A life-cycle optimization model using semi-Markov process for highway bridge maintenance

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\section*{A B S T R A C T}

Due to a variety of risks related to aging, construction, material degradation, harsh environment, increasing traffic, and insufficient capacity, a large percentage of bridges in the U.S. highway system are deteriorating beyond acceptable standards. Although significant investments are needed to bring bridges back to acceptable condition, most highway agencies lack the appropriate funding and therefore need effective methodologies for allocating limited resources efficiently and cost-effectively. This paper presents a life-cycle optimization model using a semi-Markov process and demonstrates how the proposed method can assist highway agencies to make more quantitative and explicit decisions for bridge maintenance. The 2012 National Bridge Inventory (NBI) dataset for the State of Texas was analyzed in this study to illustrate bridge structural responses and behaviors under uncertainty and risks. The proposed method is accurate when compared to real data and customized to help highway agencies to optimize their decisions on structuring bridge maintenance, and consequently, leading to cost savings and more efficient sustainability of their bridge systems. The major contribution of this research is the low-error model and process algorithm for selecting the most appropriate maintenance strategy. If employed properly, it may allow agencies to more effectively maintain an aging infrastructure system.

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1. Introduction

Highway bridges are vital links in the U.S. transportation system, but a large portion of them are deteriorating constantly due to various risks, such as high traffic volumes, wetting and drying cycles, freezing and thawing attacks, the level of exposure to deicing salts and wet environments, aging and quality/durability of construction materials, and more. Repairing these critical pieces of infrastructure would require a significant investment, but most highway agencies lack the necessary funds to address the backlog of bridge repairs. Highway agencies are therefore increasingly recognizing the need for a life-cycle optimization model to allocate limited funding in a more efficient and cost-effective way for bridge maintenance. Equipped with such models, highway agencies can forecast the long-term bridge reliability or deterioration process with consideration of the impacts of various risks, obtain information on the need and timing of maintenance activities on the planning horizon, and evaluate different maintenance strategies by life-cycle assessment and repair efficacy.

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As the essential part of a life-cycle maintenance optimization model, bridge long-term reliability or deterioration models were developed by many researchers. Despite their substantial differences in detail, existing models could be divided into three groups roughly:

- The first group establishes empirical relationships between the factors affecting bridge deterioration and the measures of a bridge’s condition. The output of such models is expressed by deterministic values, and no probabilities are involved. These deterministic models can be categorized as using straight-line extrapolation, regression, and curve-fitting methods [1–3].

- The second group describes the specific deterioration mechanisms of particular bridge components and focuses on the reliability of bridges with respect to strength limit states (load versus resistance). The deterioration models are dependent on deterministic mathematical formulas or probabilistic analyses using the Monte Carlo Simulation (MCS). For instance, by using the MCS model, Sobanjo [4] developed the mechanism of the corrosion process for the superstructure of steel bridges. Frangopol et al. [5], as pioneers in the field of life-cycle civil engineering, proposed a lifetime optimization methodology for planning the inspection and repair of deteriorating bridge structures. Estes and Frangopol [6] provided a system reliability approach for optimizing the lifetime repair strategies of highway bridges. Other studies along this line, including Estes and Frangopol [7,8], Akgül and Frangopol [9,10], and Biondini et al. [11], focus on specific types of bridges or bridge components. Recently, Bocchini et al. [12] and Furuta et al. [13] extended the analysis from an individual bridge to the network level.

- The third group considers the bridge deterioration process stochastic in nature and creates probabilistic forecasting models. The state-of-the-art stochastic model has been proposed mainly based on the classic Markov chain theory. According to the Federal Highway Administration (FHWA) condition rating for the deterioration of the Indiana Department of Highway (IDOH) bridges, Jiang and Sinha [14] developed a performance prediction model by using the Markov chain. Pontis [15], in a very well-known FHWA-sponsored Bridge Management System (BMS), utilized the Markov chain in the development of the core-element deterioration model. In the Markov-chain deterioration model, the performance level is specified as a series of discrete states, and the bridge performance changes from one state to another in accordance with a set of constant transition probabilities. Thompson and Johnson [16] performed analysis on the California bridge dataset to quantify the Markovian transition probabilities actually observed. Similar work can also be found in Puz and Radic [17], in which a probabilistic life-cycle performance model based on homogeneous Markov processes was proposed to calculate future deterioration of the infrastructure at each moment. Ng and Moses [18] proposed a relaxed time homogeneity of the Markov process in a more general stochastic model called the semi-Markov process. Other examples include the works by Cesare et al. [19] and Kleiner [20], in which they applied reliability models through the semi-Markov process in the management of bridges and large infrastructure assets.

However, these existing deterioration models and the related optimization systems still have some noticeable drawbacks, including: 1) failing to incorporate multiple relevant factors that impact the bridge deterioration process, such as climate, traffic density, material property, bridge route type, etc.; 2) lacking the ability to predict the performance of a bridge that has undergone repair or maintenance activities; 3) failing to optimize maintenance strategies from the viewpoints of economy and repair efficacy; 4) assuming discrete transition time intervals, a constant bridge population, or stationary transition probabilities in some of the Markov-based models.

