Preprint

The Impact of Service Refusal to the Supply-Demand Equilibrium in Taxicab Market

Dali Wei¹, Hongchao Liu^{2,*}

¹ Civil and Environmental Department, Texas Tech University, 10th and Akron, Lubbock, TX 79409 Email: <u>dali.wei@ttu.edu</u>; Fax: 1-(806)7424168; Telephone: 1-(806)7423972

² Civil and Environmental Department, Texas Tech University, 10th and Akron, Lubbock, TX 79409 Email: <u>hongchao.liu@ttu.edu</u>; Fax: 1- (806)7424168; Telephone: 1-(806)7422801

**Corresponding author*

Abstract: Service refusal is a significant problem in taxicab market, especially in developing countries where policies and regulations have not been well developed against this unpleasant phenomenon. Understanding the disturbance of service refusal to the demand-supply equilibrium is essential for the governing authorities to develop effective pricing policies and regulations to tackle the issue. This paper proposed a sigmoid function that depicts the service refusal behavior due to the lower than expected profit. Integrated with this service refusal function, the interrelation between the fleet size, fare, and passenger demand is well evaluated at an aggregate level. The social optimum and maximum profit solutions are examined with consideration of the presence of service refusal. It is found that in a market with non-negligible refusal problem, raising fare at a certain level would drive passenger demand as the benefit from relieved service refusal outweighs the negative impact of the markup itself. This is contrary to the common understanding that raising fare will always reduce the demand. The maximum profit and maximum social welfare achievable would drop if the service refusal becomes severer due to higher expected profits. However, at the social optimum the profit could be positive in presence of service refusal. These properties of the model are demonstrated by a numerical study.

Key Words: Service refusal, Expected profit, Demand, Social optimum, Maximum Profit

1. Introduction

Taxicab is an active mode of transportation whose flexible service is irreplaceable in metropolitan areas. In Beijing, for example, there are about 67,000 taxicabs serving nearly 6 million rides a day and in average each taxi vehicle runs approximately 400 kilometers a day (Beijing Transportation Research Center, 2011; China Statistics Bureau, 2012). While the taxi market in Beijing is under strict fare and entry regulations, refusal to service is not uncommon. A recent survey in Beijing has found as many as nearly 10,000 taxi vehicles were at rest during the afternoon peak hours, whereas only half of them were legitimate due to the shift from daytime service to nighttime service¹, the others were those attempted to evade service in congested hours (Beijing Municipal Commission of Transport, 2013; Cao et al., 2006; Institute for Energy and Environmental Research Heidelberg, 2008).

There are many reasons for the drivers to take the risk of getting punished to escape from serving during peak hours.one of the most obvious is the lower-than-expected profit. Studies to date have mostly focused on well regulated markets where service refusal is not present or at minimum with negligible impacts (Cairns and Liston-Heyes, 1996; De Vany, 1975; Douglas, 1972; Pachon and Johansen, 1989). In a regulated market, once determined the supply, i.e., the fleet size is usually considered constant and the number of taxicabs in service is independent with the taxi fare. Economic models therefore have focused on developing taxi fare structures and fleet size that maximize a selected objective such as the profits and social welfare (Chang and Chu, 2009; Kim and Hwang, 2008; Salanova et al., 2011). As most of the economic models lack consideration of the traffic impacts, Yang et al. introduced distance based and delay based taxi fare models that take into account the effects of traffic congestion on the taxi market (Yang et al., 2010a; Yang et al., 2005).

Like in many other systems, equilibrium can also be found in a taxicab system between the supply and demand. The supply-demand equilibrium and the relation between the fleet size, elastic demand, and taxi fare have been well analyzed in literatures such as Arnott (1996),

¹ Taxi drivers also prefer shifting from day-time to night-time service during the peak hours to escape the congestion due to the lower-than-expected profit.

Cairns and Liston-Heyes (1996), and Wong et al. (2001). Notably, the road network, the origin-destination based demand and the search friction were modeled by Yang and Wong (1998), Yang et al. (2002) and Yang et al. (2010b). Taking into account of the special structure of the taxi market, their models are capable of quantifying a number of system performance measures such as utilization rate and level of service quality of taxi services for different zones. Wong et al. (2008) further extended the model to account for multiple user classes and vehicle modes.

A common assumption of previous studies is that the market is well regulated and taxi drivers are willing to fully comply with various regulation policies. But little attention has been paid to the issue of service refusal as well as its impact to the supply-demand equilibrium. Refusal however may affect the demand-supply relation in many ways(von Massow and Canbolat, 2010). On one hand, it decreases the actual size of fleet in service, leading to larger waiting time. On the other hand, the degraded quality of service in terms of waiting time will affect the demand side by reducing the number of passengers willing to take the service. While governing agencies are attempting to deal with the problem from every possible aspect, understanding the underlying disturbance of this unpleasant phenomenon to the demand-supply equilibrium is essential to policy development.

Aimed at thoroughly eliminating the problem, this study was developed to investigate the topics such as the interactions between the service refusal behavior and the demand-supply equilibrium, as well as its impact to the optimal social welfare and maximum profit of cruising taxi services. The fare structure currently in use in Beijing, China is adopted in this research and a function of service refusal is introduced into the system to analyze its impact. The static effects of the regulating variables such as the fare, the fleet size, and the expected profit are integrated into a mathematic model to investigate the changes in waiting time and passenger demand as related to service refusal.

A definitional approach is provided to describe the relation between the regulating variables, the service refusal phenomena, and the variables of the taxi market. As the aggregated effects of the regulating variables and the service refusal exhibit a different equilibrium mechanism, the solutions for the maximum social welfare and maximum profit are investigated and comparison was made between the two conditions, either with or without the service refusal phenomenon. A case study is provided using the taxi fare structure currently in use in Beijing, China to demonstrate the property of the model, as well as to shed light on some thoughts that may possibly help the governing agency to make policy and regulation changes.

The paper is structured as follows: In section 2, we define a function to describe the refusal of service. In section 3, we introduce the variables and the service refusal function. In section 4, the static effects of the fare, the fleet size and the mean expected profit are investigated. In section 5, we examine the maximized social welfare solution. Section 6 addresses the maximized profit solution with service refusal. In section 6, we show the competitive solution with service refusal. Beijing's case study is provided in Section 7 to highlight the major theoretical findings. Conclusions and recommendations are given in Section 8.

