressions of the same alpha-beta relation, where we include fund size and fund age as controls. These variables proxy for the effects of scale, which is one constraint discussed prominently in recent literature (e.g., Chen et al., 2004; Pastor et al., 2015; Zhu, 2018). In particular, we observe that the coefficients on beta are not significantly affected by the inclusion of these statistically significant controls.

Panel B of Table 3 reports the results of running identical regressions as in Panel A, with active alpha as the dependent variable. The univariate regression results in Columns (1)-(4) indicate that there is a small, positive premium for per unit of beta risk. While this could suggest managers for high-beta funds being more skilled, this relation is statistically significant only using the CAPM. Using the Carhart4, PS5, or FP6 model, it is not significant at the 5% level. The multivariate regressions [Columns (5)-(8)] reveal similar results. All things considered, these results indicate that our active alpha successfully removes the beta anomaly in measuring mutual fund performance, and fund beta would not predict fund performance in significant ways, as we would naturally expect.

3.3 Persistence of active alpha

We have empirically shown that active alpha is a component of the standard alpha unaffected by the beta anomaly. In turn, it should be a measure of fund skill distinct from any passive persistence (or lack of) due to the beta anomaly. If active alpha really is a measure of managerial skill, it should be repeatable and, thus, persistent. We test this contention in Table 4. Each month t, we compute the percentile rank based on active alpha, $\widehat{\delta}_{p,t}$. of each mutual fund p. We then regress the active alpha ranks in the following month, $\widehat{\delta}_{p,t+1}$, as well as in the next two years, $\widehat{\delta}_{p,t+12}$ and $\widehat{\delta}_{p,t+24}$, on $\widehat{\delta}_{p,t}$. These regressions include controls for fund size, expense ratio, fund age, return volatility and fund flows to control for fund characteristics that could predict active alphas out into the future.

Panel A of Table 4 presents the regression results using the CAPM as the base model for calculating active alpha. We find that active alpha is highly persistent month-to-month. Specifically, the (rank) regression coefficient on $\widehat{\delta}_{p,t}$ predicting $\widehat{\delta}_{p,t+1}$ is 0.897 and is statistically significant. In other words, a fund earning a high active alpha in month t is highly likely to continue earning a high active alpha in month $t+1$. This persistence declines with time as the predictability of $\widehat{\delta}_{p,t+12}$ over one-year horizon is 0.103, which is still statistically significant. Two years out, the coefficient on $\widehat{\delta}_{p,t}$ falls to only 0.006, and is not statistically significant. The control variables in these regressions are generally insigificant with the exception of return volatility and fund flow, which show some predictive ability at longer horizons, but none of the controls are significant at the one-month horizon.

In Panel B, we see that using the Carhart4 model as the asset pricing model produces similar results. Using this model, the coefficient of $\widehat{\delta}_{p,t+1}$ on $\widehat{\delta}_{p,t}$ is 0.898, a number which is slightly higher than the coefficient in Panel A, and again indicates significant predictability of active alpha at the one-month horizon. The persistence level again declines with time to 0.15 at the one-year horizon and to 0.035 at the two-year horizon. This time, both numbers are statistically significant.

We obtain similar results in Panel C when the FP6 model is used as the base model for calculating active alpha. Using this model, the coefficient of $\widehat{\delta}_{p,t+1}$ on $\widehat{\delta}_{p,t}$ is significant 0.900. As in Panel B, statistically significant predictability of active alpha is also evident at the one- and two-year horizons, though the persistence coefficients do drop dramatically as the prediction horizon increases. Using the FP6 model, none of the control variables significantly predict active alpha at any horizon (at the 5% level). Generally, we observe that the persistence results grow consistently stronger as we account for more factor-related returns. This may be due to the fact that controlling for risk via factor models and controlling for returns related to the beta anomaly via our active alpha are jointly important for producing a clean estimate of true fund skill.

The persistence results are illustrated graphically in Figure 2. Panel A shows the persistence of active alpha when calculated using the CAPM. Differences in active alpha persist for about 8 months, though a small amount of outperformance continue to hold until about 14 months out. Consistent with the results in Table 4, the active alpha spread between the highest decile (10) and the lowest decile (1) portfolios (sorted by active alpha today) in the case of Carhart4 or FP6 model is generally larger, and is more persistent. In particular, the active alphas in decile 1 portfolio do not match those in decile 10 portfolio until about 20 months out in the Carhart4 model. When the FP6 model is used, the outperformance of the top active alpha portfolio persists for about 24 months. In summary, regardless of the asset pricing model used to calculate active alpha, the measure exhibits significant persistence, particularly at shorter horizons.

3.4 Fund performance and active alpha

Table 5 examines the characteristics of active alpha, this persistent skill measure, when benchmarked against the CAPM, the Carhart4, and the FP6 asset pricing models. We do this to better understand how active alpha relates to other mutual fund performance measures.

Panel B of Table 5 presents 10 portfolios sorted by active alpha constructed using the Carhart4 model as the benchmark. Gross returns and market-adjusted returns both increase in the level of active alpha. The 10-1 monthly return spread for both gross and market-adjusted returns is 0.30% per month. The Sharpe ratio also increases as the active alpha increases. Mutual fund monthly Sharpe ratios rise from 0.15 for the lowest active alpha portfolio to 0.21 in the highest active alpha portfolio. The information ratio results mirror the Sharpe ratio results almost exactly. Finally, we find high active alpha portfolios tend to have high standard alpha as well. This is not surprising given we benchmark active alpha against standard alpha (Equation (5)). Active alpha and standard alpha tend to be positively correlated ($\rho = 0.63$). Overall, there is a 0.22% increase in standard alpha as active alpha increases in portfolio rank from low to high.

Panel A of Table 5 presents the results using the CAPM model as the base model for calculating active alpha. Panel C of Table 4 shows the performance of mutual fund portfolios formed based on FP6 active alpha. All of the 10-1 portfolio sort differences are statistically significant and economically meaningful. What this tells us is that a higher active alpha is generally a good thing for portfolio performance. Not only are returns higher as active alpha increases, but portfolio efficiency improves as well.

3.5 Fund flows and active alpha

As discussed in the previous section, active alpha predicts superior portfolio performance (Table 5), while we have shown in section 3.3 that active alpha is persistent (Table 4). Therefore, we would expect that it is a fund characteristic cultivated by at least some sophisticated investors. If there are any such investors, they would allocate their cash flows towards those funds that exhibit high performance, as measured by active alpha. On the other hand, the fund literature finds that investors allocate their funds based on alpha measures (Barber et al., 2016; Berk and van Binsbergen, 2016). A natural question to ask then is whether there are any investors that allocate funds based on active alpha.

To investigate this question, we run panel regressions of fund flows on the lagged ranks based on annualized active alpha and standard alpha. We report the results in Table 6.

Following the prior literature on fund flows, we calculate flows for fund p in month t as:

$$Flow_{p,t} = \frac{TNA_{p,t} - TNA_{p,t-1}\,(1 + R_{p,t})}{TNA_{p,t-1}}, \tag{8}$$

so that flows represent the percentage change in the fund's net assets not attributable to its return gains or losses. Specifically, the regression specification that we utilize in Table 6 is

$$Flow_{p,t} = a + bPerformance_{p,t-1} + \mathbf{c}'\mathbf{X}_{p,t-1} + \varepsilon_{p,t}, \tag{9}$$

where $Performance_{p,t-1}$ is measured using the (lagged) percentile rank for the fund based on either its annualized active alpha $(\widehat{\delta}_{p,t-1})$ or its annualized standard alpha $(\alpha_{p,t-1})$. We include a vector of control variables $(\mathbf{X}_{p,t-1})$, which yields a vector of coefficient estimates $(\mathbf{c})$. As controls, we include lagged fund flows from month $t-13$, a lag of a fund's expense ratio, a fund's return standard deviation estimated over the prior twelve months, lagged fund size in month $t-1$ (in 2011 dollars), and the log of fund age in month $t-1$. We also include fixed effects for Morningstar Category $\times$ month.