To overcome these limitations, a life-cycle optimization model using a semi-Markov process is proposed in this study to make the decision-making process of bridge management more quantitative and explicit for highway agencies. Various factors are considered in this model, and the bridge deterioration is assumed as a semi-Markov process, which means that the durations in each condition state, also called state waiting times, are defined as random variables with probability distributions (e.g., the well-known Weibull distribution). The timing and costs of interventions (maintenance activities, rehabilitations, and repairs) in a bridge’s life-cycle are determined by the proposed model, and the most efficient and cost-effective procedure can be selected from several alternative maintenance strategies. Compared with the existing models, the proposed model has at least three unique features: the impacts of various factors, including traffic volumes (or Average Daily Traffic (ADT)), bridge types, construction materials, etc., are considered thoroughly; a semi-Markov chain that relaxes the homogeneous limitation of traditional Markov transition matrices is adopted herein; and a life-cycle cost analysis that considers different repair strategies along the planning horizon is integrated with the model. Through this model, highway agencies have great potential for optimizing their decisions on structuring the bridge life-cycle maintenance plan, leading to cost savings and more efficient sustainability of their transportation infrastructure.

The rest of this paper starts from the discussion of bridge performances according to the 2012 NBI dataset for Texas, followed by the methodological approach of the proposed life-cycle optimization model. The concepts of semi-Markov processes and deterioration states are elaborated as well in this section. Finally, a case study is provided to demonstrate how to apply the proposed model step-by-step to the maintenance practice of steel bridges in Texas.

2. Texas bridge performance according to the 2012 NBI dataset

Since the definition of semi-Markov states and the proposed model are closely related to the bridge performance recorded in the NBI database, the 2012 NBI records for Texas bridges are analyzed to demonstrate the proposed model. The 2012 NBI database contains a great deal of information related to bridge performance, including such parameters as
material types, roadway types, traffic volumes (or ADT), bridge ages, and more. The performance ratings are assigned numerically by inspectors during mostly biannual inspections. After examining the rating factors carefully, the research team selected Sufficiency Rating (SR), a method of evaluating highway bridges by calculating four separate factors (i.e. structural evaluation, functional obsolescence, etc.) to obtain a numeric value (i.e., percentage) that is indicative of bridge sufficiency to remain in service, as the main performance indicator in this study [21]. SR is essentially a comprehensive rating of a bridge’s fitness for the duty it performs based on factors derived from over 20 NBI data fields, including fields that describe its structural evaluation, functional obsolescence, and its essentiality to the public. One hundred percent represents an entirely sufficient bridge, and zero percent represents an entirely insufficient or deficient bridge.

In the 2012 NBI dataset, the records for each bridge include over one hundred pieces of information, each identified by an item number. For example, Item 029 presents the average daily traffic, Item 058 indicates the rating of deck conditions, and Item 043A groups bridges by material types. It should be noted that the deterioration process (and accordingly, the Markov transition matrix) is greatly affected by the service or site condition to which the bridge is exposed. Thus, the following actions are taken for extracting information from the 2012 NBI records for Texas and filtering the bridges into six subgroups as shown in Table 1:

1. Bridges that have undergone reconstruction or rehabilitation are removed from the inventory.
2. Bridges are filtered out by Functional Classification of Inventory Route (NBI Item 026). Only Interstate (codes 01 and 11) and Local (codes 09 and 19) bridges are considered in the analyses.
3. Only common structural materials are considered, excluding wood, masonry, or aluminum structures from the study. For each route type, bridges are divided into three subgroups due to the main structural material type: Concrete (NBI Item 043A codes 01 and 02), Steel (NBI Item 043A codes 03 and 04), and Pre-stressed Concrete (NBI Item 043A codes 05 and 06). It should be noted that a bridge may be categorized in the NBI as steel, concrete, or timber, but not all components are necessarily constructed of that material.
4. Average Daily Traffic (NBI Item 029, ADT) is considered relevant due to the direct effects of traffic loading on deterioration as well as the likely relationship between the amount of deicing salt used (harmful chloride exposure) and traffic volume. For the Interstate route type, bridges under heavy traffic conditions (ADT ≥ 5000 vehicles per day (vpd)) are included, representing the most severe service condition. For the local route type, only bridges with light traffic (ADT < 5000 vpd) are retained.
5. Sufficiency Rating (SR) is used to evaluate the bridge condition numerically, ranging from a low of 0% to a high of 100%. A rating of 90% or higher indicates excellent condition, and a rating of 20% or lower indicates a critical condition or imminent failure condition. Other ratings indicate: poor condition, fair condition, satisfactory or good condition, respectively.

Figs. 1 and 2 depict regression curves for the performance and SR values of selected interstate and local bridges, respectively. The life-cycle performance curves, as depicted in these figures, can be characterized as follows:

- The potential of the bridge deterioration process (or state transition matrix) is closely related to bridge types, construction materials, and service conditions. Once the category of a bridge is known, its transition matrix of degeneration can then be determined correspondingly.
- Regarding interstate bridges, steel and pre-stressed concrete perform better than concrete when the bridge is less than 50-year-old. However, once this age is passed, the deterioration of these materials accelerates and pasts that of concrete. Pre-stressed concrete bridges apparently have a faster deterioration rate than steel bridges, especially after the age of 60-year-old. Concrete bridges typically provide the most consistent performance during their life cycles.
- Regarding local bridges, concrete bridges present the best performance and pre-stressed concrete bridges take the second place, while steel bridges still deteriorate with the fastest rate, which is the same trend as observed from interstate bridges.
3. Life-cycle maintenance optimization model using semi-Markov process

3.1. Basic concepts of the Markov process

In a discrete-time Markov process as a stochastic process with states $X(t)$, for any $n$ time points $t_1, t_2, ..., t_n$, the conditional distribution of $X(t_n)$ for given values of $\{X(t_1), ..., X(t_{n-1})\}$ depends only on $X(t_{n-1})$, which is the most recent known value. This can be stated as:

$$P[X(t_n) \leq x_n | X(t_1) = x_1, X(t_2) = x_2, ..., X(t_{n-1}) = x_{n-1}] = P[X(t_1) \leq x_1 | X(t_1) = x_1].$$