2. Fare Structure and service refusal function

2.1. Variable definitions and fare structure

Major variables used in the paper are defined as follows:

Table 1 Variable notations and definitions			
Variable notations	Definition		
Р	Fare per ride (CNY ²)		
p_s	Initial flag-drop price (CNY)		
p_t	Congestion charge per hour (CNY)		
p_l	Charge per kilometer (km)		
L	Average length of each ride (km)		
u_c	Average traveling speed in congested network (km/hour)		
u_f	Free-flow speed (km/hour)		
\hat{T}	Trip time per ride (hour)		
С	Cost per hour of each taxicab in service (CNY)		
C_o	Cost per hour of each taxicab out of service (CNY)		
Q	Demand per hour		
Ŵ	Average waiting time (hour)		

A realistic fare structure commonly adopted in China is specified as:

$$P = p_s + p_l L + p_t t_c \tag{1}$$

which consists of an initial flag-drop charge of p_s , the charge based on the ride length $p_l L$ and the congestion based charge of $p_t t_c$. t_c is the length of the period when the taxi is traveling at a speed less than a specified threshold of u_0 indicating a congested traffic condition. For example, u_0 is set to be 12 km/hour in Beijing. t_c is then determined by the following equation:

$$t_c = \sigma L/u_j \tag{2}$$

where the parameter of σ accounts for the proportion of congested road segments, for which the traveling speed is less than u_0 . u_j is denoted as the average traveling speed on these congested road segments.

2.2. The function of service refusal and its properties

Taxi drivers' willingness to serve during the peak hours relies on the expected profit gained from each ride. If the profit per ride is less than the expected, taxi drivers are more inclined to service refusal. To depict this behavior, a sigmoid function of r(P) dependent on the fare per ride, is proposed as:

$$r(P) = 1 - \frac{1}{1 + e^{-(P - CT - s)/\mu}} \qquad \text{where } \mu > 0 \tag{3}$$

where s and μ are parameters. r(P), bounded between 0 and 1, describes the tendency of a taxi driver to refuse to serve.

Now let's explore the properties of this function, as well as the physical meanings of the parameter s and μ . To examine how the service refusal rate (frequency of service refusal) changes with respect to the fare, the partial derivative of r(P) with respect to P is:

$$\delta \equiv \frac{\partial r}{\partial P} = -\frac{(1-r)^2}{\mu} e^{\frac{P-CT-s}{\mu}}$$
(4)

Obviously, $\partial r/\partial P$ is negative, which indicates that a higher value of fare per ride would result in a lower rate of service refusal as the profit gained per ride increases with the fare per ride.

The parameter of s actually implies the mean expected profit per ride since the service refusal rate equals 0.5 if the profit per ride (P - CT) equals s. A larger value of s would result in a higher service refusal rate given the same fare per ride. This can be easily verified by the partial derivative of r with respect to s, which is in the form of:

 $^{^2}$ The Renminbi (code: CNY) is the currency of the People's Republic of China (PRC), whose principal unit is Yuan.

$$\gamma \equiv \frac{\partial r}{\partial s} = \frac{(1-r)^2}{\mu} e^{-(P-CT-s)/\mu}$$
(5)

since μ is positive as defined in Eq. (3), $\partial r/\partial s$ is always greater than zero, which implies a positive correlation between the service refusal rate and the mean expected profit *s*. This is illustrated in Figure 1, where service refusal curves with respect to the fare per ride shift to the right given a higher mean expected profit of *s*.



Figure 1: Service refusal rate with respect to the congestion price and the parameter of s

Meanwhile, the parameter of μ indicates the smoothness of the service refusal rate with respect to the fare per ride as shown in Fig. 2. The parameter of μ can be explained from two perspectives. Firstly, it implies the diversity of expected profits among heterogeneous taxi drivers. This is because a smoother curve of the service refusal rate with a larger value of μ covers a wider range of the fare per ride but the mean expected profit remains unchanged. Secondly, the parameter of μ also represents the sensitivity of the service refusal behavior with respect to the fare per ride. Taxi drivers are more sensitive to the fare per ride with smaller values of μ . It implies a small change in the fare per ride would give rise to a large increase or decrease in service refusal rate and the size of the fleet in service.



Figure 2: Service refusal rate with respect to the congestion price and the parameter of s

3. Impact of the service refusal on the equilibrium in taxicab market

With the service refusal function defined in Section 2, this section investigates its impact on the interaction between the demand and the supply. Particularly, the analysis will focus on how the demand, the customer waiting time, and the consumer surplus change with respect to the fare per ride, the fleet size and the mean expected profit.

3.1. Basic assumptions

Let's assume the demand of Q is in the form of:

0

$$= f(P, W, T) = \tilde{Q}e^{-\alpha(P+\kappa W+\tau T)}$$
(6)

where α , κ, τ and \tilde{Q} are positive parameters. \tilde{Q} is the potential customer demand. κ and τ are the monetary values of unit waiting time and in-vehicle travel time, respectively. α is a scaling parameter, which implies the sensitivity of demand to the full trip price. Eq. (6) indicates the demand would decrease with a higher fare per ride, a longer waiting time and a longer travel time.

The customer waiting time of W, as a measure of the service quality, depends on the number of vacant taxis N_{ν} and can be derived as (Douglas, 1972):

$$W = \frac{A}{SN_{\nu}} \tag{7}$$

where A is a constant presenting the number of street miles and SN_v is the cursing vacant taxi-hours. The travel time of T can be calculated by:

$$T = \frac{L\sigma}{u_j} + \frac{L(1-\sigma)}{u_f}$$
(8)

The number of vacant taxis N_{ν} , the demand Q, and the travel time T are related by:

$$N_{\nu} = (1 - r)N - QT \tag{9}$$

where (1 - r)N is the number of taxis in service. As one could identify from Eq. (6) to Eq. (9), the demand without service refusal is a special case when the mean expected profit tends to be negative infinity and the service refusal rate tends to be one.

3.2. Effects of the service refusal on demand

To facilitate the discussion of the equilibrium properties, let f_P be the partial derivative of Q with respect to P, where W is treated as an independent variable. f_W is denoted as the partial derivative of Q with respect to W. From the definition of Q in Eq. (3), f_P and f_W are both negative.