The results of estimating equation (9) using the overall performance rank as the regressor are presented in Panels A and B of Table 6. We present the regression coefficients of fund flow on performance rank, where the standard alpha and active alpha are estimated using three different asset pricing models: the CAPM, the Carhart4 model, and the FP6 model. Since our active alpha is, by construction, a component of standard alpha, we begin in Panel A by estimating the effects of standard alpha and active alpha independently to better understand the strength of each measure in attracting fund flows.[1] We see that fund flows are significantly positively related to past performance as measured by either standard alpha or active alpha. For both performance measures, the flow-performance relation weakens slightly as we add more factors to our performance benchmarks, but in all six regressions, standard alpha and active alpha significantly attract fund flows.[2] In particular, the coefficients on active alpha are approximately two-thirds of the magnitude of those on standard alpha. This result implies that the skill component of standard alpha generate significant flows, but also that there is a significant fraction of investors who allocate flows to the passive component of standard alpha. The larger coefficients on standard alpha and in turn, its relative strength in predicting fund flows is not surprising, as standard alpha is the more familiar performance measure.

Of course, one might be tempted to argue that the results in Panel A is consistent with the story that investors only chase standard alpha, with the coefficients on active alpha being significantly positive, yet relatively small is primarily driven by its correlation with standard alpha, which is what matters for the investors.[3] We further investigate whether there are any investors attending to active alpha by jointly estimate the effects of standard alpha and active alpha in attracting fund flows. To address potential concerns about multicollinearity in this specification, we compute the variance inflation factor (VIF) for the active alpha percentile rank. Across alternative asset-pricing models and across performance rank choices, the VIF

[1] See below, Table 7, which presents an analysis of the relative importance of the two components of standard alpha in attracting fund flows.

[2] This is consistent with the recent literature (e.g., Berk and van Binsbergen, 2016; Barber et al., 2016) that finds alphas from more sophisticated models explain mutual fund flows worse than CAPM alpha.

[3] Indeed, the correlation between active alpha and standard alpha is between 0.60 and 0.70 across asset pricing model we use to calculate the alpha measures and whether we compare the raw alphas or the percentile ranks.

turns out to be at most 2, which suggests that multicollinearity is not an issue.[4] Besides, when we run our regression model in Panel B with small modifications, we obtain consistent evidence that fund flows respond to active alpha in and beyond standard alpha (see Panel D for example, where we obtain even stronger results). Furthermore, in the appendix, we present a simple model that formally justifies identifying for the existence of investors chasing active alpha by testing the significance of the *partial* coefficient associated with active alpha. Across alternative asset-pricing models, we find that the partial coefficient on active alpha is appreciably smaller than two-thirds of the magnitude of that on standard alpha, which indicates that indeed the large coefficient on active alpha in Panel A is partly due to its high correlation with active alpha. On the other hand, fund flows are jointly significantly positively related to both standard alpha and active alpha. The results indicate that there exists a subset of investors, who are apparently aware that passive alpha should not necessarily be rewarded and do allocate capital based on active alpha.

Panels C and D are identical to Panels A and B except that, following the practice of Sirri and Tufano (1998), they replace the overall performance rank with the within-category performance rank as the regressor. Specifically, funds are ordered within the nine categories corresponding to Morningstar's 3×3 stylebox based on their active alphas or standard alphas. This allows us to test whether our results in Panels A and B are simply driven by mutual fund investors chasing styles, rather than them chasing fund performance per se. However, we continue to find that both standard alpha and active alpha continue to generate similar flow responses, and the coefficients on active alpha is smaller in magnitude than those on

[4] As a rule of thumb, a regression model may be subject to multicollinearity worries if a variable has VIF values greater than 10 (or 5 to be conservative).

standard alpha. Interestingly, this alternative ranking only renders the gap between active alpha and standard alpha in attracting fund flows smaller.

When we look at the results in Table 6 it is apparent that any flows chasing the standard alpha measure, regardless of the assumed return generating process, are attracted by either the passive alpha component of standard alpha or the active alpha component. Therefore any flows allocated to alpha are either allocated to the passive alpha obtained from the beta anomaly or the active alpha component. We maintain that any flows allocated to passive alpha are not rewarding managerial outperformance, instead they are rewarding the inability of the asset pricing model to fully control for the beta anomaly. Table 7 estimates how fund flows are associated with the two components of standard alpha, the active alpha and the passive alpha, as well as the six factors in the FP6 asset pricing model.

The results presented in Table 7 illustrate how all returns, whatever the source, tend to attract fund flows. These results suggest that mutual fund investors allocate some of their flows based on potential asset pricing factors as in Barber et al. (2016),but our results indicate that both the liquidity and betting against beta factor also attract fund flows, factors that were not examined in Barber et al. (2016) Active alpha and passive alpha both attract statistically significant flows into the fund at approximately the same rate when included as regressors in the same specification for fund flows. The coefficient on active alpha is 0.151 and the coefficient on passive alpha is 0.150. This result indicates that after controlling for factor returns, some investors are allocating flows based on the passive alpha component of standard alphas, a measure that we argue should not be attributed to managerial skill. Looking at the performance of specific factors in attracting fund flows, the market return has the weakest coefficient on fund flows, while the more exotic returns associated with the momentum and liquidity factors attract funds at the greatest rate.

3.5.1 Investor sophistication and active alpha

Thus far, we have provided evidence that some investors allocate flows to managers who exhibit high active alpha performance. In this section, we test and find support for the conjecture that sophisticated investors tend to use active alpha measure. As in Evans and Fahlenbrach (2012), we use institutional share class as a proxy for investor sophistication.

We test the impact of institutional share class on the flow-return relations. To do so, we first classify a mutual fund share class as institutional if Morningstar share class is INST or CRSP institution dummy is 1. For each mutual fund, we measure the flow to its institutional class as the value-weighted flow across fund's multiple institutional classes. Similar, the flow to fund's retail share class is the value-weighted flow across fund's retail classes. Finally, we modify the main flow-return regression Equation (9) by including an interaction term between active alpha and the institution share class dummy.

Table 8 presents regression coefficient estimates from panel regressions of monthly flow to institution/retail share class (dependent variable) on lagged rank of annualized active alpha and the interaction term with institution share class dummy variable. Panel A reports the regression results for all of the mutual funds in our sample over the period 1984 to 2014. The interaction term between active alpha and institutional share class dummy is significant at 1% level. In Panel B, we restrict the sample to mutual funds with both institutional and retail share classes. We consistently find that investors in the institutional share classes respond more to active alpha than do investors in the retail share classes. These results are consistent with the notion that investors in the retail share classes are less sophisticated in their assessment of funds performance than are investors in the institutional share class. Overall, these results confirm our hypothesis that sophisticated investors allocate flows to mutual funds that exhibit high active alpha performance.

3.6 Explaining the role of active alpha in generating fund flows

We began our empirical investigation of active alpha by establishing that it is a persistent fund characteristic that can be used to pick mutual funds with higher risk-adjusted performance, in turn arguing that it is a measure of fund skill. Consistent with this argument, we then showed that active alpha attracts fund flows, and fund flows respond to active alpha in and beyond active alpha. We have also shown that the strength of active alpha (relative to that of standard alpha) in garnering fund flows is higher in the case of institutional share classes.