(1)
When the Markov process goes from state $X_i$ to state $X_j$, it is said to transition from state $i$ to state $j$. A Markov transition probability matrix denotes the transition probability from state $i$ to state $j$ during time $t$, as shown below:

$$
p^{t+1} = \begin{bmatrix}
p_{11}^{t+1} & p_{12}^{t+1} & \cdots & p_{1n}^{t+1} \\
p_{21}^{t+1} & p_{22}^{t+1} & \cdots & p_{2n}^{t+1} \\
\vdots & \vdots & \ddots & \vdots \\
p_{n1}^{t+1} & \cdots & \cdots & p_{nn}^{t+1}
\end{bmatrix} : p_{ij}^{t+1} \geq 0(i, j = 1, 2, \ldots, n); \sum_{j=1}^{n} p_{ij}^{t+1} = 1(i = 1, 2, \ldots, n). \quad (2)
$$

In an $n$-state space discrete Markov process, the state of the process at any time $t$ is typically stochastic and is defined by a Probability Mass Function (PMF) that is denoted by an $n$-dimensional vector $A(t)$.

$$
A(t) = \left\{ a_1^t, a_2^t, \ldots a_n^t \right\} \left( \sum_{i=1}^{n} a_i^t = 1 \right). \quad (3)
$$

Member $a_i^t$ denotes the probability that the process is in state $i$ at time $t$. The PMF of the process at time $(t + k)$ is obtained:

$$
A(t + k) = A(t){p}^{t+1}{p}^{t+2}\ldots{p}^{t+k-1}{p}^{t+k}. \quad (4)
$$

For a stationary Markov chain, the above equation can be reduced to:

$$
A(t + k) = A(t){p}^k. \quad (5)
$$

In which the Markov chain is stationary in time (or homogeneous) and the Markov transition probability matrices are constant.

Usually, under the condition of no interventions (renewals or rehabilitations), the process of bridge degradation is unidirectional, i.e., if state 1 denotes good as new and state $n$ denotes failure, then the process can move only from state 1 to state $j$. Further, it is often assumed that a bridge can deteriorate only one state at a time, that is, the bridge will deteriorate from state 1 to state 2, then to state 3, and so on to failure. The process thus cannot jump from state 1 to state 3 without passing through state 2. If a bridge is expected to deteriorate very fast, one can shorten transition periods to the extent that realistically only single state deterioration is possible in one period. The time period is assumed to be one or two years, since highway agencies normally inspect bridges every other year to update the NBI dataset. This leads to a relatively simple Markov transition probability matrix:

$$
p^{t+1} = \begin{bmatrix}
p_{11}^{t+1} & p_{12}^{t+1} & \cdots & 0 \\
p_{21}^{t+1} & p_{22}^{t+1} & \cdots & 0 \\
0 & 0 & \ddots & \cdots \\
0 & 0 & \cdots & p_{nn}^{t+1}
\end{bmatrix} \quad (6)
$$

### 3.2. Semi-Markov process

In the real practice, a bridge deterioration process in which sojourn (or waiting) times in any given state are time-dependent distributed random variables cannot be captured by a discrete time Markov chain. In this regard, a semi-Markov chain with various transition probability matrices can relax this limitation and is more suitable to capture the bridge’s real degradation process. A semi-Markov process is a class of stochastic processes that moves from one state to another with the successive states visited forming a Markov chain. The process stays in a particular state for a random length of time, and its distribution depends on the state and on the next state to be visited [18].

In a semi-Markov process, $T_1, T_2, \ldots, T_{n-1}$ are random variables denoting the sojourn times in states $\{1, 2 \ldots n-1\}$, respectively. Their corresponding Probability Density Functions (PDFs), Cumulative Density Functions (CDFs), and Survival Functions (SFs) are thus denoted $f_1(t), F_1(t)$, and $S_1(t)$. $T_{1-k}$ is the time it will take the process to go from state $i$ to state $k$. In addition, $f_{1-k}T_{1-k}, F_{1-k}(T_{1-k}), S_{1-k}(T_{1-k})$ are the PDF, CDF, and SF of $T_{1-k}$, respectively. If the deterioration process is in state 1 at time $t$, the conditional probability that it will transit to the next state in the next time step $\Delta t$ is given by:

$$
Pr[X(t + 1) = 2|X(t) = 1] = p_{1,2}(t) = \frac{f_1(t)\Delta t}{S_1(t)}. \quad (7)
$$

where $t = 0$ is the time when the process enters into state 1 (i.e., new asset – in most cases). It should be noted that $\Delta t$ is assumed to be small enough to exclude a two-state deterioration. In this case study, $\Delta t$ is assumed to be one unit (year) and is thus omitted. Similarly, if the process is in state 2 at time $t$, the conditional probability that it will transit to the next state in the next time step $\Delta t$ is given by:

$$
Pr[X(t + 1) = 3|X(t) = 2] = p_{2,3}(t) = \frac{f_{1-2}(t)}{S_{1-2}(t) - S_1(t)}. \quad (8)
$$
Noted that in Eq. (8), the PDF $f_1 \rightarrow_2 t$ pertains to $T_1 \rightarrow_2$, which is the random variable denoting the sum of waiting times in states 1 and 2. Further, the denominator expresses the simultaneous condition that $T_1 \rightarrow_2 < t$ and $T_1 < t$, which is equivalent to the condition $X(t) = 2$. In a general form, if the deterioration process is in state $i$ at time $t$, the conditional probability that it will transit to the next state $i + 1$ in the next time step $\Delta t$ (usually one unit (year)) is given by:

$$
\Pr[X(t + \Delta t) = i + 1 | X(t) = i] = p_{i,i+1}(t) = \frac{s_{i,i+1}(t)}{s_{i,i}(t)} (i = 1, 2, ..., n - 1). \tag{9}
$$

Eq. (9) provides all the transition probabilities $p_{i,i+1}(t)$ to generate the transition probability matrix for the semi-Markov process. It is noted that these transition probabilities are assumed to be time-dependent, and thus the process is non-stationary (or non-homogeneous). Once the transition probability matrix is established, the deterioration process can be modeled to obtain the PMF of the process at any time $t$.