Let's first look at the impact on the demand Q of three variables of interest, specifically the fare per ride P, the fleet size N, and the mean expected profit s. From Eqs. (3), (4) and (6), the partial derivative of Q with respect to P can be derived as:

$$Q_P \equiv \frac{\partial Q}{\partial P} = f_P + f_W \frac{\partial W}{\partial N_v} \left(-N \frac{\partial r}{\partial P} - Q_P T \right)$$
(10)

By letting w_{N_v} denote $\partial W / \partial N_v$, and reorganizing, we have:

$$Q_P = \frac{f_P - f_W w_{N_v} N \delta}{1 + f_W w' T} \tag{11}$$

Eq. (12) shows an aggregate effect of the fare per ride on the demand. The first term of f_P in the numerator is negative, which represents the decreased demand if the fare per ride rises by one unit. But this is compensated by a positive term of $-f_W w_{N_v} N\delta$ ($\delta < 0, f_2 < 0$ and $w_{N_v} < 0$). This term implies an increase in demand due to the decreased waiting time, which is attributed to the decreased service refusal rate and the increased number of taxis in service.

Therefore, the sign of Q_P is undetermined. Increasing the price at a certain level could drive the demand. In that case, customers are willing to accept the markup since the benefit from decreased service refusal rate is more significant compared with the markup itself. This is fundamentally different from the market without service refusal, where Q_P would be simplified from Eq. (12) to $f_P/(1 + f_W w_{N_p} T)$, which is apparently negative. That implies the demand always decreases as the fare rises in the market without service refusal.

The impact of the fare per ride on the demand also implies that the trip cost, which is the sum of the full trip price, the cost of the waiting time and travel time, would not necessarily increase given a higher fare per ride. The benefit from the decreased waiting time due to the increased number of operating taxis could outweigh the increase in the fare. It is interesting to note that the service refusal behavior is only reflected through δ , the derivative of service refusal rate with respect to the fare per ride. That means if the service refusal rate is constant or irrelevant to the fare ($\delta = 0$), the impact on the demand would be the same as in the market without service refusal³.

Now let's examine the impact of the fleet size on the demand with respect to service refusal. The partial derivative of the demand with respect to the fleet size N can be derived similarly as:

$$Q_N \equiv \frac{\partial Q}{\partial N} = \frac{f_W w_{N_v} (1-r)}{1 + f_W w_{N_v} T}$$
(12)

which is obviously positive and implies that an increase in fleet size would result in an increase in the demand. Without the service refusal, it would be simplified to $f_W w_{N_v} / (1 + f_W w_{N_v} T)$, which is also positive. Hence, the effects of the fleet size for a market, either with or without the service refusal, have the same trend. But the service refusal does shrink the impact of the fleet size with the coefficient of (1 - r) between 0 and 1. This is easy to understand since only a proportion (1 - r) of the change in the fleet size would actually affect the market.

The expected mean profit is directly related to the service refusal rate and the level of severity of service refusal in a market. The partial derivative of the demand with respect to the mean expected profit can be derived similarly as:

$$Q_s \equiv \frac{\partial Q}{\partial s} = \frac{-\gamma N f_W w_{N_v}}{1 + f_W w_{N_v} T}$$
(13)

 Q_s is negative since $f_W w_{N_v}$ and γ are both positive. This implies that the demand is decreasing if taxi drivers expect a higher profit per ride. As examining the term of $\gamma N f_W w_{N_v}$ in Eq. (10), one would identify that the decrease in demand is essentially attributed from the higher service refusal rate implied by γ , the decreased number of taxis in service δN and finally the increased waiting time of $\delta N f_W w_{N_v}$.

³ If the service refusal rate is irrelevant of the fare per ride, the number of operating taxis is independent of the fare and the benefit from the increased taxis in service would vanish.

Generally, in the market with service refusal, an increase in the fare would not necessarily decrease the demand and increase the trip cost. It actually depends on the interaction between the benefit from increased number taxis in service and the negative impact of the increased price itself. This is the fundamental difference compared to the market with taxi drivers' full compliance. Also, the service refusal would weaken the regulating impact of the fleet size on the demand. Last but not least, severe service refusal brought by the high expected profits per ride would hurt the demand due to the long waiting time.

3.3. Effects of the service refusal on waiting time

The average customer waiting time, as a measure of the service quality, is investigated in this section. Using Eqs. (3), (4) and (6), the partial derivative of W with respect to P is:

$$W_P \equiv \frac{\partial W}{\partial P} = w_{N_v} \frac{N\delta - f_P T}{1 + f_W w_{N_v} T}$$
(14)

which is clearly negative given $\delta > 0$, $f_P < 0$ and $w_{N_v} < 0$. The first term of $N\delta$ implies more vacant taxis are available due to the increased fare per ride and the lower service refusal rate. The second term of f_PT implies the decreased number of occupied taxi-hours due to the lower demand brought by the increased fare. The combined effects from the above two terms would result in a decrease in waiting time as the fare rises.

Without considering the service refusal, W_P would only include the second term in the numerator while the benefit from increased number of taxis in service vanishes. The impact of the fare on the waiting time is actually enhanced due to the additional impact from the increased number of taxis in service as the fare rises. Similar to Eq. (8), the service refusal is only reflected through the derivative δ , which indicates that the impact of service refusal on the waiting time would vanish if the service refusal and the fleet size actually in service is irrelevant to the fare per ride.

Similarly, we obtain the derivate of the waiting time with respect to the fleet size N as:

$$W_N \equiv \frac{\partial W}{\partial N} = w_{N_v} \frac{1-r}{1+f_W w_{N_v} T}$$
(15)

which is apparently negative indicating the waiting time would decrease given a larger fleet size. For the market without service refusal (r = 0), Eq. (12) would be simplified to $w_{N_v}/(1 + f_W w_{N_v} T)$. Obviously, the impact of the fleet size in presence of service refusal is weakened by a coefficient of (1 - r) since only a proportion of the fleet is actually in service.

Now let's examine how the mean expected profit affects the waiting time. The partial derivative is:

$$W_s \equiv \frac{\partial W}{\partial s} = w_{N_v} \frac{-\delta N}{1 + f_W w_{N_v} T} \tag{16}$$

which is positive implying that the waiting time becomes longer as the expected profit rises. This is due to the higher service refusal rate and less taxis in service. It indicates that the service refusal would degrade the service quality and lead to longer customer waiting time.