We contend that these empirical evidence is consistent with an active management industry, which serves investors of varying sophistication. While investor sophistication has many dimensions, our empirical results point to heterogeneous ability on the part of investors to account for the beta anomaly in evaluating mutual fund performance.

In the appendix, we formalize this argument by describing a simple model with the presence of both sophisticated and naive investors. In addition to the actively managed fund, sophisticated investors has available an alternative investment opportunity in its passive benchmark (with the same risk). Naive investors make risky investments only with the mutual fund. In this model, sophisticated investors care about difference in alphas between fund alpha (i.e., standard alpha) and passive bnehcmark (i.e., passive alpha), which corresponds to the empirically defined active alpha because they can short the passive benchmark portfolio, they only care if the manager can provide risk-adjusted return in and beyond that offered by passive benchmark. On the other hand, naive investors do not care what the source of alpha is.

3.6.1 Calibration

We have shown that the empirical evidence is consistent with a simple model of heterogeneous investor sophistication, in which two types of investors coexist. In this section, we use the model to obtain guidance as to how large active alpha chasers as a group might be by calibrating the model to the data. We find that 10% of investors are sophisticated, which is quite large considering that active alpha is a novel measure and suggests reasonably high sophistication on the part of mutual fund investors.

In the model, both components of the fund's alpha are assumed to evolve as AR(1):

$$\delta_t = \left(1-\phi^A\right)\delta^* + \phi^A \delta_{t-1} + \tau_t, \tag{10}$$

$$\theta_t = \left(1-\phi^P\right)\theta^* + \phi^P \theta_{t-1} + \upsilon_t, \tag{11}$$

where τ_t and υ_t are *i.i.d.*, respectively, $N\left(0,\sigma_\tau^2\right)$ and $N\left(0,\sigma_\upsilon^2\right)$. δ^* and θ^* represent, respectively, the unconditional expectations of active alpha and passive alpha.

We begin by tying down the model parameters that can be inferred directly from estimating (10)-(11). Regressing annualized active alpha or annualized passive alpha on its lag with fund fixed effects, the autoregressive coefficients are estimated around 0.91, so we use $\phi^A = 0.91$ and $\phi^P = 0.91$. Berk and Green (2004) infer the parameters that govern the distribution of skill level (mean of the prior ϕ_0 and prior standard deviation γ) by calibrating their model. They report that the flow-performance relationship is consistent with high average levels of skills ($\phi_0 = 6.5\%$ per year, or 0.5% per month, and γ is similar in magnitude to ϕ_0). So we will draw both components of the fund's average alpha (δ^* and θ^*) from a lognormal distribution with mean 0.25% per month and standard deviation 0.25% per month, which implies the fund's average alpha is drawn from a distribution with mean 0.5% per month. These numbers are consistent with both components of the fund's average alpha contributing equally[5] and being drawn from a diffuse distribution.

We also set $\sigma_\tau = 0.4$ percent per month, which is the estimate of the standard deviation of residuals obtained from estimating (10). We note that we would set $\sigma_\upsilon = 3$ percent (per month), if we were to use the same reasoning for the fund's passive alpha, but instead we will use σ_υ that allows us to match the empirical correlation between the sign of the change in active alpha and the sign of the change in standard alpha (see below).

In the model, the excess return on the actively managed fund is $r_t = \alpha_t + \epsilon_t$, while the passive benchmark portfolio's excess return has mean θ_t and the same risk as the fund, so $r_t^P = \theta_t + \epsilon_t$. That is, ϵ_t is all return component . That is, ϵ_t represents the sum of all the return components other than alpha, whether active or passive. For simplicity, we assume $\epsilon_t = \beta^* r_t^M$, where r_t^M is the excess return on the market portfolio, and β^* is the fund

[5] The average value of the fixed effects from estimating (10) is higher than that from estimating (11), so apparently δ^* is drawn from a distribution with lower mean than θ^*. Our use of equal means for δ^* and θ^* is intended really for efficiency sake of our simulation exercise. In each sample, we assume that it ends whenever investor expects the fund's alpha or the active alpha negative going forward, so a slightly higher average for δ^* allows us more samples that can be used given the number of simulations.

beta. In other words, this is a CAPM world, and idiosyncratic risk is negligible, as we are analyzing diversified U.S. equity mutual funds. To determine the parameter β^*, we appeal to Proposition 1 from Frazzini and Pedersen (2014), which shows that a security's alpha with respect to the market is $\alpha_t^s = \psi_t (1 - \beta_t^s)$ (in equilibrium), running the Fama-MacBeth regression of annualized alpha on one minus beta in the sample of U.S. equities, suppressing the constant term. The estimated coefficient is around 0.05, so we infer the beta of a fund with average passive alpha of θ^* as $\beta^* = 1 - \theta^*/0.05$. Finally, the volatility of monthly market excess returns is higher than 4 percent, so we specify $Std\left(r_t^M\right) = 0.05$, or 5 percent, and in turn, specify $\sigma_\epsilon = \beta^* \times Std\left(r_t^M\right)$.

We simulate 400,000 samples of this economy for 500 months. We assume that the sample ends whenever investors' expectation of the fund's alpha or active alpha is rendered negative. This yields typically 2,500 samples of average lenght over 100 months. In this case, the model obtains that

$$Flow_t = q\frac{\widehat{\delta}_{t+1|t} - \widehat{\delta}_{t|t-1}}{\widehat{\delta}_{t|t-1}} + (1-q)\,\frac{\widehat{\alpha}_{t+1|t} - \widehat{\alpha}_{t|t-1}}{\widehat{\alpha}_{t|t-1}}.$$

where q is the fraction of sophisticated investors. In other words, fund flows is determined by changes in investors' expectation of the fund's alpha or active alpha. The relative importance of the two expectations is determined by q. To do away with potential misspecification issues, we follow Berk and van Binsbergen (2015), and we focus on signs of the relevant variables. Specifically, we compute $Sign\left(Flow_t\right)$, $Sign\left(\widehat{\delta}_{t+1|t} - \widehat{\delta}_{t|t-1}\right)$, and $Sign\left(\widehat{\alpha}_{t+1|t} - \widehat{\alpha}_{t|t-1}\right)$. We use σ_v and q to match two moments. First, we use σ_v to match the empirical correlation between $Sign\left(\widehat{\delta}_{t+1|t} - \widehat{\delta}_{t|t-1}\right)$ and $Sign\left(\widehat{\alpha}_{t+1|t} - \widehat{\alpha}_{t|t-1}\right)$, which is about 0.41 in the data. The parameter σ_v governs the innovation volatility to passive alpha and in turn, allows us to control the said correlation. Setting the parameter $\sigma_v = 1.3$ percent per month yields an average simulated correlation around 0.41. We then vary q, or the fraction of sophisticated investors, to match

$$\frac{Corr\left(Sign\left(Flow_t\right), Sign\left(\widehat{\delta}_{t+1|t} - \widehat{\delta}_{t|t-1}\right)\right)}{Corr\left(Sign\left(Flow_t\right), Sign\left(\widehat{\alpha}_{t+1|t} - \widehat{\alpha}_{t|t-1}\right)\right)}.$$

This represents how well the change in active alpha expectation correlates with the sign of fund flows relative to how well the change in the fund's alpha expectation correlates with the sign of fund flows. Intuitively, we would expect this number to be higher as the fraction of sophisticated investors increases. In the data, this number is slightly bigger than 0.6. In the model, it turns out that this number is consistent with about 10% of investors being sophisticated.