The waiting time $T_i$ of the process in any state $i$ is arbitrarily modeled as a random variable with a two-parameter Weibull distribution. Since the PDF $F_i(t)$ is assumed to be arbitrarily distributed random variables $T_{i-k}$ according to Eq. (10).

$$
F_i(t) = \Pr[T_i \leq t] = 1 - e^{-(\lambda_i t)^\upsilon_i} \tag{10}
$$

$$
S_i(t) = 1 - F_i(t) = e^{-(\lambda_i t)^\upsilon_i} \tag{11}
$$

$$
\frac{\delta F_i(t)}{\delta t} = \lambda_i \upsilon_i (\lambda_i t)^{\upsilon_i - 1} e^{-(\lambda_i t)^\upsilon_i}. \tag{12}
$$

The parameters for the deterioration model are based on historical observations and condition assessments of bridges since there are sufficient historical records in the NBI database. The parameters are then derived by Eq. (13) [20]:

$$
\begin{align*}
S_i(u) &= e^{-(\lambda_i u)^\upsilon_i} \\
S_i(v) &= e^{-(\lambda_i v)^\upsilon_i} \\
\ln[S_i(u)] &= -\lambda_i u^\upsilon_i \\
\ln[S_i(v)] &= -\lambda_i v^\upsilon_i \\
\beta_i &= \ln(\frac{\ln(S_i(u)) - \ln(S_i(v))}{\ln(u) - \ln(v)}) \\
\lambda_i &= \frac{1}{u} (-\ln[S_i(u)])^{1/\upsilon_i}.
\end{align*} \tag{13}
$$

where

$$
\beta_i = \ln(\frac{\ln[S_i(u)] - \ln[S_i(v)]}{\ln(u) - \ln(v)}), \quad \lambda_i = \frac{1}{u} (-\ln[S_i(u)])^{1/\upsilon_i}. \tag{14}
$$

In the above equations, if the probability of $S_i(u)$ and $S_i(v)$ of being in state $i$ at the $u$th and $v$th years are known, parameters $\beta_i$ and $\lambda_i$ can be derived. Once parameters $\beta_i$ and $\lambda_i$ are known, the transition probability matrix can be established. The determination of these parameters will be presented in detail in the case study given later.

### 3.3. The definition of semi-markov states

According to the FHWA bridge condition rating [21], five semi-Markov states for the degradation of bridges can be defined as follows: “State 1” is good as new (above 90% sufficiency), “State 2” (80–90% sufficiency), “State 3” (50–80% sufficiency), “State 4” (20–50% sufficiency), and “State 5” (less than 20% sufficiency). Also, repair strategies are correspondingly determined to be “Do-Nothing”, “Preventative Maintenance, i.e., “Repainting”, “Minor Repair”, “Major Repair”, and “Replacement,” for these five states, respectively.

### 3.4. Prediction of life-cycle costs

The Markov chain can be used to predict future costs of bridge rehabilitation activities within the life-cycle term. The expected cost of failure at age $t$ is the product of the cost of failure and the probability of failure at age $t$ [19].

$$
E[C^f(t)] = C^f a^t_r. \tag{15}
$$

It is assumed that the rehabilitation or repair can be modeled by forcing the current state of the bridge back to its previous states. This operation can be done systematically with the following Markov transition matrix $[R_e]$:

$$
R_e = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0.9 & 0 & 0.1 & 0 & 0 \\
0.8 & 0 & 0 & 0.2 & 0 \\
0.5 & 0 & 0 & 0 & 0.5 \\
\end{bmatrix}.
$$

It should be noted that the repair effects are subject to the restrictions of the bridge’s current conditions (i.e., the bridges in State 2 should have a greater likelihood to return to State 1 than those in State 3) and repair strategies (i.e., a major repair
activity should have a greater likelihood to restore bridge conditions than a minor repair activity). Therefore, it is reasonable to assume that there are 90%, 80%, and 50% probabilities to return to State 1 for bridges in States 3, 4, and 5, respectively. Based on these probabilities, the expected costs can be calculated by the following equation considering the effects of repair activities:

\[
E[C^F(t)] = \sum C^F A_i T^i R_e (1 + i)^{-t},
\]

where \(E[C^F(t)]\) denotes the expected cost of failure at age \(t\); \(R_e\) denotes the repair effect matrix; and \(i\) denotes the discount rate. The life-cycle costs of bridge maintenance based on the proposed model will be discussed in detail in the case study given later.