In contrast to the demand, the effects of the regulating variables on the waiting time are consistent for both markets with and without service refusal. The waiting time rises as the fare decreases and the fleet size increases. However, the magnitude of the impact is different. Compared with the market without service refusal, the impact of the fare per ride on the waiting time is enhanced since the service refusal rate imposes additional influence on the number of vacant vehicles. Meanwhile, the impact of fleet size is weakened as only a proportion of the fleet is actually in service. Moreover, the service refusal would prolong the customer waiting time as the mean expected profit rises.

3.4. Effects of the service refusal on consumer surplus

The consumer surplus is an economic measure of consumer satisfaction, which is calculated by analyzing the difference between what consumers are willing to pay for the service relative to its market price (Flores-Guri, 2003). As the waiting time enters the demand curve, the consumer surplus is obtained by integrating under a demand curve in which the waiting time is held fixed while the fare varies (Cairns and Liston-Heyes, 1996). The demand Q is rewritten as the function of $f(\omega) = f(P + \kappa W + \tau T)$, and the consumer surplus is obtained as:

$$S_c(\omega) = \int_{\rho}^{\infty} f(\omega) d\omega$$
 (17)

The partial derivative of S_c with respect to the fare can be derived as:

$$S_{c,P} \equiv \frac{\partial S_c}{\partial P} = -Q \frac{\partial \omega}{\partial P}$$
(18)

Given the fact that

$$Q_P = f_\omega \frac{\partial \omega}{\partial P} \tag{19}$$

Eq. (18) can be written as:

$$S_{c,P} = -Q \frac{Q_P}{f_{\omega}} = -Q \left[1 + w_{N_v} \kappa (-\delta N - Q_P T) \right]$$
(20)

where f_{ω} is the derivative of f with respective to ω .

Since the sign of Q_P is undetermined from Eq. (11), the consumer surplus is not necessarily decreases if the fare rises. Hence in a market with service refusal, the fare at which the customers can obtain the maximum surplus is not necessarily at zero. Instead, the price at the optimal consumer surplus would also maximize the demand because $Q_P = 0$ also makes $S_{c,P} = 0$. In contrast, in a market without service refusal or with a constant service refusal rate where $\delta = 0$, the consumer surplus would decrease as fare rises and the fare at the optimal consumer surplus is always zero.

Similarly, the impact on consumer surplus regarding the change of fleet size N can be examined by deriving the following partial derivative:

$$S_{c,N} = -Q \frac{\partial \omega}{\partial N} = -Q \frac{Q_N}{f_\omega} = -Q \{ w_{N_v} \kappa [(1-r) - Q_N T] \}$$
(21)

Apparently, $S_{c,N}$ is positive meaning the consumer surplus would increase as more vacant taxis are available and the waiting time is shortened. However, the service refusal behavior would weaken the effects of fleet size in the same way as it does to the demand since only a proportion of the change in the fleet size actually affects the market.

Now let's check how the expected profit would impact the consumer surplus. The partial derivative of S_c with respect to s is:

$$S_{c,s} = -Q \frac{\partial \omega}{\partial s} = -Q \frac{Q_s}{f_\omega} = -Q \{ w_{N_v} \kappa [-\gamma N - Q_s T] \}$$
(22)

 $S_{c,N}$ is negative as it has the same sign as Q_s . Obviously, it states that the consumer surplus would shrink if taxi drivers expect higher expected profits and exhibit a higher service refusal rate.

3.5. Summary of the static effects of relative variables

The above analysis examines the static effects on the demand, waiting time and consumer surplus regarding variables of the fare, the flee size, and the expected profit per ride, where the service refusal phenomena has been explicitly considered by integrating a sigmoid refusal function. The results are summarized in Table 2.

Table 2: Comparative static effects in markets with and without service refusal ⁴				
	With service refusal	Without service refusal		
With respect to fare per ride <i>P</i>				
Demand	$\frac{f_1 - f_2 w_{N_v} N \delta}{1 + f_2 w' T} (+/-)$	$\frac{f_1}{1+f_2w'T}$ (-)		
Waiting time	$W_{N_v} \frac{N\delta - f_1 T}{1 + f_2 W_{N_v} T} (-)$	$W_{N_{v}} \frac{f_{1}T}{1+f_{2}W_{N_{v}}T}$ (-)		
Consumer surplus	$-Q\left[1+w_{N_v}\kappa(-\delta N-Q_PT)\right] (+/-)$	$-Q[1-w_{N_v}\kappa Q_P T](-)$		
With respect to fleet size N				
Demand	$w_{N_{v}} \frac{N\delta - f_{1}T}{1 + f_{2}w_{N_{v}}T} (+)$	$W_{N_{v}} \frac{-f_{1}T}{1+f_{2}W_{N_{v}}T} (+)$		
Waiting time	$W_{N_v} \frac{1-r}{1+f_2 W_{N_v} T}$ (-)	$W_{N_{v}} \frac{1}{1 + f_{2} W_{N_{v}} T}$ (-)		
Consumer surplus	$-Q\left\{w_{N_v}\kappa[(1-r)-Q_NT]\right\} (+)$	$-Q[w_{N_v}\kappa(1-Q_NT)] (+)$		
With respect to mean expected profit s				
Demand	$\frac{-\gamma N f_2 w_{N_{\mathcal{V}}}}{1 + f_2 w_{N_{\mathcal{V}}} T} (-)$	0		
Waiting time	$W_{Nv} \frac{-\delta N}{1+f_2 W_{Nv} T} (+)$	0		
Consumer surplus	$-Q\left\{w_{N_{v}}\kappa[-\gamma N-Q_{s}T]\right\} (-)$	0		

In summary, the impacts exhibit following unique features in comparison with the market without service refusal.

The most distinguished characteristic is that both the demand and the consumer surplus are not monotonously decreasing function with respect to the fare. The interaction between the opposite impacts brought by the increased fare and the shortened waiting time determines whether or not the demand and consumer surplus would increase. Counter-intuitively, it implies that raising the fare to a decent level could actually drive the demand when the benefit from the shorted waiting time due to the decreased service refusal rate outweighs the negative impact from the markup. This is different from the market without service refusal, where the demand is decreasing and the maximum demand is always achieved when the fare equals zero.