4 Conclusion

Mutual fund managers can earn positive alphas passively by allocating resources to low beta assets to take advantage of the low-beta anomaly. This positive relation between beta and standard alpha is significant over a number of different asset pricing models, including a six-factor model that includes the four factors in the Carhart (1997) model plus a liquidity factor and the betting against beta factor of Frazzini and Pedersen (2014). To correct for the passive alphas that can be recorded regardless of the asset pricing model, we develop a measure of alpha called active alpha that subtracts the outperformance from a beta-matched portfolio from the fund's standard alpha. We contend that active alpha is a useful measure of managerial skill since it isolates outperformance that is distinct from the outperformance that can be obtained from the low-beta anomaly.

A high active alpha is associated with positive portfolio properties including overall returns, market-adjusted returns and high Sharpe ratios. Active alpha is also predictable, in that past active alphas are significantly correlated with future active alphas for at least 12 months into the future. Given the positive properties of high active alpha portfolios and the fact that it is to some extent predictable, sophisticated investors should allocate their capital to high active alpha funds. We find evidence that active alpha does attract cash flows, particularly from more sophisticated investors who are presumably aware of the low-beta anomaly.

5 Appendix

In this appendix, we present a simple model to highlight the impact of the relative composition of sophisticated vs. naive investors on the flow-performance relation, and to use in section 3.6.1 to infer the fraction of sufficiently sophisticated investors (who allocate money based on active alpha, rather than standard alpha) in the data.

There is an actively managed mutual fund, whose manager has (potential) ability to generate expected returns in excess of those provided by a passive benchmark—an alternative investment opportunity available to some investors with the same risk as the manager's portfolio. The expected passive alpha on this benchmark and the manager's ability to beat it are unknown to investors, who learn about this ability and the passive alpha by observing the histories of the managed portfolio's returns and the benchmark returns. Let $r_t = \alpha_t + \epsilon_t$

denote the return, in excess of the risk-free rate, on the actively managed fund. This is not the performance attributable to managerial ability, which is α_t net of passive alpha (see below). The parameter α_t is the fund's expected alpha. The error term, ϵ_t, is normally distributed with mean zero and variance σ^2 and is independently distributed through time. We further assume that this uncertainty is systematic: investors cannot diversify away this risk. The passive benchmark portfolio's excess return has mean θ_t and the same risk as the fund, i.e., $r_t^P = \theta_t + \epsilon_t$. Note that the model is partial equilibrium.[6]

In the model,

$$\begin{aligned} \delta_t &= \alpha_t - \theta_t \\ &= r_t - r_t^P \end{aligned}$$

which is the risk-adjusted return to investors over what would be earned on the passive benchmark, corresponds to active alpha. Of course, active alpha is the same as the standard alpha measure if the passive benchmark has zero alpha, but empirical evidence suggests otherwise (e.g., Frazzini and Pedersen, 2014). Note that α_t, θ_t (and in turn δ_t) are taken to vary over time. Specifically, both components of the alpha are assumed to evolve as AR(1):

$$\begin{aligned} \delta_t &= \left(1 - \phi^A\right) \delta^* + \phi^A \delta_{t-1} + \tau_t, \\ \theta_t &= \left(1 - \phi^P\right) \theta^* + \phi^P \theta_{t-1} + \upsilon_t, \end{aligned}$$

[6] The benchmark portfolio's returns are assumed to be exogenously given, and we do not model the source of successful managers' abilities. In that sense, our approach is similar to that in Berk and Green (2004) and Huang et al. (2012). We are describing the simplest model, which produces the sensitivity of mutual fund flows not only to the standard alpha measure, but also to active alpha that is an alternative measure of fund manager skill controlling for passive alpha.

where τ_t and υ_t are *i.i.d.*, respectively, $N(0, \sigma_\tau^2)$ and $N(0, \sigma_\upsilon^2)$. δ^* and θ^* represent, respectively, the unconditional expectations of the active alpha and the passive alpha.

There are two types of investors: a fraction q of investors are *sophisticated*, indexed by s, who allocate money across all assets (the risk-free asset and the active fund, as well as its passive benchmark). The remaining $1 - q$ fraction of investors are *naive*, indexed by n, who are inexperienced. They only allocate money between the active fund and the risk-free asset. We note that the behavior of naive investors is consistent with the empirical evidence on limited market participation.

On date $t-1$, investors have priors about δ_t and θ_t. These investors form their posterior expectations of the fund manager's ability as well as of the passive alpha through Bayesian updating. On date t, after observing the period t excess return r_t, they update their priors about δ_t and θ_t, which in turn imply their beliefs about δ_{t+1} and θ_{t+1}. Investors' prior beliefs are assumed to be normally distributed:

$$\begin{bmatrix} \delta_1 \\ \theta_1 \end{bmatrix} \sim N\left(\begin{bmatrix} \widehat{\delta}_{1|0} \\ \widehat{\theta}_{1|0} \end{bmatrix}, \begin{bmatrix} V_{1|0}^{\delta} & 0 \\ 0 & V_{1|0}^{\theta} \end{bmatrix}\right). \tag{A1}$$

Assume that $V_{1|0}^{\theta} = \sigma_\upsilon^2$. Then, it is straightforward to show by using standard Bayesian results for updating the moments of a normal distribution that their posterior expectations after observing the history $\left\{r_u, r_u^P\right\}_{u=1}^t$ are:

$$\begin{bmatrix} \delta_{t+1} \\ \theta_{t+1} \end{bmatrix} \Big| \left\{r_u, r_u^P\right\}_{u=1}^t \sim N\left(\begin{bmatrix} \widehat{\delta}_{t+1|t} \\ \widehat{\theta}_{t+1|t} \end{bmatrix}, \begin{bmatrix} \sigma_\tau^2 & 0 \\ 0 & \sigma_\upsilon^2 \end{bmatrix}\right)$$

where

$$\widehat{\delta}_{t+1|t} = \left(1-\phi^{A}\right)\delta^{*}+\phi^{A}\left(r_{t}-r_{t}^{P}\right) \tag{A2}$$

$$\widehat{\theta}_{t+1|t} = \left(1-\phi^{P}\right)\theta^{*}+\phi^{P}\left(w\widehat{\alpha}_{t|t-1}^{P}+(1-w)\,r_{t}^{P}\right) \tag{A3}$$

and $w=\sigma_{\epsilon}^{2}/\left(\sigma_{\upsilon}^{2}+\sigma_{\epsilon}^{2}\right)$. Similarly, this implies the posterior about α_{t+1} is normally distributed with a mean of $\widehat{\alpha}_{t+1|t}=\left(\widehat{\delta}_{t+1|t}+\widehat{\theta}_{t+1|t}\right)$ and a variance of $\left(\sigma_{\tau}^{2}+\sigma_{\upsilon}^{2}\right)$.

We consider an overlapping-generations (OLG) economy in which investors of type $i\in\{s,n\}$ are born each time period t with wealth $W_{i,t}$ and live for two periods. Each time period t, young investors have a constant absolute risk aversion (CARA) utility over their period $t+1$ wealth, $e^{-\gamma_{i}W_{i,t+1}}$, where $W_{i,t+1}=W_{i,t}+X_{i,t}r_{t+1}+X_{i,t}^{P}r_{t+1}^{P}$, $X_{i,t}$ is the dollar allocation to the mutual fund at time t, and $X_{i,t}^{P}$ is the dollar allocation to the passive benchmark. Since naive investors are assumed to make risky investments only with the mutual fund, $X_{n,t}^{P}=0$.