3.5. Algorithm

The algorithm of the proposed model is summarized by the following step-by-step procedures and also illustrated in a flowchart in Fig. 3 for better understanding:

- **Step 1**: Define the semi-Markov states in the bridge deterioration process according to the FHWA bridge condition rating.
- **Step 2**: Determine the parameters of the Weibull distributions of state waiting times by using the NBI historical data or Monte-Carlo Simulation if historical data is not readily available.
- **Step 3**: Establish the semi-Markov transition probability matrix based on the known Weibull-distributions of state waiting times.
- **Step 4**: Make bridge maintenance strategies based on existing bridge repair techniques and sound engineering judgment.
- **Step 5**: For each strategy, determine the PMF at the year when repair activity occurs by using the transition probability matrices.
- **Step 6**: Calculate the repair costs according to the PMF of the year when repair activity occurs.
- **Step 7**: Update the PMF of that year by using the repair matrix to reflect the repair effects.
- **Step 8**: Repeat Step 5–7 if there is another repair activity. Otherwise, sum the costs of all repair activities to obtain the life-cycle costs of each strategy.
- **Step 9**: Evaluate the repair effects of maintenance strategies and then determine the most optimal strategy based on cost and structural effect. Within the constraint of limited funding, the most optimal strategy should produce the highest structural performance in the whole lifespan of bridge while generating the minimum life-cycle costs.

4. Case study

4.1. Purpose

This case study focuses on the performance and maintenance of steel bridges carried by local routes in Texas, and its purpose is to illustrate: 1) how to apply the proposed model to formulate the semi-Markov chain and then to predict the bridge deterioration process based on the model; 2) how to calculate the life-cycle costs of different maintenance strategies based on the semi-Markov probability matrices; 3) how to evaluate the effects of rehabilitation activities and the post-repair behaviors of bridges; and 4) how to optimize the life-cycle bridge maintenance based on efficiency (or post-repair performance) and funding. In this example, only steel bridges without reconstruction are retrieved from the 2012 NBI dataset for Texas, which makes the deterioration of bridges in the service life a unidirectional decreasing curve. A discount rate of 3.5% is assumed, and the analysis period is 75 years, which is long enough to incorporate the cost of at least one rehabilitation activity for all alternatives [22]. The detailed analysis follows the step-by-step procedures discussed previously in Section 3.5.

4.2. Analysis procedure

In this case study, five maintenance strategies have been proposed. However, for illustrative purposes, only Strategy A is presented in detail. Other strategies can be analyzed in the same way, so only the results are presented. The analysis procedure includes:

- **Step 1**: Define five semi-Markov states as discussed previously: "State 1" is good as new (above 90% sufficiency), "State 2" (80–90% sufficiency), "State 3" (50–80% sufficiency), "State 4" (20–50% sufficiency), and "State 5" (less than 20% sufficiency).
- **Step 2**: Determine the parameters of the Weibull distributions of state waiting times: the probabilities of any two different years for each state can be determined from the 2012 NBI dataset for Texas, and then the parameters of the Weibull-distributed holding times are summarized in Table 2.

Next, the waiting times \(T_i\) in every state \(i\) and the sums of waiting times in the various states, \(T_{1:k}\), are then calculated by these parameters and, (9) and (10), as shown in Figs. 4 and 5 for the PDFs and SDFs, respectively.
Fig. 3. The flowchart of the proposed model.
Table 2
The parameters of weibull-distributed random variables $T_{i,k}$.

<table>
<thead>
<tr>
<th>State $i$</th>
<th>$u$</th>
<th>$P_{i,u}$ (%)</th>
<th>$v$</th>
<th>$P_{i,v}$ (%)</th>
<th>$\beta_i$</th>
<th>$\lambda_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>18.92</td>
<td>20</td>
<td>3.70</td>
<td>0.9856</td>
<td>0.1677</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>59.46</td>
<td>20</td>
<td>33.33</td>
<td>1.0471</td>
<td>0.0535</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>51.85</td>
<td>40</td>
<td>25.64</td>
<td>1.0511</td>
<td>0.0335</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>41.03</td>
<td>50</td>
<td>33.33</td>
<td>0.9398</td>
<td>0.0221</td>
</tr>
</tbody>
</table>

\[ \text{State } i \quad \text{u} \quad \text{P}_{i,u}(\%) \quad \text{v} \quad \text{P}_{i,v}(\%) \quad \beta_i \quad \lambda_i \]

![Probability Density Functions (PDFs) of Cumulative Waiting Time in Various States](image1)

**Fig. 4.** Cumulative waiting times in various states.

![Survival Density Functions (SDFs) of Cumulative Waiting Time in Various States](image2)

**Fig. 5.** Survival distribution functions of cumulative waiting times in various states.

- Step 3: Once the Weibull distributions of state waiting times are determined, the age-dependent transition probability matrices can be calculated by Eq. (9) from Year 1 to Year 75. A typical transition probability matrix, i.e., the transition
matrix between Year 2 and Year 3, is shown below:

\[
p^{2,3} = \begin{bmatrix}
0.9991 & 0.0009 & 0 & 0 & 0 \\
0 & 0.9994 & 0.0006 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]

If these transition probabilities are determined, the deterioration process can be formulated as:

\[
A(t) = A(t_0) \prod_{k=0}^{t-1} p_{ij}^{k,k+1}.
\]

where \(A(t)\) is the PMF of the bridge at any time \(t\) in the life span.

For example, the PMF of Year 5 is:

\[
A(5) = A(1) \prod_{k=0}^{4} p_{ij}^{k,k+1} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
0.9996 & 0.0004 & 0 & 0 & 0 \\
0 & 0.9996 & 0.0004 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
0.9991 & 0.0009 & 0 & 0 & 0 \\
0 & 0.9994 & 0.0006 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
0.9983 & 0.0017 & 0 & 0 & 0 \\
0 & 0.9993 & 0.0007 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
0.9938 & 0.0062 & 0 & 0 & 0 \\
0 & 0.9971 & 0.0029 & 0 & 0 \\
0 & 0 & 0.9988 & 0.0012 & 0 \\
0 & 0 & 0 & 0.8981 & 0.1019 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Similarly, the PMFs \((A(t))\) for year 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, and 75 are calculated, as shown in Table 3.