The waiting time appears similar decreasing trend as the fare rises in both markets with and without service refusal. The difference lies in the fact that the service refusal has an additional effect in terms of the increased taxis available due to the decreased service refusal rate. Hence, the waiting time would decrease more rapidly compared with the market without service refusal as the fare rises.

⁴ (+), (-) and (+/-) means the derivative is positive, negative or undetermined, respectively.

With respect to the fleet size, the service refusal would weaken its impact on the market in terms of the demand, the waiting time and the consumer surplus by a coefficient of (1 - r) since only a proportion of the changes in the fleet size would actually be in service and have an effect on the market.

Lastly, if taxi drivers expect higher profits per ride, the service refusal rate would rise. That would result in severer service refusal behavior in the taxicab market, and lead to a contraction in both demand and the consumer surplus. The waiting time would also increase since fewer taxis are in service.

4. Optimal social welfare solution

As discussed in the previous section, the service refusal behavior imposes different impacts to the taxicab market. This section explores conditions of optimal social welfare with service refusal regarding the two regulating variables: the fare and the fleet size.

The social welfare comprises both the consumer surplus of Eq. (17) and the taxicab profits. Here, we need to specify the equation of calculating the taxicab profits. To note that, the costs associated with the taxi operation normally consist of the gas cost and the license fee. In China, taxi drivers normally have to pay a certain amount of license fee per month to their companies. Hence, the profit is the income minus the gas cost and the license fee for operating taxis, while the taxi drivers out of service would face a loss of the license fee.

The profit of taxis is formulated as:

$$S_t = PQ - (1 - r)CN - rC_0N$$
(23)

where C_0 accounts for the license fee per hour for taxis out of service. C is the cost for operated taxis including both the license fee C_0 and the gas cost.

With Eq. (23), the social welfare is specified as:

$$S(P,N) = \int_{\rho}^{\infty} f(\omega)d\omega + PQ - (1-r)CN - rC_0N$$
(24)

The partial derivative of S regarding P is:

$$\frac{\partial S}{\partial P} = -Q \frac{Q_P}{f_\omega} + Q + PQ_P + \delta CN - \delta C_0 N \tag{25}$$

The partial derivative of S regarding N is:

$$\frac{\partial S}{\partial N} = -Q \frac{Q_N}{f_\omega} + Q + PQ_N - (1-r)C - rC_0$$
(26)

The necessary condition of the maximum social welfare is $\partial S/\partial P = 0$ and $\partial S/\partial N = 0$. Using Eq. (11) and Eq. (20), Eq. (25) can be written as:

$$P = -Qw_{N_v}\kappa T - \delta N \frac{(C - C_0) + Q\kappa w_{N_v}}{Q_p}$$
(27)

Similarly, Eq. (26) can be written as:

$$P = -Qw_{N_{v}}\kappa T + (1-r)\frac{(C-C_{0}) + Q\kappa w_{N_{v}}}{Q_{N}} + \frac{C_{0}}{Q_{N}}$$
(28)

Combing Eq. (27) and Eq. (28), one would obtain:

$$(C - C_0) + Q\kappa w_{N_v} = \frac{-C_0 Q_P}{\delta N Q_N + (1 - r) Q_P}$$
(29)

Taking Eq. (30) into Eq. (28), we have:

$$P = (C - C_0)T + \frac{C_0(Q_P T + \delta N)}{\delta N Q_N + (1 - r)Q_P}$$
(30)

Using Eq. (11) of Q_p and Eq. (12) of Q_N , Eq. (31) can be further simplified to: $C_1 + f_2 = T_1 + S_1 + S_2$

$$P = (C - C_0)T + \frac{C_0(1 + f_W w_{N_p} T)(Q_P T + \delta N)}{(1 - r)f_P}$$

= $(C - C_0)T + \frac{C_0}{1 - r} \left(T + \frac{\delta N}{f_P}\right)$
= $CT + C_0 \left[\frac{r}{1 - r}T + \frac{\delta N}{(1 - r)f_P}\right]$ (31)

Eq. (31) is the central result of the optimal social welfare condition with service refusal behavior. The first term of CT is the operation costs for the operating taxis. If without the service refusal behavior (r = 0 and $\delta = 0$), the fare would equal to CT. That means the fare per ride just covers the operation costs during the trip and taxis are operated at a loss equal to the cost of vacant taxi-hours (Yang et al., 2005). This is exactly the same result derived by Arnott (1996). Also, if the license fee cost C_0 is ignorable, the fare equation is simplified to CT. That implies if there are no loss to the taxi drivers out of service, the fare at the optimal social welfare would also equal to that in a market without service refusal.

As one would observe from Eq. (31), with the impact of service refusal, the fare at the optimal social welfare covers an additional term regarding the cost for idle taxis. Since $\delta > 0$ and $f_P < 0$, this term implies an extra charge, which compensates the costs of idle taxis. Hence, the fare per ride at social optimum is higher due to the service refusal behavior. This brought a possibility that the profit of the taxis firms may not be negative at the social optimum in a market with service refusal. These properties will be verified and discussed in the numerical study.

5. Maximum profit solution

In this section, the focus is to regulate the fleet size and the fare in a monopoly taxi market to obtain the maximum profit. The comparison to the market without service refusal behavior is discussed.

The profit of the taxi firm is given by:

$$I(P,N) = PQ_N - (1-r)C - rC_0$$
(32)

including the costs of both operating taxis and taxis out of service. To maximize the profit, the first-order conditions have to be satisfied, which is the partial derivatives of I with respect to P and N should equal to zero.