Given CARA utility, it is easy to show that the optimal mutual fund holdings are

$$X_{s,t} = \frac{\widehat{\delta}_{t+1|t}}{\gamma_{s}\sigma_{\tau}^{2}}$$

$$X_{n,t} = \frac{\widehat{\alpha}_{t+1|t}}{\gamma_{n}\left(\sigma_{\tau}^{2}+\sigma_{\upsilon}^{2}+\sigma_{\epsilon}^{2}\right)}$$

Imposing the restriction that $X_{s,t}$ and $X_{n,t}$ are nonnegative (no shorting of funds), we have

$$X_{s,t} = \frac{\max\left(\widehat{\delta}_{t+1|t},0\right)}{\gamma_{s}\sigma_{\tau}^{2}} \tag{A4}$$

$$X_{n,t} = \frac{\max\left(\widehat{\alpha}_{t+1|t},0\right)}{\gamma_{n}\left(\sigma_{\tau}^{2}+\sigma_{\upsilon}^{2}+\sigma_{\epsilon}^{2}\right)} \tag{A5}$$

Intuitively, when choosing their optimal allocation to the fund, sophisticated investors will consider only active alpha, since they have the ability to short the passive benchmark portfolio and in turn extract only the performance truly attributable to managerial ability. On the other hand, naive investors will attend to the standard alpha measure, since they cannot short sell the benchmark asset and in turn care equally about all sources of alpha.

We define the flow into the fund from investors of type i on date t as

$$F_{i,t} = \frac{X_{i,t} - X_{i,t-1}}{X_{i,t-1}}.$$

The total net flow into the fund is then

$$\begin{aligned} F_t &= qF_{s,t} + (1-q)\,F_{n,t} && \text{(A6)} \\ &= q\frac{\max\left(\widehat{\delta}_{t+1|t}\,,0\right)}{\max\left(\widehat{\delta}_{t|t-1}\,,0\right)} + (1-q)\,\frac{\max\left(\widehat{\alpha}_{t+1|t}\,,0\right)}{\max\left(\widehat{\alpha}_{t|t-1}\,,0\right)} - 1 && \text{(12)} \end{aligned}$$

For simplicity, we assume the history of observed returns is such that $\widehat{\delta}_{t|t-1}\,,\widehat{\alpha}_{t|t-1} > 0$. Hence, both types of investors started with positive dollar holdings, $X_{s,t-1}, X_{n,t-1} > 0$, in the mutual fund at time $t-1$.

Looking forward, it is useful to note two facts that follows immediately from equation A6. If all investors are naive, i.e., $q = 0$, then the flow sensitivity to active alpha is $-1/\gamma < 0$. On the other hand, if all investors are sophisticated, i.e., $q = 1$, then the flow sensitivity to the standard alpha measure is $-1/\eta\,(1-w) < 0$. Quintessentially, an empirical observation that flows respond positively to both active alpha and the standard alpha measure would suffice to show that at least some investors are sophisticated and not all investors are sophisticated, i.e., $q \in (0,1)$. Moreover, how strongly flows respond to active alpha vs. the standard alpha measure would be informative of the fraction of investors who are sophisticated, i.e., how big q is.

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Table 1: Summary Statistics

This table summarizes the statistics across fund-month observations from Jan. 1983 to Dec. 2014. Panel A reports fund characteristics such as net return, flows, fund size, expense ratio, age, and return volatility. Percentage fund flow is percentage change TNA from month t-1 to t adjusted for the fund return in month t. Return volatility is calculated as the standard deviation of prior 12 month fund returns. All variables are winsorized at the 1% and 99% levels. Panel B presents the estimated alphas from 36-month rolling regressions using various factor models. Panel C presents of estimated active alphas using various factor models.

	#obs	Mean	SD	25th perc	Median	75th perc
Panel A: Fund Characteristics						
Monthly net return	298,055	0.779%	5.130%	-1.870%	1.250%	3.830%
Percentage fund flow	298,055	-0.097%	4.160%	-1.490%	-0.409%	0.832%
Fund size ($mil)	298,055	1,277	2,838	105.5	324.6	1,041
Expense ratio (per month)	298,055	0.098%	0.110%	0.078%	0.096%	0.117%
Age (months)	298,055	198.9	161.7	92	147	238
Return volatility (t-12 to t-1)	298,055	4.635	2.093	3.03	4.231	5.769
Fund Beta	298,055	0.998	0.165	0.909	0.998	1.083
Panel B: Fund Performance - Standard Alpha (per month)						
CAPM alpha	298,055	0.059%	2.310%	-0.997%	0.019%	1.050%
FF3 alpha	298,055	0.005%	1.830%	-0.866%	-0.001%	0.860%
Carhart4 alpha	298,055	-0.006%	1.800%	-0.856%	-0.007%	0.840%
PS5 alpha	298,055	0.003%	1.830%	-0.854%	0.004%	0.859%
Panel C: Fund Performance - Active Alpha (per month)						
CAPM active alpha	297,926	0.011%	3.020%	-1.530%	-0.019%	1.500%
FF3 active alpha	297,977	-0.048%	2.620%	-1.450%	-0.046%	1.370%
Carhart4 active alpha	297,981	-0.038%	2.640%	-1.380%	-0.032%	1.350%
PS5 active alpha	297,970	-0.050%	2.620%	-1.410%	-0.026%	1.340%

Table 2: Beta Anomaly in Mutual Fund Returns

Panel A reports average annualized abnormal return (alpha) for deciles of mutual funds sorted according to market beta exposures. We estimate the beta for a mutual fund using each of the four performance evaluation models. Market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. Panel B reports average annualized active alphas for deciles of mutual funds sorted according to market beta exposures. Each month, alphas and active alphas are estimated according to Section xx for each of the four performance evaluation models. We use Newey-West (1987) standard errors with 18 lags; t-statistics are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

	Panel A: Alpha				Panel B: Active Alpha			
Beta Group	CAPM	FF3	Carhart4	PS5	CAPM	FF3	Carhart4	PS5
1 (low)	2.50%	1.80%	1.80%	1.81%	-1.02%	-1.29%	-1.42%	-1.27%
2	1.56%	1.10%	0.92%	0.94%	-1.52%	-1.01%	-0.74%	-0.49%
3	1.06%	0.91%	0.55%	0.77%	-1.40%	-1.46%	-0.18%	-1.37%
4	0.72%	0.70%	0.46%	0.64%	-0.34%	-1.04%	0.17%	-1.10%
5	0.52%	0.54%	0.35%	0.48%	-0.69%	-0.50%	-0.26%	-0.43%
6	0.60%	0.19%	0.03%	0.19%	0.35%	-0.13%	-0.83%	-0.45%
7	0.50%	0.08%	-0.16%	0.03%	0.97%	0.30%	-0.82%	-0.12%
8	-0.04%	-0.18%	-0.44%	-0.19%	0.57%	0.07%	-0.74%	0.04%
9	-0.36%	0.19%	-0.19%	-0.17%	1.36%	0.67%	0.06%	0.42%
10 (high)	-0.99%	-0.73%	-1.18%	-0.89%	2.29%	1.19%	0.59%	0.30%
High-Low	-3.48%	-2.53%	-2.98%	-2.69%	3.31%	2.48%	2.01%	1.56%