In Fig. 6, the semi-Markov deterioration process and the regression curve are compared to validate the proposed model. It clearly shows that the predicted deterioration process matches well with the regression curve from the 2012 NBI dataset for Texas. The Mean Absolute Percent Error (MAPE) for these curves was approximately 10.8%, demonstrating a good model fit.

- Step 4: Make bridge maintenance strategies based on available techniques and existing experiences. In this case study, five bridge maintenance strategies (Strategies A, B, C, D and E) are considered, and the detailed repair plans are shown in Fig. 7. For the illustrative purpose, only the minor repair cost is included, and other costs are assumed to be zero: \(C^{A}=[0, 0, 29,000, 0, 0]^{T}\), \(C^{B}=[0, 0, 68,000, 0, 0]^{T}\), \(C^{C}=[0, 0, 32,000, 0, 0]^{T}\), \(C^{D}=[0, 0, 44,000, 0, 0]^{T}\), \(C^{E}=[0, 0, 144,000, 0, 0]^{T}\), for Strategies A, B, C, D and E, respectively. Noted that the relevant information and cost data are obtained from a real bridge repair project and more details can be referred to the author’s work [23].
- Step 5: For each strategy, determine the PMF at the year when the repair occurs by using transition probability matrices. Using Strategy A as the example, the first repair activity occurs in Year 10 and the PMF of five states in Year 10 can be found in Table 2:

\[
A(10) = \begin{bmatrix}
0.9192 & 0.0798 & 0.0010 & 0 & 0 \\
0.9684 & 0.0315 & 0.0001 & 0 & 0 \\
0 & 0 & 0.9917 & 0.0079 & 0.0004 \\
0 & 0 & 0 & 0.7975 & 0.2025 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]

- Step 6: Assuming the discount rate is 3.5%, calculate the repair cost by the PMF of the year when the repair occurs. Based on Strategy A, the expected cost in Year 10 can be calculated as:
Table 3
The PMFs of steel bridges in the 75-year service life.

|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|

Fig. 6. Validation of the proposed deterioration model and the 2012 NBI historical records for Texas.
\[ E[C^{10}] = A(10)C^A(1 + 3.5\%)^{-10} \]

\[
\begin{bmatrix}
0.9192 & 0.0798 & 0.0010 & 0 & 0 \\
0 & 0.9684 & 0.0315 & 0.0001 & 0 \\
0 & 0 & 0.9917 & 0.0079 & 0.0004 \\
0 & 0 & 0 & 0.7975 & 0.2025 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
29.000 \\
0 \\
0 \\
\end{bmatrix} (1 + 3.5\%)^{-10} = 3,561.
\]

- Step 7: Update the PMF of that year by using the repair matrix to reflect the repair effect. After the repair of Year 10, the new PMF of states will be improved as:

\[
A(10)^{\text{New}} = A(10)R = \begin{bmatrix}
0.9192 & 0.0798 & 0.0010 & 0 & 0 \\
0 & 0.9684 & 0.0315 & 0.0001 & 0 \\
0 & 0 & 0.9917 & 0.0079 & 0.0004 \\
0 & 0 & 0 & 0.7975 & 0.2025 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0.9 & 0 & 0.1 & 0 & 0 \\
0.8 & 0 & 0 & 0.2 & 0 \\
0.5 & 0 & 0 & 0 & 0.5 \\
\end{bmatrix}.
\]
At Year 30, the second repair activity will be applied and the PMF of states can be calculated based on the new PMF of Year 10:

\[
A(30) = A(10)^{\text{New}} \sum_{k=10}^{29} p_{ij}^{k,k+1} = \begin{bmatrix}
0.2078 & 0.3881 & 0.3537 & 0.0442 & 0.0062 \\
0.2072 & 0.3869 & 0.3549 & 0.0446 & 0.0066 \\
0.1869 & 0.3490 & 0.3938 & 0.0586 & 0.0118 \\
0.1536 & 0.2869 & 0.2614 & 0.1067 & 0.1913 \\
0.1039 & 0.1941 & 0.1768 & 0.0221 & 0.5031 
\end{bmatrix}.
\]

Then, the expected repair cost in Year 30 can be calculated as:

\[
E[c^{30}] = A(30)^*c^A (1 + 3.5%)^{-30} = $676.
\]

After the repair of Year 30, the new PMF of states will be improved as:

\[
A(30)^{\text{New}} = A(30)^*R = \begin{bmatrix}
0.2078 & 0.3881 & 0.3537 & 0.0442 & 0.0062 \\
0.2072 & 0.3869 & 0.3549 & 0.0446 & 0.0066 \\
0.1869 & 0.3490 & 0.3938 & 0.0586 & 0.0118 \\
0.1536 & 0.2869 & 0.2614 & 0.1067 & 0.1913 \\
0.1039 & 0.1941 & 0.1768 & 0.0221 & 0.5031 
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0.9 & 0 & 0.1 & 0 & 0 \\
0.8 & 0 & 0 & 0.2 & 0 \\
0.5 & 0 & 0 & 0 & 0.5 
\end{bmatrix} = \begin{bmatrix}
0.9527 & 0 & 0.0354 & 0.0088 & 0.0031 \\
0.9524 & 0 & 0.0355 & 0.0089 & 0.0032 \\
0.9430 & 0 & 0.0394 & 0.0117 & 0.0059 \\
0.8569 & 0 & 0.0261 & 0.0213 & 0.0957 \\
0.7263 & 0 & 0.0177 & 0.0044 & 0.2516 
\end{bmatrix}.
\]

In Year 50, the third repair activity will occur and the PMF of states can be calculated based on the new PMF of Year 30:

\[
A(50) = A(30)^{\text{New}} \sum_{k=30}^{49} p_{ij}^{k,k+1} = \begin{bmatrix}
0.5651 & 0.2587 & 0.1223 & 0.0444 & 0.0095 \\
0.5649 & 0.2587 & 0.1224 & 0.0445 & 0.0096 \\
0.5593 & 0.2561 & 0.1235 & 0.0477 & 0.0134 \\
0.5082 & 0.2327 & 0.1069 & 0.0468 & 0.1054 \\
0.4308 & 0.1973 & 0.0881 & 0.0287 & 0.2551 
\end{bmatrix}.
\]

The expected costs in Year 50 can be calculated as:

\[
E[c^{50}] = A(50)^*c^A (1 + 3.5%)^{-50} = $287.
\]

After the repair of Year 50, the new PMF of states will be improved as:

\[
A(50)^{\text{New}} = A(50)^*R = \begin{bmatrix}
0.5651 & 0.2587 & 0.1223 & 0.0444 & 0.0095 \\
0.5649 & 0.2587 & 0.1224 & 0.0445 & 0.0096 \\
0.5593 & 0.2561 & 0.1235 & 0.0477 & 0.0134 \\
0.5082 & 0.2327 & 0.1069 & 0.0468 & 0.1054 \\
0.4308 & 0.1973 & 0.0881 & 0.0287 & 0.2551 
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0.9 & 0 & 0.1 & 0 & 0 \\
0.8 & 0 & 0 & 0.2 & 0 \\
0.5 & 0 & 0 & 0 & 0.5 
\end{bmatrix} = \begin{bmatrix}
0.9741 & 0 & 0.0122 & 0.0089 & 0.0047 \\
0.9741 & 0 & 0.0122 & 0.0089 & 0.0048 \\
0.9714 & 0 & 0.0123 & 0.0095 & 0.0067 \\
0.9273 & 0 & 0.0107 & 0.0094 & 0.0527 \\
0.8579 & 0 & 0.0088 & 0.0057 & 0.1276 
\end{bmatrix}.
\]

At Year 70, the fourth repair activity will be applied, and the PMF of states can be calculated based on the new PMF of Year 50:

\[
A(70) = A(50)^{\text{New}} \sum_{k=50}^{69} p_{ij}^{k,k+1} = \begin{bmatrix}
0.8820 & 0.0762 & 0.0213 & 0.0152 & 0.0054 \\
0.8819 & 0.0762 & 0.0213 & 0.0152 & 0.0055 \\
0.8795 & 0.0760 & 0.0213 & 0.0158 & 0.0074 \\
0.8395 & 0.0725 & 0.0196 & 0.0149 & 0.0534 \\
0.7767 & 0.0671 & 0.0175 & 0.0107 & 0.1280 
\end{bmatrix}.
\]

The expected costs in Year 70 can be calculated as:

\[
E[c^{70}] = A(70)^*c^A (1 + 3.5%)^{-70} = $54.
\]

After the repair of Year 50, the new PMF of states will be improved as:
Fig. 8. Life-cycle costs for all five strategies in the 75-year bridge service life.

\[
A(70)^{\text{new}} = A(70) \cdot R = \begin{bmatrix}
0.8820 & 0.0762 & 0.0213 & 0.0152 & 0.0054 \\
0.8819 & 0.0762 & 0.0213 & 0.0152 & 0.0055 \\
0.8795 & 0.0760 & 0.0213 & 0.0158 & 0.0074 \\
0.8395 & 0.0725 & 0.0196 & 0.0149 & 0.0534 \\
0.7767 & 0.0671 & 0.0175 & 0.0107 & 0.1280
\end{bmatrix} \begin{bmatrix}
0.9920 \\
0.9921 \\
0.9910 \\
0.9684 \\
0.9321
\end{bmatrix}
\]

The PMFs and repair costs of Strategies B, C, D and E can be calculated according to their own repair plans in the same way as illustrated by Strategy A.

- Step 8: Sum the costs of all repair activities to obtain the life-cycle costs for each repair strategy (Strategies A, B, C, D, and E), as shown in Fig. 8. The order of life-cycle costs from the highest to the lowest is: Strategy E > B > D > C > A.

- Step 9: Evaluate the repair effects of all maintenance strategies and then determine the most optimal strategy considering both the repair effect (or post-repair performance) and funding (the one with the lowest costs). Since the states in the proposed model are defined by a Sufficiency Rating (SR) that is mainly comprised of structural adequacy, it is reasonable to assume that the bridge, while in different states, has different load capacities, i.e., the bridge in State 1 has 100% of the original design capacity, while the bridge in State 5 only supports 20% of the design load capacity. The load capacities for all five states can be written in the form of a matrix: \( R_e = [100\%, 80\%, 60\%, 40\%, 20\%]^T \). If the PMF \( A(t) \) of the bridge at the year when the repair occurs is known, the repair effect can be evaluated by the following equation based on the restored load capacity:

\[
P^{\text{new}} = A(t) R_e
\]

In which \( p^{\text{new}} \) is the restored load capacity after the repair, \( R_e \) is the defined repair effect matrix in terms of load capacity, and \( A(t) \) is the PMF of the year when the repair is applied.

Again using Strategy A as an example, the load capacity of the bridge in Year 1 can be calculated as:

\[
P = \text{sum}(A(1)[R_e]/300\%) = \begin{bmatrix}
1 & 0 & 0 & 100\%
0 & 1 & 0 & 80\%
0 & 0 & 1 & 60\%
0 & 0 & 0 & 40\%
0 & 0 & 0 & 20\%
\end{bmatrix} = 100\%.
\]

It should be noted that \( P \) is normalized to 100% of the design capacity by the division of 300%.

At Year 10 before the repair, the load capacity is decreased by the deterioration:

\[
P = \text{sum}(A(10)[R_e]/300\%) = \begin{bmatrix}
0.9192 & 0.0798 & 0.0010 & 0 & 0 & 100\%
0 & 0.9684 & 0.0315 & 0.0001 & 0 & 80\%
0 & 0 & 0.9917 & 0.0079 & 0.0004 & 60\%
0 & 0 & 0 & 0.7975 & 0.2025 & 40\%
0 & 0 & 0 & 0 & 1 & 20\%
\end{bmatrix} = 97.84\%.
\]
When the first repair is applied, the PMF of the bridge condition is improved, and thus the load capacity is increased correspondingly:

$$p^{new} = \text{sum}(A(10)^{new}[R_c]/300\%) = \begin{bmatrix} 0.9999 & 0 & 0.0001 & 0 & 0 \\ 0.9968 & 0 & 0.0032 & 0 & 0 \\ 0.8991 & 0 & 0.0992 & 0.0016 & 0.0002 \\ 0.7392 & 0 & 0 & 0.1595 & 0.1013 \\ 0.5000 & 0 & 0 & 0 & 0.5000 \end{bmatrix} \begin{bmatrix} 100\% \\ 80\% \\ 60\% \\ 40\% \\ 20\% \end{bmatrix} = 146.04\%.$$  

For other repair activities aside from Strategy A, the bridge load capacities after the repair can be calculated similarly. Then, the repair effects for all five strategies are calculated and presented in Fig. 9.

According to Fig. 9, the bridge load capacity decreases continuously during the bridge deterioration process without any repair activities. If the repair is applied, strategies A, B, C and D can restore the bridge load capacity to its original design capacity or even increase it to a higher level during the whole life-cycle. The figure indicates that Strategy A improves the bridge to its highest load capacity at the end of its service life. However, Strategy E cannot restore the bridge to its original capacity during its whole life-cycle. The figure also shows that the bridge maintained by Strategy E has a lower load capacity than it did originally around 45 years after its construction. Considering the life-cycle costs of each strategy, Strategy A is still the most optimal.

5. Conclusions

This study proposed a life-cycle optimization model using a semi-Markov process for highway bridge maintenance. The 2012 NBI dataset for the state of Texas was analyzed in this paper. A case study was provided to show the feasibility of this model in real practice. In conclusion, the major findings can be described as follows:

- Bridge deterioration is a complicated process affected by various factors, i.e., aging, construction material, environmental conditions, traffic density, and more. The proposed model, by including these variables, assuming that it is generally applicable, could use bridge-specific information to realistically model the service life deterioration behavior of bridges in a specific environment. The resulting model was validated by the regression curve from the NBI 2012 historical records for Texas (Fig. 6) and matches with only a 10% mean absolute percent error.

- Since the traditional Markov process cannot capture the real time at which the bridge state transitions, it is suitable to model the bridge deterioration as a semi-Markov process in which the state transition has the Markov property and the holding time in each state is assumed to follow a Weibull distribution.

- The proposed model can predict the repair effects of the maintenance actions and hence capture the post-maintenance performance of the bridge, which enables it to evaluate maintenance strategies based on both funding and efficacy.

- Although only steel bridges over the Texas local routes were analyzed in the case study, the proposed model is generally applicable to other types of bridges constructed with other materials. The proposed model is also applicable to optimize the maintenance of bridge components, i.e., decks, superstructures, and substructures.

- The proposed model and decision algorithm will allow for more effective decision-making in terms of using limited maintenance funds to repair critical infrastructure.
• The authors did not consider any user costs (or indirect costs, such as the user delay costs, vehicle operation costs, and detouring costs, etc.) in the case study of this paper. However, it is necessary to consider both agency costs and user costs if repair activities cause any impact to the daily traffic through the bridge.

• In this paper, the authors only focused on the uncertainty of bridge structural performance in its life-cycle. However, the LCCA of repair strategies can be impacted by many cost uncertainty factors. For example, repair costs may not remain constant as predicted, or a discount rate may fluctuate over time during the bridge service life, or traffic demand may not follow the projected path with a bridge service life shortened by heavy traffic. As such, a sensitivity study on primary cost uncertainty factors or a probabilistic LCCA should be included in the final decision making process. Actually, the authors have already conducted some studies on the cost uncertainty by using either sensitivity analysis or probabilistic LCCA [23–25]. In the future work, the authors will improve the proposed model and incorporate both the bridge performance uncertainty and the cost uncertainty into the revised model.

Annotation

The following symbols are used in this paper:

\[ X(t) \] Markov process state for any time \( t \)

\[ P(X_{t+1}) \] Markov transition probability matrix

\[ A(t) \] Probability Mass Function (PMF) at any time \( t \)

\[ F_i(t) \] Cumulative Density Function (CDF) of waiting time at any time \( t \)

\[ S_i(t) \] Survival Function (CDF) of waiting time at any time \( t \)

\[ f_i(t) \] Probability Density Function of waiting time at any time \( t \)

\[ R \] Repair Effect Matrix

\[ \beta_i, \lambda_i \] Parameters of Weibull Distributions

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