The partial derivative of the profit with respect to P is:

$$\frac{\partial I}{\partial P} = Q + Q_P P + CN\delta - C_O N\delta = 0 \tag{33}$$

and further written as:

$$P = \frac{CN\delta - C_0N\delta - Q}{Q_P} \tag{34}$$

The partial derivative of the profit with respect to N is:

$$\frac{\partial I}{\partial N} = Q_N P - C(1-r) - C_0 r = 0 \tag{35}$$

which can be written in the similar form to Eq. (34):

$$P = \frac{C(1-r) + C_0 r}{Q_N}$$
(36)

Combining Eq. (34) and Eq. (36), one can obtain:

$$\frac{C(1-r) + C_0 r}{Q_N} = \frac{CN\delta - C_0 N\delta - Q}{Q_P}$$
(37)

Taking Q_N from Eq. (11) and Q_P from Eq. (12), we have:

$$rC_o + C(1-r) = \frac{f_2 w_{N_v} [C_o N\delta - Q(1-r)]}{f_1}$$
(38)

Now taking Q_N into Eq. (36), one obtains:

$$P = [C(1-r) + C_0 r] \frac{1 + f_2 w_{N_v} T}{f_2 w_{N_v} (1-r)}$$

= $\frac{C(1 + f_2 w_{N_v} T)}{f_2 w_{N_v}} + \frac{C_0 r(1 + f_2 w_{N_v} T)}{f_2 w_{N_v} (1-r)}$
= $CT + \frac{C_0 r}{(1-r)} T + \frac{1}{f_2 w_{N_v} (1-r)} [rC_0 + C(1-r)]$ (39)

Applying Eq. (38) to Eq. (39), we have:

$$P = CT + \frac{C_0 r}{(1-r)}T + \frac{C_0 N\delta}{(1-r)f_1} + \frac{Q}{f_1}$$
(40)

By comparing the fare at the social optimum, the charge per ride at the maximum profit is apparently higher than the fare at the social optimum by an additional amount of Q/f_1 , which represents the consumer's marginal net willingness-to-pay for a ride. If ignoring the service refusal ($r = 0, \delta = 0$), the consumer would be charged at a lower fare of $CT + Q/f_1$, which is the same result derived in Yang et al. (2005). However, with the presence of the service refusal, the fare does need to be raised in order to achieve maximum profit.

6. A numerical study and discussion

To demonstrate the analytical analysis obtained so far and further explore numerical properties of the market equilibrium with service refusal, a case study is presented and discussed in this section. The basic parameters, such as the average trip distance, operation cost, license fee, congested speed and free flow speed, are directly obtained and derived from the Annual Report of Beijing Transportation (Beijing Transportation Research Center, 2011), while the other parameters such as the sensitivity parameter of α , value of waiting time and value of travel time are referred to travel surveys and studies of Beijing (Beijing Municipal Commission of Transport, 2013; Jia et al., 2007; Liu et al., 2008).

Table 3 Values of parameters		
Parameters	Values	
Initial flag-drop price p_s	12 CNY	
Charge per kilometer (km) p_l	2 CNY	
Charge per waiting wait p_t	55 CNY per hour	
Average trip length L	8 km	
Average traveling speed in congested network (km/hour)	10 km per hour	
Free flow speed u_f	40 km per hour	
Cost per hour C for taxis in service	40 CNY	
Cost per hour C_o of taxis out of service	10 CNY	
Potential demand \bar{Q}	1,100,000	
Proportion of congested segment σ	0.6	
Value of waiting time κ	60 CNY/hour	
Value of trip time τ	35 CNY/hour	
Customer's choice behavior α	0.03	
Waiting time parameter A	1000 veh·h	
Driver diversity parameter μ	10	

6.1. Impact on demand

Figure 3 illustrates the demand curves with respect to fare per ride in markets with and without service refusal. The solid line clearly demonstrates that the demand would decrease as fare rises and the maximum demand is achieved when the fare equals zero if the service refusal is ignored.

But this is essentially different from demand curves in markets with service refusal as illustrated by dotted lines in Figure 3. It can be seen that the demand curve exhibits a convex shape as the fare rises. The maximum demand is achieved when the fare is set to a certain threshold that makes Q_P equal zero. If the fare is below this threshold, raising the price would not hurt the demand. Instead, the benefit brought by the shortened waiting time due to increased operating taxis outweighs the negative impact from the raised price, which would boost the demand. This stage corresponds to the ascent phase of the curve. However, when the fare is higher than the threshold, raising the fare would hurt the demand as the benefit from increased number taxis in service is less significant than the impact of the markup. As the fare rises, the demand curves in markets with and without service refusal will eventually overlap when the service refusal rate tends to zero.



Figure 3: Demand curves with respect to fare per ride in market with and without service refusal

Comparing demand curves between markets with and without service refusal, one would observe that given the same fare and the same fleet size the demand is apparently lower with service refusal. That implies by solely regulating the fare, either decreasing or increasing, cannot eliminate the negative impact on the demand if the desired demand is beyond the maximum demand attainable in the market with service refusal.

Furthermore, if the taxi drivers expect greater profit per ride, the service refusal rate would rise, which further shrinks the demand as seen in the figure. Meanwhile, the threshold fare that maximizes the demand would also rise since the peak of the curve shifts to right. The maximum demand decreases in the market with severer service refusal. These observations are consistent with and further extend our analytical discussion in Section 3.

Figure 4 plots demand curves with respect to the fleet size. Obviously, the demand grows as the fleet size increases in both markets with and without service refusal. However, the service refusal renders this growing at a slower rate. This coincides with our analysis in Section 3, and the reason is that only a proportion (1 - r) of the increase in the fleet size would have an effect on the market. As the expected profit and the service refusal rate rise, the growing rate would be further decreased. That implies to achieve the same demand while keeping the fare unchanged, the fleet size has to be increased to compensate taxis out of service.



Figure 4: Demand curves with respect to fleet size in market with and without service refusal

6.2. Impact on waiting time

The passenger waiting time both with and without service refusal is illustrated in Figure 5 and Figure 6. Apparently it decreases as the fare rises, which is the same regardless of service refusal. However, the waiting time decreases at a faster rate in presence of service refusal. Based on the analysis in section 3, this is because both the shrinking demand and the increased vacant taxis contribute to the decrease in the waiting time. Given the same price and fleet size, service refusal deteriorates the service quality since the waiting time is much longer. As the fare rises, the waiting times in the taxicab markets with and without service refusal would overlap when the service refusal rate tends to zero.

Furthermore, greater expected profits and higher service refusal rates would make the decreasing curve steepen, implying a longer waiting time. This can be explained by the positive sign of W_s . To achieve the same service quality (the same waiting time) as that in a market absence of service refusal with the same fleet size, the fare has to be raised. This markup has to be higher in the market with severer service refusal.

The impact of the fleet size on the waiting time is illustrated in Figure 6. Similarly, the waiting time in a market either with or without service refusal exhibits the same decreasing

trend as the fleet size increases. But in presence of the service refusal, the waiting time is longer. These observations are consistent with the derivation and discussion of W_P , W_N and W_S .



Figure 5: Waiting time with respect to fare in market with and without service refusal



Figure 6: Waiting time with respect to fleet size in market with and without service refusal

6.3. Impact on consumer surplus

Effects of the service refusal on the consumer surplus are demonstrated in Fig. 7 and Fig. 8 in terms of the fare and the fleet size. By comparing Fig. 8 with Fig. 4, one would easily identify that curves in both figures exhibit exactly the same trends. This can be easily verified from Eq. (19). Since f_w equals αQ based on the exponential demand function of Eq. (6), $S_{c,P}$ would equal to $-Q_P/\alpha$. Apparently, $S_{c,P}$ is the partial derivative of Q times a coefficient, which is the opposite of the reciprocal of the sensitivity parameter.

Hence, the features exhibited in Fig. 7 and Fig. 8 can be explained in a similar way as the characteristics of the demand, which are also consistent with the analysis in Section 3.4. The consumer surplus in presence of service refusal shows a convex form as the fare rises. But it will eventually overlap with the curve in the market without service refusal when the service refusal rate tends to zero. When the fare is relatively lower, raising the fare could increase the consumer surplus as consumers could benefit more from the decrease in waiting time due to the lowered refusal rate than the negative effect from the markup itself. As the expected profit rises, the demand curves shift to the right and the maximum demand achievable also decreases. Increasing the fleet size would raise the consumer surplus. But with service refusal phenomena, the impact has been weakened.



Figure 7: Consumer surplus with respect to the fare in market with and without service refusal



Figure 8: Consumer surplus with respect to the fleet size in market with and without service refusal

6.4. Social optimum

The social optimums for market with and without service refusal are illustrated in Fig. 9 to Fig. 12. The service refusal clearly imposes significant impact and renders the contour of the social welfare exhibits a different pattern.



Figure 9: Social welfare contour in market without service refusal



Figure 10: Social welfare contour in market with service refusal (s=5)



Figure 11: Social welfare contour in market with service refusal (s=15)



Figure 12: Social welfare contour in market with service refusal (s=30)

First of all, service refusal would apparently lower the maximum social welfare achievable in the market. The maximum social welfare ignoring the service refusal is 8.7 million CNY as seen in Fig. 10. But in presence of the service refusal when the expected profit is set to be 5 CNY, the maximum social welfare would be lowered to 7.5 million CNY. A higher expected profit results in severer service refusal, which would further decrease the maximum social welfare. As seen in Fig. 13, the maximum social welfare drops to 5.7 million CNY when the expected profit rises to 30 CNY.

Secondly, consistent with our analysis in section 4, the fare should be raised to achieve the social optimum accounting for the service refusal. As seen in Fig. 9, the optimal fare is 22 CNY for market without service refusal, which just covers the cost for the ride time. However, it needs to be raised to 35 CNY in presence of service refusal when the expected profit is 5 CNY. And the fare at the social optimum keeps growing as the expected profit rises. When the expected profit is 30 CNY per ride, the fare has to be raised to 52 CNY. Even so, the maximum social welfare is still much lower compared with market in absence of service refusal. This trend is also demonstrated in Fig. 9.

In contrast to the increase in the fare, the fleet size at the social optimum is decreasing as the service refusal becomes severer. The fleet size is controlled at 190 thousands to achieve the social optimum if ignoring the service refusal. But this size has to be limited to 150 thousands accounting for service refusal behavior when the mean expected profit is set to be 30 CNY per ride.

Furthermore, by comparing Fig. 9 to Fig. 12, one would observe that contour tends to shift upward as the service refusal becomes severer. That implies to maintain the same amount of social welfare as that in the market without service refusal the fare has to be raised if keeping the fleet size stable. Meanwhile, the contour also tends to skew towards the right as the expected mean profit rises. That indicates the fleet size has to be enlarged to achieve the same amount of social welfare if the fare keeps unchanged. Generally, contours for the same social welfare shrinks in presence of service refusal.



Figure 13: Fare and fleet size at the social optimal with respect to s

6.5. Maximum profit

The properties of the maximum profit are examined at this section. Figs. 14 to 16 present the contour of the profit in markets with and without service refusal. Important characteristics of effects of the service refusal can be identified.



Figure 14: Profit contour without service refusal









Figure 17: Profit contour with service refusal (s = 30)

Consistent with the analysis in section 5, in a market with service refusal the fare should be raised to achieve the maximum profit comparing the market with drivers' full compliance. From Fig. 14, the fare at the maximum profit is 56 CNY per ride, but it rises to 64 CNY in presence of service refusal when the mean expected profit is 30 CNY. However, the fleet size at the optimal profit does not show a significant change for market either with or without service refusal. For the study case, it is around 69 thousands. Furthermore, the maximum profit achievable decreases as the mean expected profit rises and service refusal become severer. The maximum profit in market without service refusal cannot be achieved by regulating fare and fleet size if considering the service refusal. This can be viewed as an interesting faster-is-slower phenomena as found in many dynamic systems (Helbing and Mazloumian, 2013; Parisi and Dorso, 2007; Wei and Liu, 2013). That implies if drivers expect higher profits and refuse to serve when the profit cannot match the expected, the maximum profit they could achieve would actually decrease.

By observing the profit contour, a similar trend as the social welfare has been found. The contour for the same amount of profit would shift upwards and skew toward the right as the mean expected profit rises. That implies in order to achieve the same amount of profit as in the market free from service refusal, the price has to be raise if the fleet size keeps unchanged. Similarly, the fleet size has to be enlarged if it is the only regulated variable. The social optimal price and fleet size are also marked in the contour. In terms of the fleet size and the fare, the gap between the maximum profit and the optimal social welfare is shortened as the service refusal becomes severer.



Figure 18: Fare and fleet size at the social optimal with respect to s

Lastly, let's examine the profitability at the social optimum. Figure 19 plots the curves of the optimal social welfare, as well as the corresponding profit and loss for operating taxis and taxis out of service, respectively. Apparently, in the market without service refusal $(s \rightarrow -\infty)$, the profit is negative at the social optimum. However, in the market in presence of the service, the profit of the taxi firms at the social profit could be positive. The profit of the operating vehicles increases as the service refusal becomes severer, and the loss to the vehicles out of service also rises. However, the total profit increases at the social optimum as the mean expected profit rises. It implies as the service refusal in a market becomes severer, the optimal social welfare would decrease but the profit at the social welfare would actually increase.



Figure 19: Fare and fleet size at the social optimal with respect to s

7. Conclusion and future work

Taxicab market is a multi-actor system that involves different levels of decision-makers such as travelers, taxi drivers and governing agencies. In a real-world market, it is common that taxi drivers do not fully comply with regulating policies especially in developing countries.

Service refusal behavior due to the lower-than-expected profit leads to a new equilibrium mechanism of the taxi market, where the supply is not solely determined by the regulated fleet size but also relies on the service refusal behavior. In this paper, the equilibrium properties are investigated through introducing a sigmoid function to depict the service refusal behavior. The solutions for the social optimum and maximum profit are examined.

It is found that the demand could be boosted by raising the fare at a decent level so that the benefit from the decreased service refusal rate outweighs the markup itself. This would also benefit to the consumer surplus and result in shorter waiting time and higher service quality. This is contrary to the common understanding that raising fare would always reduce the demand and customer surplus. The ability to regulate the market via tuning the fleet size is weakened since only a proportion of the changes in the fleet size would actually have an effect on the market.

To obtain the social optimum, both the fare and the fleet size need to be raised. Even so, the maximum social welfare achievable cannot match that in the market without service refusal. However, the profit at the social optimum could be positive in presence of the service refusal. To achieve the maximum profit, the fare has to be raised as well but the fleet size can be controlled at a stable size. An interesting faster-is-slower phenomenon is found that the maximum profit would be lowered if drivers expect higher profits per ride and exhibit severer service refusal in the market.

The future work would be focused on the trip specified service refusal, which accounts for the fact that taxi drivers prefer trips that could bring higher profit. Network models will be adopted to tackle this issue.

References

Arnott, R. (1996) Taxi Travel Should Be Subsidized. *Journal of Urban Economics* 40, 316-333.

Beijing Municipal Commission of Transport (2013) Report of Public Transport Beijing.

- Beijing Transportation Research Center (2011) Annual report of transportation of Beijing. Beijing.
- Cairns, R.D., Liston-Heyes, C. (1996) Competition and regulation in the taxi industry. *Journal of Public Economics* 59, 1-15.
- Cao, G., Zeng, K., Qiu, P. (2006) Analysis of service refusal problem. *Road Traffic & Safety* 6, 35-37.
- Chang, S.K.J., Chu, C.-H. (2009) Taxi vacancy rate, fare, and subsidy with maximum social willingness-to-pay under log-linear demand function. *Transportation Research Record: Journal of the Transportation Research Board* 2111, 90-99.
- China Statistics Bureau (2012) China statistical yearbook. *Beijing: Chinese Statistical Bureau*.
- De Vany, A.S. (1975) Capacity utilization under alternative regulatory restraints: an analysis of taxi markets. *The Journal of Political Economy*, 83-94.
- Douglas, G.W. (1972) Price regulation and optimal service standards: The taxicab industry. *Journal of Transport Economics and Policy*, 116-127.
- Flores-Guri, D. (2003) An economic analysis of regulated taxicab markets. *Review of Industrial Organization* 23, 255-266.
- Helbing, D., Mazloumian, A. (2013) Operation Regimes and Slower-is-Faster-Effect in the Control of Traffic Intersections. *Modelling and Optimisation of Flows on Networks*. Springer, pp. 357-394.
- Institute for Energy and Environmental Research Heidelberg (2008) *Transport in China:Energy Consumption and Emissions of Different Transport Modes*. Heidelberg.
- Jia, H.-f., Gong, B.-w., Zong, F. (2007) Disaggregate modeling of traffic mode choice and its application. *Journal of Jilin University (Engineering and Technology Edition)* 37, 1288-1293.
- Kim, Y.-J., Hwang, H. (2008) Incremental discount policy for taxi fare with price-sensitive demand. *International Journal of Production Economics* 112, 895-902.
- Liu, B., Juan, Z., Li, Y., Gong, B. (2008) Development of a Multinomial Logit Model for Trave Mode Choice of Residents. *Journal of Highway and Transportation Research and Development* 25, 116-120.
- The World Bank (1989) Pricing and regulatory issues in urban transport.
- Parisi, D.R., Dorso, C.O. (2007) Why "Faster is Slower" in Evacuation Process. *Pedestrian and Evacuation Dynamics 2005* eds Waldau, N., Gattermann, P., Knoflacher, H., Schreckenberg, M. Springer Berlin Heidelberg, pp. 341-346.

- Salanova, J.M., Estrada, M., Aifadopoulou, G., Mitsakis, E. (2011) A review of the modeling of taxi services. *Procedia Social and Behavioral Sciences* 20, 150-161.
- von Massow, M., Canbolat, M.S. (2010) Fareplay: An examination of taxicab drivers' response to dispatch policy. *Expert Systems with Applications* 37, 2451-2458.
- Wei, D., Liu, H. (2013) Analysis of asymmetric driving behavior using a self-learning approach. *Transportation Research Part B: Methodological* 47, 1-14.
- Wong, K.I., Wong, S.C., Yang, H. (2001) Modeling urban taxi services in congested road networks with elastic demand. *Transportation Research Part B: Methodological* 35, 819-842.
- Wong, K.I., Wong, S.C., Yang, H., Wu, J.H. (2008) Modeling urban taxi services with multiple user classes and vehicle modes. *Transportation Research Part B: Methodological* 42, 985-1007.
- Yang, H., Fung, C., Wong, K., Wong, S. (2010a) Nonlinear pricing of taxi services. *Transportation Research Part A: Policy and Practice* 44, 337-348.
- Yang, H., Leung, C.W.Y., Wong, S.C., Bell, M.G.H. (2010b) Equilibria of bilateral taxi–customer searching and meeting on networks. *Transportation Research Part B: Methodological* 44, 1067-1083.
- Yang, H., Wong, S. (1998) A network model of urban taxi services. *Transportation Research Part B: Methodological* 32, 235-246.
- Yang, H., Wong, S.C., Wong, K. (2002) Demand–supply equilibrium of taxi services in a network under competition and regulation. *Transportation Research Part B: Methodological* 36, 799-819.
- Yang, H., Ye, M., Tang, W.H., Wong, S.C. (2005) Regulating taxi services in the presence of congestion externality. *Transportation Research Part A: Policy and Practice* 39, 17-40.