Table 3: Beta Anomaly in Mutual Fund Returns

Panel A shows the Fama-MacBeth (1973) estimates of annualized alpha regressed on market beta exposure. We estimate the beta for a mutual fund using each of the four performance evaluation models. Panel B shows the Fama-MacBeth (1973) estimates of annualized active alpha regressed on market beta exposure. Market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. The controls in Column (1) - (4) are nine Morningstar category dummies. The controls in Column (5) - (8) are nine Morningstar category dummies, size, and age. Standard errors are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: Alpha	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	CAPM	Carhart4	PS5	FP6	CAPM	Carhart4	PS5	FP6
Beta	-0.050**	-0.060***	-0.058***	-0.031**	-0.051**	-0.061***	-0.059***	-0.032***
	(0.020)	(0.012)	(0.012)	(0.013)	(0.020)	(0.012)	(0.012)	(0.012)
Size					0.002***	0.002***	0.002***	0.002***
					(0.000)	(0.001)	(0.001)	(0.000)
Age					-0.003**	-0.004***	-0.004***	-0.004***
					(0.001)	(0.000)	(0.001)	(0.001)
Constant	0.063***	0.067***	0.066***	0.0422***	0.062***	0.072***	0.073***	0.053***
	(0.023)	(0.011)	(0.011)	(0.012)	(0.022)	(0.010)	(0.010)	(0.011)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES	YES
Observations	270,266	270,266	270,266	270,266	269,714	269,714	269,714	269,714
R-squared	0.360	0.148	0.143	0.122	0.371	0.160	0.156	0.134

Panel B: Active Alpha	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	CAPM	Carhart4	PS5	FP6	CAPM	Carhart4	PS5	FP6
Beta	0.036***	0.032*	0.029	0.021	0.035***	0.030	0.027	0.020
	(0.013)	(0.019)	(0.020)	(0.019)	(0.013)	(0.019)	(0.019)	(0.019)
Size					0.002***	0.002***	0.002***	0.002***
					(0.000)	(0.000)	(0.001)	(0.001)
Age					-0.004***	-0.004***	-0.004***	-0.004***
					(0.001)	(0.001)	(0.001)	(0.001)
Constant	-0.030**	-0.029	-0.027	-0.019	-0.025	-0.024	-0.019	-0.007
	(0.014)	(0.019)	(0.021)	(0.020)	(0.016)	(0.018)	(0.019)	(0.020)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES	YES
Observations	270,119	270,179	270,159	270,153	269,567	269,627	269,607	269,601
R-squared	0.222	0.110	0.104	0.099	0.232	0.122	0.114	0.110

Table 4: Active Alpha Persistence

This table reports the results of Fama-MacBeth regressions of the future annualized active alpha rank on the past annualized active alpha rank. A fund's rank represents its percentile performance relative to other funds within the same Morningstar-style box during each month. The rank ranges from 0 to 1. Column (2) reports the monthly cross-sectional regression of annualized active alpha rank on prior month's annualized active alpha rank. We use Newey-West (1987) standard errors with twenty-four lags for column (2). Column (3) reports the monthly cross-sectional regression of annualized active alpha rank on prior year's annualized active alpha rank. We use Newey-West (1987) standard errors with eighteen lags for column (3). Column (4) reports the monthly cross-sectional regression of annualized active alpha on prior two year's annualized active alpha rank. We use Newey-West (1987) standard errors with twelve lags for column (4). In Panel A, active alpha is based on CAPM model. In Panel B, active alpha is based on four-factor model. In Panel C, active alpha is based on six-factor model. Standard errors are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: CAPM Active Alpha

	Active Alpha$_t$	Active Alpha$_{t+11}$	Active Alpha$_{t+23}$
Active Alpha$_{t-1}$	0.897***	0.103***	0.006
	(0.004)	(0.013)	(0.014)
Log Fund size	0.000	0.000	0.002
	(0.000)	(0.003)	(0.003)
Log Exp. Ratio	-0.079	-0.118	0.342
	(0.116)	(0.785)	(0.816)
Log Age	-0.001	-0.004	-0.001
	(0.001)	(0.003)	(0.004)
Return Volatility	0.0017	0.012**	0.011**
	(0.001)	(0.005)	(0.005)
Flow	-0.013	-0.342***	-0.094**
	(0.024)	(0.132)	(0.043)
Constant	0.049***	0.424***	0.443***
	(0.006)	(0.037)	(0.036)
R-squared	0.817	0.056	0.039
Observations	267,316	245,578	221,225
Correlation	0.901	0.103	0.018

Table 4 continued

Panel B: Carhart4 Active Alpha

	Active Alpha$_t$	Active Alpha$_{t+11}$	Active Alpha$_{t+23}$
Active Alpha$_{t-1}$	0.898***	0.150***	0.0347***
	(0.003)	(0.014)	(0.010)
Log Fund size	0.000	0.0013	0.002
	(0.000)	(0.003)	(0.003)
Log Exp. Ratio	-0.154	-1.154	-1.233
	(0.100)	(0.729)	(0.819)
Log Age	-0.001	-0.009**	-0.011**
	(0.001)	(0.004)	(0.004)
Return Volatility	0.001	0.007	0.004
	(0.001)	(0.005)	(0.005)
Flow	0.005	-0.168**	-0.099
	(0.02)	(0.084)	(0.060)
Constant	0.052***	0.453***	0.540***
	(0.005)	(0.028)	(0.038)
R-squared	0.817	0.068	0.043
Observations	267,374	245,616	221,260
Correlation	0.901	0.146	0.040

Panel C: FP6 Active Alpha

	Active Alpha$_t$	Active Alpha$_{t+11}$	Active Alpha$_{t+23}$
Active Alpha	0.900***	0.152***	0.059***
	(0.004)	(0.014)	(0.011)
Log Fund size	0.000	-0.001	-0.001
	(0.000)	(0.002)	(0.003)
Log Exp. Ratio	-0.059	-0.359	-0.763
	(0.114)	(0.830)	(0.720)
Log Age	-0.001	-0.006	-0.006
	(0.001)	(0.004)	(0.005)
Return Volatility	0.000	0.0054	0.005
	(0.001)	(0.004)	(0.004)
Flow	0.017	-0.144*	-0.019
	(0.015)	(0.079)	(0.068)
Constant	0.054***	0.447***	0.495***
	(0.005)	(0.027)	(0.035)
R-squared	0.820	0.064	0.042
Observations	267,349	245,616	221,260
Correlation	0.900	0.130	0.047

Table 5: Active Alpha Sort Portfolio

This table reports performance of active-alpha sorted calendar-time mutual fund portfolios. Each month, mutual funds are assigned to one of ten deciles mutual fund portfolios based on prior month's annualized active alpha. Panel A reports CAPM active alpha sort results. Panel B reports Carhart4 active alpha sort results. Panel C reports FP6 active alpha sort results. All mutual funds are equally weighted within a given portfolio, and the portfolios are rebalanced every month to maintain equal weights. column (2) - column (5) report mutual fund portfolio's time series average of gross return, market adjusted return, Sharpe ratio, information ratio, and Carhart4 alpha. We use Newey-West (1987) standard errors with eighteen lags; t-statistics are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: CAPM Active Alpha

Active Alpha	Gross Ret	MAR	Shar. R.	Info. R.	Carhart4 Alpha
1 (low)	0.901%	-0.175%	0.1484	0.1479	-0.045%
2	0.949%	-0.122%	0.1653	0.1649	-0.030%
3	0.958%	-0.111%	0.1683	0.1679	-0.018%
4	0.959%	-0.110%	0.1669	0.1666	-0.025%
5	0.973%	-0.096%	0.1689	0.1685	-0.018%
6	0.988%	-0.081%	0.1740	0.1737	0.010%
7	1.026%	-0.044%	0.1859	0.1857	0.021%
8	1.064%	-0.006%	0.1914	0.1911	0.037%
9	1.110%	0.038%	0.2036	0.2034	0.068%
10 (high)	1.199%	0.120%	0.2042	0.2039	0.116%
High-Low	0.298%***	0.295%***	0.0557**	0.0560**	0.161%
t-stats	(2.636)	(2.602)	(2.057)	(2.064)	(0.949)

Panel B: Carhart4 Active Alpha

Active Alpha	Gross Ret	MAR	Shar. R.	Info. R.	Carhart4 Alpha
1 (low)	0.906%	-0.173%	0.1483	0.1479	-0.058%
2	0.975%	-0.097%	0.1643	0.1638	-0.006%
3	0.991%	-0.079%	0.1722	0.1719	-0.017%
4	0.983%	-0.086%	0.1731	0.1728	-0.015%
5	0.995%	-0.073%	0.1738	0.1735	0.006%
6	0.999%	-0.070%	0.1748	0.1744	0.009%
7	1.002%	-0.066%	0.1820	0.1817	-0.002%
8	1.026%	-0.044%	0.1859	0.1856	0.023%
9	1.044%	-0.027%	0.1889	0.1886	0.012%
10 (high)	1.206%	0.128%	0.2112	0.2110	0.166%
High-Low	0.300%***	0.301%***	0.0629***	0.0631***	0.224%***
t-stats	(3.000)	(3.022)	(2.906)	(2.911)	(2.673)

Table 5 continued

Panel C: FP6 Active Alpha

Active Alpha	Gross Ret	MAR	Sharpe R	Info. R	Carhart4 Alpha
1 (low)	0.976%	-0.101%	0.1639	0.1635	0.001%
2	0.990%	-0.080%	0.1697	0.1693	0.002%
3	0.986%	-0.083%	0.1758	0.1754	0.000%
4	0.984%	-0.085%	0.1685	0.1681	-0.037%
5	0.961%	-0.107%	0.1704	0.1701	-0.015%
6	0.996%	-0.073%	0.1770	0.1767	-0.011%
7	0.983%	-0.086%	0.1731	0.1728	-0.022%
8	1.005%	-0.065%	0.1769	0.1766	-0.001%
9	1.061%	-0.012%	0.1941	0.1939	0.063%
10 (high)	1.182%	0.102%	0.2056	0.2053	0.134%
High-Low	0.205%***	0.203%***	0.0417***	0.0419***	0.133%**
t-stats	(2.658)	(2.639)	(2.461)	(2.459)	(1.967)

Table 6: Fund Flows and Competing Measures

This table presents regression coefficient estimates from panel regressions of monthly fund flow (dependent variable) on the lagged rank of annualized active alpha and the lagged rank of annualized alpha. In Panel A and Panel B, a mutual fund's rank represents its percentile performance relative to all other mutual funds during the same month. In Panel C and Panel D, a fund's rank represents its percentile performance relative to other funds within the same Morningstar-style box during the same month. The rank ranges from 0 to 1. We calculate the rank based on annualized alpha and annualized active alphas. Controls include lagged fund flows from month t-13, lagged values of log of fund size, log of fund age, expense ratio, return volatility, and style-month fixed effects. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Dependent Variable: Flow

Panel A (1): Overall Alpha Percentile Rank				Panel A (2): Overall Active Alpha Percentile Rank			
	CAPM	Carhart4	FP6		CAPM	Carhart4	FP6
Alpha	0.039***	0.029***	0.027***	Active Alpha	0.025***	0.020***	0.019***
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Observations	269,610	269,610	269,610		269,463	269,523	269,497
R-squared	0.243	0.232	0.225		0.219	0.213	0.209
Style-month fixed effects	YES	YES	YES		YES	YES	YES
Controls	YES	YES	YES		YES	YES	YES

Panel B : Overall Alpha Percentile Rank & Overall Active Alpha Percentile Rank			
	CAPM	Carhart4	FP6
Active Alpha Percentile Rank	0.006***	0.003***	0.002***
	(0.001)	(0.001)	(0.001)
Alpha Percentile Rank	0.035***	0.028***	0.025***
	(0.001)	(0.001)	(0.001)
Observations	269,463	269,523	269,497
R-squared	0.244	0.232	0.226
Style-month fixed effects	YES	YES	YES
Controls	YES	YES	YES

Table 6 continued

Panel C (1): Alpha Percentile Rank within Style

	CAPM	Carhart4	FP6
Alpha	0.031***	0.027***	0.025***
	(0.001)	(0.001)	(0.001)
Observations	269,610	269,610	269,610
R-squared	0.243	0.231	0.225
Style-month fixed effects	YES	YES	YES
Controls	YES	YES	YES

Panel D(2): Active Alpha Percentile Rank within Style

	CAPM	Carhart4	FP6
Active Alpha	0.022***	0.019***	0.018***
	(0.001)	(0.001)	(0.001)
Observations	269,463	269,523	269,497
R-squared	0.219	0.212	0.209
Style-month fixed effects	YES	YES	YES
Controls	YES	YES	YES

Panel D : Alpha Percentile Rank within Style & Active Alpha Percentile Rank within Style

	CAPM	Carhart4	FP6
Active Alpha Percentile Rank	0.006***	0.003***	0.003***
	(0.001)	(0.001)	(0.001)
Alpha Percentile Rank	0.028***	0.025***	0.023***
	(0.001)	(0.001)	(0.001)
Observations	269,463	269,523	269,497
R-squared	0.244	0.232	0.225
Style-month fixed effects	YES	YES	YES
Controls	YES	YES	YES

Table 7: Fund Flow Response to Fund Return Components

This table presents regression coefficient estimates from panel regressions of monthly fund flow (dependent variable) on the components of a fund's return - a funds' active alpha, passive alpha, and six factor-related return. Factor-related returns are estimated based on the funds' factor exposure and the factor return. The six factors include the market, size, value, momentum, liquidity, and betting-against-beta. Controls include lagged fund flows from month t-13, lagged values of log of fund size, log of fund age, expense ratio, return volatility, and style-month fixed effects. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Dependent Variable: Flow	
Active Alpha	0.151***
	(0.006)
Passive Alpha	0.150***
	(0.007)
Mkt Ret	0.051***
	(0.008)
Size Ret	0.117***
	(0.013)
Value Ret	0.092***
	(0.010)
Mom Ret	0.121***
	(0.008)
Liquidity Ret	0.196***
	(0.014)
Bab Ret	0.112***
	(0.011)
Observations	269,497
R-squared	0.246
Style-month fixed effects	YES
Controls	YES

Table 8: Investor Sophistication and Flow-Active Alpha Relationship

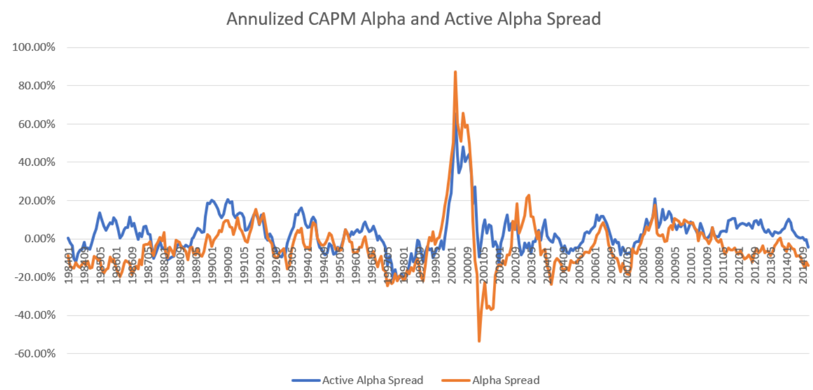
This table presents regression coefficient estimates from panel regressions of monthly flow to institution/retail share class (dependent variable) on lagged rank of annualized active alpha and the interactions with institution share class dummy variable. A share class is defined as institutional share class if Morningstar share class is INST or CRSP institution fund dummy is 1. For each mutual fund, the flow to its institutional class is the value-weighted flow across fund's multiple institutional classes. Similar, the flow to fund's retail share class is the value-weighted flow across fund's retail classes. Panel A the regression result for all of the mutual funds in our sample over the period 1984 to 2014. In Panel B, we restrict the sample to mutual funds with both institutional and retail share classes. Controls include lagged rank of annualized alpha, lagged fund flows from month t-13, lagged values of log of fund size, log of fund age, expense ratio, return volatility, and style-month fixed effects. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Dependent Variable: Flow

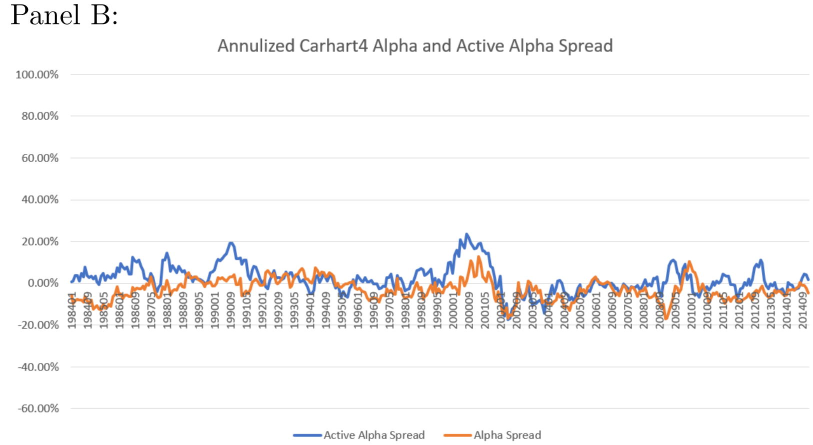
Panel A: Full Sample				Panel B: Smaller Sample		
	CAPM	Carhart4	FP6	CAPM	Carhart4	FP6
Active Alpha	0.020***	0.016***	0.014***	0.019***	0.014***	0.012***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
Inst. Class * Active Alpha	0.024***	0.022***	0.023***	0.025***	0.023***	0.024***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Log Fund size	0.001***	0.001***	0.001***	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Log Exp. Ratio	0.450***	0.507***	0.468***	0.313*	0.389**	0.348**
	(0.090)	(0.091)	(0.090)	(0.162)	(0.163)	(0.162)
Log Age	-0.002***	-0.002***	-0.002***	-0.001	-0.001	-0.001
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
Return Volatility	-0.002***	-0.002***	-0.002***	-0.003***	-0.002***	-0.002***
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
Lagged Flow	0.155***	0.152***	0.152***	0.185***	0.182***	0.182***
	(0.007)	(0.007)	(0.007)	(0.010)	(0.011)	(0.011)
Constant	0.008	-0.023	0.100***	0.012	-0.000	-0.167
	(0.031)	(0.035)	(0.003)	(0.021)	(0.000)	(0.222)
R-squared	0.058	0.055	0.055	0.040	0.037	0.037
Observations	382,780	382,867	382,815	226,635	226,689	226,637
Style-month fixed effects	YES	YES	YES	YES	YES	YES

Figure 1: Time series of spreads between high beta and low beta mutual fund portfolios
This figure plots the annualized alpha and annualized active alpha spreads between highest beta portfolio and lowest beta decile portfolio (decile 10 - decile1). Each month, mutual funds are ranked into equal-weight decile portfolios based on market beta exposures estimate. Mutual fund alphas, active alphas, and market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. In Panel A, we report annualized alpha and active alpha spreads based on CAPM; in the middle Panel B, we report annualized alpha and active alpha spreads based on Carhart four-factor model; and in Panel C, we report annualized alpha and active alpha spreads based on FP six-factor model.

Panel A:



Panel B:



Panel C:

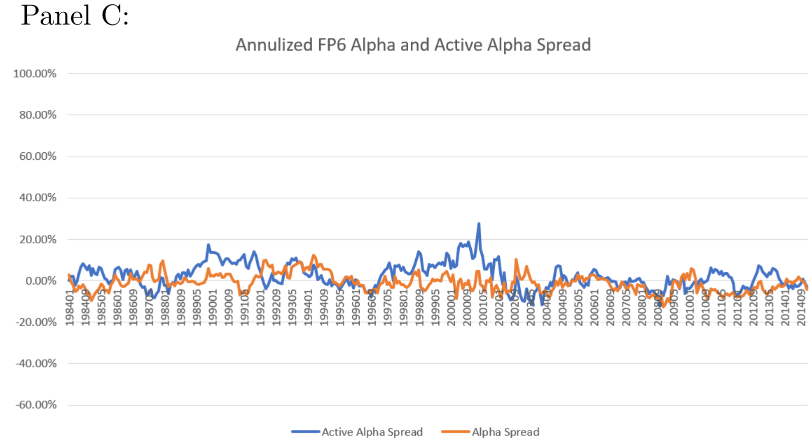
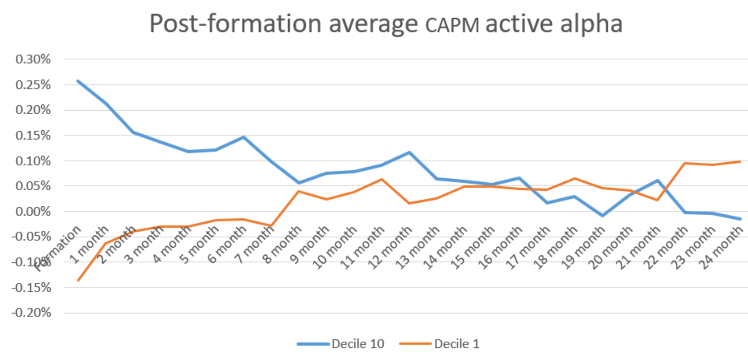


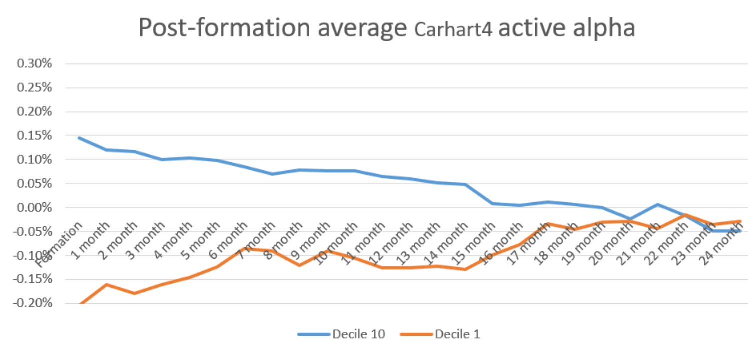
Figure 2: Persistence of Active Alpha

This figure depicts the average monthly active alpha of portfolios tracked over a 2-year period between 1984 and 2014. The portfolios are formed by sorting all the funds into deciles according to lagged annualized active alpha. Subsequently, the top and bottom decile portfolios are tracked over the next 2-year period. The portfolios are equally weighted each month, so the porfolios are readjusted whenever a fund disappears from the sample. In Panel A, we report the CAPM active alphas; in Panel B, we report the Carhart4 active alpha; and in Panel C, we report the FP6 active alpha.

Panel A:



Panel B:



Panel C: