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Enhancing Realism in Modeling Merge Junctions in Analytical Models for System Optimal Dynamic Traffic Assignment

Wei-Hua Lin and Hongchao Liu

Abstract — The existing analytical system optimal dynamic traffic assignment (SO-DTA) model formulated with the linear programming (LP) approach usually assumes system control over vehicles in the entire network. This property would give rise to unreasonable priorities at merge junctions that are sometimes physically impossible to realize for the given roadway configuration. In this paper, we demonstrate that models with and without considering merge priority ratio would exhibit very different traffic patterns and route choice behavior. To realistically model traffic flow on a transportation network, one should properly distinguish the level of control by drivers, roadway geometry, and system providers. This paper also attempts to develop a linear programming module that explicitly considers the merge priority ratio of a merge junction and can potentially be incorporated into the existing LP formulation of the SO-DTA problem based on the cell transmission model. By modeling more realistically the behavior of vehicles at merge junctions, the solution obtained can be used as a benchmark to compare control strategies developed without considering explicitly the merge priority ratio at merge junctions or strategies developed with heuristic approaches.

Index Terms**—Dynamic Traffic assignment; System optimum; Traffic control; Traffic Management System; Intelligent Transportation Systems; Mathematical Programming.**

I. BACKGROUND

ONE of the objectives of dynamic traffic assignment is to predict traffic flow patterns over time and space on a transportation network for a given set of time-varying origin-destination (OD) demands based on some predefined conditions. The system optimal state corresponds to the condition for distributing vehicles on a network in such a way that minimizes the total vehicle travel time or total vehicle delay in the system. System optimal control aimed to achieve system optimum can be employed to develop control strategies used for daily operations. Moreover, traffic patterns generated under the system optimum condition would help system providers identify the location of the potential bottleneck in a network and thus effectively allocate limited resources to alleviate congestion.

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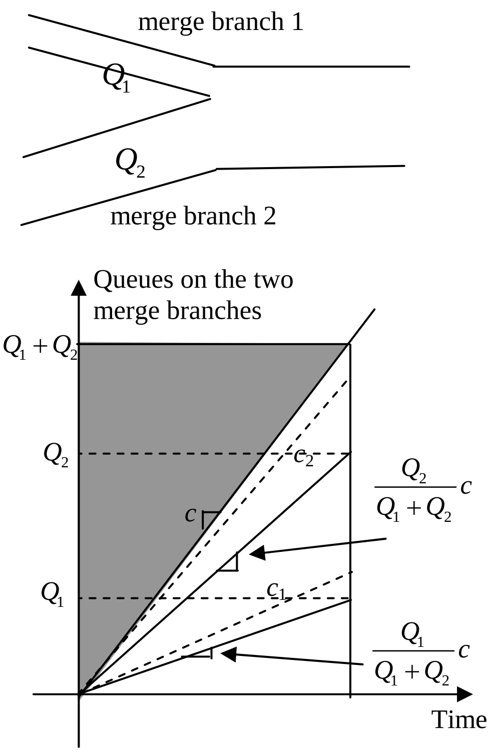
The analytical system optimal dynamic traffic assignment (SO-DTA) models are conventionally formulated as a mathematical programming problem. The pioneering work in this area was due to Merchant and Nemhauser [1, 2] who formulated the problem as a discrete-time, nonlinear, nonconvex mathematical program for the single destination program. Much of the later work along this line sought to simplify the formulation [3, 4], to develop more efficient solution algorithms [5], and to extend the formulation to handle many-to-many networks [6]. Though models proposed by others in more recent years vary in detail, the modeling approach adopted was fundamentally the same. The underlying traffic flow model often uses exit functions or link performance functions in which the exit flow of a link depends only on the content of the link. The exit function could yield flow patterns that would never arise in real life [7]. It was also shown in the work by Addison and Heydecker [8] that the results from models with exit functions differ substantially from those obtained by the kinematic wave model or the LWR model [9, 10]. The LWR model is capable of capturing the dynamic process of physical queues. For the past decade, various models have been proposed [11, 12] to formulate the SO-DTA problem as a linear programming problem based on the cell transmission model [13, 14], which is a discrete version of the LWR model.

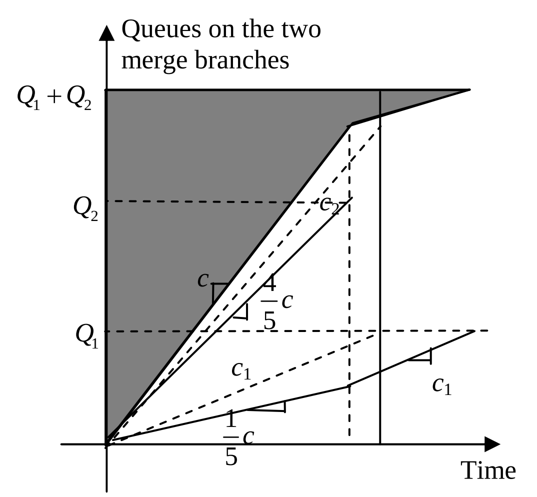
Though models based on the cell transmission model are more realistic in representing traffic dynamics than their counterparts developed with link performance functions, they tend to simplify the treatment of the merge junction in the cell transmission model by assuming system control over vehicles in the entire network. This simplification was made in part because the original merge model in the cell transmission model is inherently nonlinear [14]. The simplification is reasonable if the network problem is both a design and an operation problem that one can exercise control over the entire network. For ITS deployment, however, many times such a full-level of system control is not possible since we are only able to impose limited system control over a network. The roadway configuration in many parts of a network cannot be easily altered. In such cases, merge priority at some merge junctions should be preserved as they are and treated as given parameters. If we allow merge priority ratio at every merge junction to be dynamic, determined in the course of system optimization, it could potentially lead to some inappropriate merge rules. In some situations, the solution would even give rise to priorities that are physically impossible to realize due to the given roadway configuration.

Realism in modeling the merge junction is very important. In practice, there are a variety of different forms of traffic control at merge junctions, including ramp metering, yield and stop signs, lane barriers, variable message signs, etc. The objectives of this type of control could vary substantially, ranging from enhancing safety to ensuring equity to all vehicles in sharing the roadway facilities at a local level. Minimizing the total system delay is many times considered in conjunction with other objectives. Other forms of "control" at merge junctions can be a result of the existing roadway configuration, such as the design of a merge junction on a freeway where vehicles entering the freeway would merge with freeway traffic from an on-ramp. When queues are present both on the ramp and on the freeway, it was found that a reasonable merge priority ratio for freeway and ramp traffic would be $(2m-1):1$ for a freeway with m lanes and a ramp with a single lane [15]. Though this type of constraint imposed by the roadway geometry can be easily captured in a simulation model, it is, in fact, quite challenging to enforce such a rule in an analytical model. It is not readily available in many existing analytical DTA models.

The type of control imposed on a merge junction could ultimately affect the development of queues upstream of the merge junction both temporally and spatially. For example, for the merge junction shown in Fig. 1, suppose the capacity for the main branch is c. The capacities for the two merge branches are c_1 and c_2, respectively. $c_1 < c$, $c_2 < c$, and $c_1 + c_2 > c$. Suppose also that the merge priority is determined by a merge ratio, which is 1:4. Denote the queues on branches 1 and 2 to be Q_1 and Q_2, respectively. For illustration purposes, we assume also that the incoming flows to the two branches are 0. We need to determine the exit flows from the two merge branches to achieve a system optimum condition. Without the presence of any constraints to explicitly enforce the 1:4 merge ratio, the system optimal solution would keep the main branch discharging vehicles at the highest rate possible until both queues are dissipated. This would lead to a queue dissipation process during which flows from the two merge branches would advance to the main branch in such a proportion that both queues would eventually vanish at exactly the same time. Let the flow discharging rate be q_1 and q_2 for merge branches 1 and 2. From Fig. 1(a), it follows that $q_1 = \frac{Q_1}{Q_1+Q_2}c$ and $q_2 = \frac{Q_2}{Q_1+Q_2}c$, independent of the merge ratio. By considering the merge ratio, merge branch 1 would operate at $q_1 = \frac{1}{5}c$ before Q_2 is fully discharged and $q_1 = c_1$ afterwards. Merge branch 2 would operate at $q_2 = \frac{4}{5}c$. The solution is independent of the queue length, assuming that $\frac{1}{5}c < c_1$ and $\frac{4}{5}c < c_2$. Q_2 is discharged ahead of Q_1 as shown in Fig. 1(b). In the former case, the system control overrides local control, whereas in the latter case the system control is apparently governed by the local control, the "control" from merge priority resulting from the configuration of the merge junction.



(a) Delay (the shaded area) and service rate (merge priority rule is not preserved)



(b) Delay (the shaded area) and service rate (merge priority rule is preserved)

Fig. 1. The Impact of Merge Control in Total System Delay.

It is not difficult to see that total delay in Fig. 1(a) represented by the shaded area is the smallest possible one. By considering the "local control," the total system delay is increased. In a SO-DTA model, different levels of control as discussed above should be properly distinguished and incorporated into the model since they may lead to very different queuing patterns and route choice behavior. Vehicle holdings should be considered in conjunction with the constraint imposed by roadway geometry. In this example, we assume for simplicity that both capacities c_1 and c_2 for the two merge branches are inactive. The treatment can be more complicated when this requirement is removed. The network merge model in the cell transmission model covers a full range of the possible relationship between demand and capacity at merge junctions.

In this paper, we present a linear programming module for the merge junction based on the cell transmission model. The network model for the merge junction in the cell transmission model is more realistic than many other models. It is an extension of the kinematic wave model of traffic flows. The merge model in the cell transmission model, however, is non-linear, making it difficult to be incorporated into other existing linear models for the SO-DTA problem. In particular, our formulation generates a solution that preserves the proper merge priority rule and replicates exactly the feature in the cell transmission model for this particular application. The linear form of our formulation can be potentially integrated into an analytical SO-DTA model for a many-to-one network.

It should be noted that the analytical DTA models are usually difficult to implement for real-time operations. However, they can be useful to serve as a benchmark for evaluating the performance of various optimization heuristic developed for traffic control at different levels. In the following section, we will describe in detail our formulation of the merge component. Following that, a numerical example will be presented in Section III to demonstrate that the solution of a SO-DTA model can be very different with and without explicitly distinguishing control at different levels at merge junctions.

II. Enhancement to Modeling Merge Junctions

In this section, we show first how the merge model in the cell transmission model (CTM) can be represented by a linear programming module. The network representation in the cell transmission model differs from the traditional link-node representation [13, 14]. In the cell transmission model, freeway segments are represented by a series of cells. A merge branch i can be characterized by three cells as shown in Fig. 2, including cell i for the main branch, and two companion cells, indexed by $u_1(i)$ and $u_2(i)$, representing the two upstream merge branches.

The geometry of a cell is characterized by its capacity and density. The length of a cell is chosen such that it takes a single time step to traverse a cell at the free flow speed. In this section, we will first describe briefly the merge model in the CTM and the corresponding mathematical programming formulation proposed in the past that has been used widely for SO-DTA. We then present the linear programming module we developed that specifically takes into consideration the merge priority in the CTM. The following is a notation list for variables used in model formulation

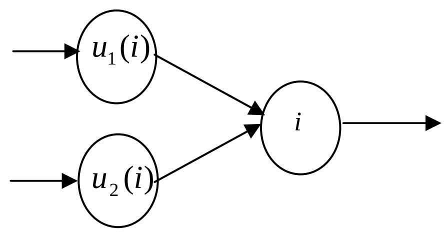


Fig. 2. Representation of a merge junction in the cell transmission model.

Decision variables:

$n_i(t)$: number of vehicles in cell i at time t.

$y_i(t)$: number of vehicles entering cell i in $[t, t+1)$.

$z_i(t)$: number of vehicles leaving cell i in $[t, t+1)$.

$p_i(t)$: dummy variables that linearize the constraints for preserving merge priority ratio.

$q_i(t)$: dummy variables that linearize the constraints for preserving merge priority ratio.

Endogenous variables:

α: wave coefficient $\left(=\frac{v_f}{w}\right)$, where v_f is free flow speed and w is wave speed.

$Q_i(t)$: (capacity) maximum number of vehicles that can enter cell i in $[t, t+1)$.

$N_i(t)$: (jam density) maximum number of vehicles that can reside in cell i in $[t, t+1)$.

λ_i: the merge priority for vehicles entering cell i from cells $u_1(i)$ and $u_2(i)$.

P_1, P_2: Penalty terms.

In many analytical models based on the cell transmission model, flows exiting from the two merge branches, $z_{u_1(i)}(t)$, $z_{u_2(i)}(t)$, and entering into the merge cell i, $y_i(t)$, are usually modeled by the following set of constraints:

$$z_{u_1(i)}(t)+z_{u_2(i)}(t)\le Q_i(t) \tag{1a}$$

$$z_{u_1(i)}(t)+z_{u_2(i)}(t)\le \alpha\left(N_i(t)-n_i(t)\right) \tag{1b}$$

$$z_{u_1(i)}(t)\le Q_{u_1(i)}(t) \tag{1c}$$

$$z_{u_2(i)}(t)\le Q_{u_2(i)}(t) \tag{1d}$$

$$z_{u_1(i)}(t)\le n_{u_1(i)}(t) \tag{1e}$$

$$z_{u_2(i)}(t)\le n_{u_2(i)}(t) \tag{1f}$$

$$y_i(t)=z_{u_1(i)}(t)+z_{u_2(i)}(t) \tag{1g}$$

The constraints above are a simplification of the network model in the CTM. They do not explicitly capture the merge priority described in the original model. In the cell transmission model, for a merge junction represented by cell i and two merge cells $u_1(i)$ and $u_2(i)$ shown in Fig. 2, if cell i has sufficient room to receive all vehicles from the two merge branches, then all vehicles residing in these two cells should advance. The actual sending and receiving flows can be calculated by:

$$z_{u_1(i)}(t)=S_{u_1(i)}(t)\text{ ,and } z_{u_2(i)}(t)=S_{u_2(i)}(t),$$
$$\text{if } R_i(t)>S_{u_1(i)}(t)+S_{u_2(i)}(t); \tag{2a}$$

where $S_i(t)$ and $R_i(t)$ are the maximum sending and receiving flows at time *t*, respectively, calculated for cell i.

If cell i does not have enough room to admit all of the vehicles from the two upstream merge cells, then flows advancing from the two upstream cells should follow the rules below as defined in the cell transmission model [14]:

$$z_{u_1(i)}(t)=\text{mid}\left\{S_{u_1(i)}(t),R_i(t)-S_{u_2(i)}(t),\frac{1}{1+\lambda_i}R_i(t)\right\} \tag{2b}$$

$$z_{u_2(i)}(t)=\text{mid}\left\{S_{u_2(i)}(t),R_i(t)-S_{u_1(i)}(t),\frac{\lambda_i}{1+\lambda_i}R_i(t)\right\} \tag{2c}$$

$$\text{if } R_i(t)\le S_{u_1(i)}(t)+S_{u_2(i)}(t);$$

In both cases, the flow into cell *i* is $y_i(t)=z_{u_1(i)}(t)+z_{u_2(i)}(t)$.

The above equations are non-linear. In the following, we propose an alternative version of the merge constraints equivalent to Equation (2) that can be easily linearized. Consideration reveals that in order to preserve the merge priority, the flow entering cell i from the two merge cells at every time step should satisfy the following two conditions: (1) The total flow advanced, i.e. $z_{u_1(i)}(t)+z_{u_2(i)}(t)$, is maximized and (2) the absolute value of $z_{u_1(i)}(t)-\lambda_i z_{u_2(i)}(t)$ is always minimized.

Condition 1 ensures that a maximum flow, bounded by R_i, is sent from the two merge branches. Note that this condition would also eliminate the unintended vehicle holding at the merge junction. Condition 2 determines the proper flow mix from the two merge branches. In other words, if there are multiple solutions that satisfy Condition 1, Condition 2 should be satisfied as well. Clearly, Condition 1 should be enforced on top of condition 2. The resulting solution can be shown graphically with Fig. 3. Fig. 3 is an alternative representation that maps the possible sending flow for a given traffic to the actual sending flow from each branch. In the Figure, the relationship between the possible sending flow (S_1 and S_2) and the actual sending flow (z_1 and z_2) is represented by a directional arc incident from (S_1 and S_2) to (z_1 and z_2).

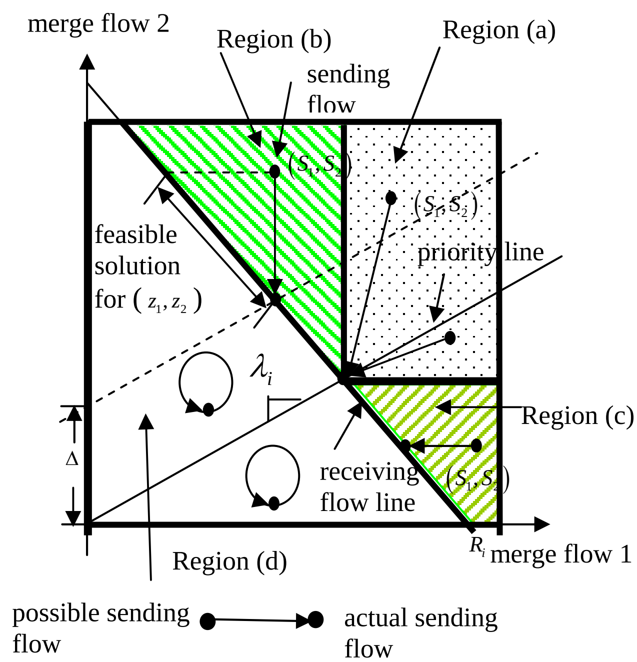


Fig. 3. An alternative representation of the relationship between sending and receiving flow in a merge junction.

As shown in the Figure, the merge priority line has a slope of λ_i, which is defined as the merge priority ratio for merge junction *i*. It governs the flow mix that enters the merge cell from the two merge branches when $R_i(t)\le S_{u_1(i)}(t)+S_{u_2(i)}(t)$, corresponding to area right to the receiving flow line shown in the figure. The area can be further partitioned into three regions, regions (a), (b) and (c), corresponding to conditions (a) $S_{u_1(i)}(t)\ge\frac{1}{1+\lambda_i}R_i(t)$, $S_{u_2(i)}(t)\ge\frac{\lambda_i}{1+\lambda_i}R_i(t)$, (b) $S_{u_1(i)}(t)<\frac{1}{1+\lambda_i}R_i(t)$, $S_{u_2(i)}(t)\ge\frac{\lambda_i}{1+\lambda_i}R_i(t)$, and (c) $S_{u_1(i)}(t)\ge\frac{1}{1+\lambda_i}R_i(t)$, $S_{u_2(i)}(t)<\frac{\lambda_i}{1+\lambda_i}R_i(t)$, as shown in the Fig. The mapping from the maximum sending flow to the actual sending flow based on the cell transmission model is represented by the two dots connected with an arrow line in each region. In region (a), the actual sending flow is unique. All sending flow mix (e.g. S_1 and S_2 as shown in the figure) in that region is mapped to the point where the receiving line and the priority line intercepts. In regions (b) and (c), the number of feasible solutions for a sending flow mix are infinite as indicated in the figure. The resulting sending flow mix satisfying all equations defined in the cell transmission model is unique. It is the one with the smallest $|\Delta| = \left|z_{u_2(i)}(t)-\lambda_i z_{u_1(i)}(t)\right|$ as shown in the figure. When $R_i(t) \ge S_{u_1(i)}(t)+S_{u_2(i)}(t)$, the possible sending flow and the actual sending flow are both in region (d), suggesting that all vehicles can advance since the capacity is greater than the demand.

For Conditions 1 and 2 to be satisfied in all four regions, the objective function of an equivalent mathematical programming formulation can be expressed as:

$$\max\ P_1\left[z_{u_1(i)}(t)+z_{u_2(i)}(t)\right]-P_2\left|z_{u_2(i)}(t)-\lambda_i z_{u_1(i)}(t)\right|$$

where P_1 and P_2 are penalty terms and $P_1 >> P_2$ (to ensure that condition 1 is preserved on top of condition 2). The first term, corresponding to condition 1, ensures that the actual sending flow is equal to the maximum receiving flow, and the second term, corresponding to condition 2, keeps the flow mix closest to the priority line. The objective function can be linearized. The linear programming formulation for the merge junction is thus:

[LP1]

$$\max P_1\left[z_{u_1(i)}(t)+z_{u_2(i)}(t)\right]-P_2\left[p_i(t)+q_i(t)\right]$$

subject to

$$z_{u_1(i)}(t)+z_{u_2(i)}(t)\le Q_i(t) \tag{3a}$$

$$z_{u_1(i)}(t)+z_{u_2(i)}(t)\le \alpha\left(N_i(t)-n_i(t)\right) \tag{3b}$$

$$z_{u_1(i)}(t)\le Q_{u_1(i)}(t) \tag{3c}$$

$$z_{u_2(i)}(t)\le Q_{u_2(i)}(t) \tag{3d}$$

$$z_{u_1(i)}(t)\le n_{u_1(i)}(t) \tag{3e}$$

$$z_{u_2(i)}(t)\le n_{u_2(i)}(t) \tag{3f}$$

$$z_{u_2(i)}(t)-\lambda_i z_{u_1(i)}(t)= p_i(t)-q_i(t) \tag{3g}$$

$$y_i(t)= z_{u_1(i)}(t)+z_{u_2(i)}(t) \tag{3h}$$

All variables are non-negative.

We show now that the above formulation is equivalent to the nonlinear model in the original merge model of the CTM. We can prove the equivalency between the two by exhausting all possible cases that would arise under different capacity and demand conditions. In fact, there are a total of four possible cases which can be identified in the graph given in Fig. 3. We define the sending flow in the same way as that defined in the CTM; the sending flows for merge branches, indexed by $u_1(i)$ and $u_2(i)$, are defined by $S_{u_1(t)}(t)=\min\left\{Q_{u_1(t)}(t), n_{u_1(t)}(t)\right\}$ and $S_{u_2(i)}(t)=\min\left\{Q_{u_2(i)}(t), n_{u_2(i)}(t)\right\}$, respectively. The receiving flow, $R_i(t)$, is defined by inequalities (3a) and (3b), i.e., $R_i(t)=\min\left\{Q_i(t), \alpha\left(N_i(t)-n_i(t)\right)\right\}$. The actual flow should then satisfy $z_{u_1(i)}(t)\le S_{u_1(i)}(t)$ (represented by (3c) and (3e)), $z_{u_2(i)}(t)\le S_{u_2(i)}(t)$ (represented by (3d) and (3f)), and $z_{u_1(i)}(t)+z_{u_2(i)}(t)\le R_i(t)$ (represented by (3a) and (3b)). The remaining constraints are used for preserving a proper merge priority. The four cases correspond to regions (a) to (d) shown in the Figure.

Case 1: $S_{u_1(i)}(t)\ge \frac{1}{1+\lambda_i}R_i(t)$ and $S_{u_2(i)}(t)\ge \frac{\lambda_i}{1+\lambda_i}R_i(t)$ (corresponding to region (a)).

The optimal solution to [LP1] is $z_{u_2(i)}(t)=\frac{\lambda_i}{\lambda_i+1}R_i(t)$, $z_{u_1(i)}(t)=\frac{1}{\lambda_i+1}R_i(t)$, and $p_i(t)=q_i(t)=0$. The objective function is maximized at $z^*=P_1R_i(t)$. This is the case we have $\Delta=0$, since the solution falls exactly on the priority line as shown in Fig. 3. In order to show that the optimal solution is indeed equivalent to the one given by the cell transmission model, we need to show that $z_{u_2(i)}(t)=\frac{\lambda_i}{\lambda_i+1}R_i(t)$ falls between $R_i(t)-S_{u_1(i)}(t)$ and $S_{u_2(i)}(t)$. By definition, $z_{u_2(i)}(t)=\frac{\lambda_i}{\lambda_i+1}R_i(t)\le S_{u_2(i)}(t)$. Also, $R_i(t)-S_{u_1(i)}(t)\le R_i(t)-\frac{1}{\lambda_i+1}R_i(t)=\frac{\lambda_i}{\lambda_i+1}R_i(t)$. Thus, $z_{u_2(i)}(t)=\frac{\lambda_i}{\lambda_i+1}R_i(t)$. Likewise, we can show that $z_{u_1(i)}(t)=\frac{1}{\lambda_i+1}R_i(t)$ is the mid term of $R_i(t)-S_{u_2(i)}(t)$, $\frac{1}{\lambda_i+1}R_i(t)$, and $S_{u_1(i)}(t)$. Thus, the solution obtained from the linear programming module [LP1] agrees with the solution from the merge model in the cell transmission model.

Case 2: $S_{u_1(i)}(t)\le \frac{1}{1+\lambda_i}R_i(t)$ and $S_{u_2(i)}(t)\ge \frac{\lambda_i}{1+\lambda_i}R_i(t)$, and $S_{u_1(i)}(t)+S_{u_2(i)}(t)\ge R_i(t)$ (corresponding to region (b)).

In this case, one of the two constraints, (3a) or (3b), must be binding. Thus, the first term in the objective function is maximized at $P_1R_i(t)=\min\{Q_i(t), \alpha(N_i(t)-n_i(t))\}$. In order to make the second term as small as possible, we need to keep the absolute value of $z_{u_2(i)}(t)-\lambda_i z_{u_1(i)}(t)$ as small as possible. Since in region (b) $z_{u_2(i)}(t)-\lambda_i z_{u_1(i)}(t)\ge 0$, the optimal solution is clearly to keep $z_{u_1(i)}(t)$ as large as possible. This can be achieved by making $z_{u_1(i)}(t)$ binding at its upper bound, i.e., $z_{u_1(i)}(t)=S_{u_1(i)}(t)$ and $z_{u_2(i)}(t)=R_i(t)-S_{u_1(i)}(t)$. The objective function is maximized at $z^*=P_1R_i(t)-P_2\left(R_i(t)-S_{u_1(i)}(t)(1+\lambda_i)\right)$. (In the extreme case, when $S_{u_1(i)}(t)=\frac{1}{1+\lambda_i}R_i(t)$ and $S_{u_2(i)}(t)=\frac{\lambda_i}{1+\lambda_i}R_i(t)$, we have $z^*=P_1R_i(t)$. The result is the same as that in case 1.) In this case, we need to show that $S_{u_1(i)}(t)$ is the mid term of $R_i(t)-S_{u_2(i)}(t)$, $\frac{1}{\lambda+1}R_i(t)$, and $S_{u_1(i)}(t)$ from Eq. (2b). From the graphical solution given in Fig. 3, $S_{u_1(i)}(t)\le\frac{1}{\lambda+1}R_i(t)$; but $S_{u_1(i)}(t)\ge R_i(t)-S_{u_2(i)}(t)$. Thus, $S_{u_1(i)}(t)$ is indeed the mid term of the three. We then have $z_{u_1(i)}(t)=S_{u_1(i)}(t)$. Similarly, we can show that $z_{u_2(i)}(t)=R_i(t)-S_{u_1(i)}(t)$. The solution obtained is consistent with Eq. (2c).

Case 3: $S_{u_1(i)}(t)\ge\frac{1}{1+\lambda_i}R_i(t)$ and $S_{u_2(i)}(t)\le\frac{\lambda_i}{1+\lambda_i}R_i(t)$, and $S_{u_1(i)}(t)+S_{u_2(i)}(t)\ge R_i(t)$ (corresponding to region (c)).
This case is symmetric to case 2. The proof given in case 2 applies to this case as well.

Case 4. $S_{u_1(i)}(t)+S_{u_2(i)}(t)\le R_i(t)$ (corresponding to region (d)). In this case, the optimal solution is achieved at $z_{u_1(i)}(t)=S_{u_1(i)}(t)=\min\left\{Q_{u_1(i)}(t),n_{u_1(i)}(t)\right\}$ and $z_{u_2(i)}(t)=S_{u_2(i)}(t)=\min\left\{Q_{u_2(i)}(t),n_{u_2(i)}(t)\right\}$. Clearly, the solution is consistent with (2a). Therefore, in all four cases, the solutions from the linear module (LP1) are consistent with the nonlinear merge model defined in the cell transmission model.

III. An Illustrative Example

In the following, we present a numerical example to show that the solution of SO-DTA models with a mathematical programming formulation can be very different with and without distinguishing control at different levels and imposed by different entities. We use the nine-cell one-to-one network of Fig. 4, with one diverge junction and one merge junction.

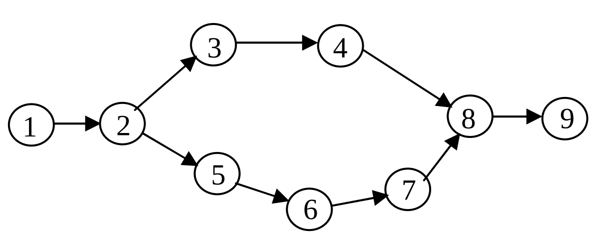


Fig. 4 The example network

The capacity and density for each cell are given in Tables I and II. The capacity for cell 6 changes over time. It is reduced to 0.2 for three time slices, representing the presence of an incident. The demand for each time slice entering the network is given in Table III. The merge priority is 3:1 for traffic from cells 4 and 7 to merge into cell 8. We also assume that the wave coefficient $\alpha=1$.

We compare the results from two different solution approaches. In solution approach 1, we solve the problem in a conventional way in which merge priority or the unintended vehicle holdings are not explicitly treated. In the second approach, we use the linear programming module discussed in this paper. The problem was solved by the CPLEX Linear Optimizer. The results are given in Tables IV and V. For each entry in the two tables, two values are displayed as the output of the model. The first value is the number of vehicles in cell i at time t, $n_i(t)$ .The second one is the outflow at time t, $z_i(t)$.

The solution obtained by using the conventional approach (Table IV) exhibits vehicle holdings in a number of places. For example, at t = 9, cell 1 has 5.80 units of vehicles and its downstream cell, cell 2, has 8.60 units of vehicles. The exit flow from cell 1 is 0 as shown in the Table. According to the cell transmission model, the exit flow from cell 1 at $t=9$ should be 3.4 ($\min\{5.80,6.00,6.00,12.00-8.60\}$) units instead of 0. Thus, 3.4 units of vehicles were held for an additional time step in cell 1. For vehicle movements at the merge junction, since one is unable to incorporate the 3:1 merge priority into the conventional formulation, the merge priority behaves like all-or-nothing assignment most of the time. In each time step, only vehicles from one of the merge cells advance even though there are queues for vehicles in both cells (see the boxes for cells 4 and 7 from t = 11 to t = 16).

The solution generated from the proposed formulation (Table V) exhibits no vehicle holdings. The merge priority at the merge junction is preserved as specified. When both of the merge cells are congested, the 3:1 merge ratio is maintained for the vehicles coming from cells 4 and 7 (see the entries for corresponding cells for $t\ge 9$).

Interestingly, the solutions obtained with and without considering merge priority would also affect the route choice decisions. For solution 1, 64% of the vehicles choose route 1-2-3-4-8-9. For solution 2, only 45% of the vehicles choose the same route.

TABLE I
Parameters for Each Cell in the Example Network

Cell number i	Capacity Q_i	Density N_i
1	6	12
2	6	12
3	3	6
4	3	6
5	3	6
6	see Table II	6
7	3	6
8	3	6
9	3	6

TABLE II

TIME DEPENDENT CAPACITY FOR CELL 6

Time	Capacity
1	3
2	3
3	3
4	3
5	0.2
6	0.2
7	0.2
8	3
9	3
10	3
11	3
12	3
13	3
14	3
15	3
16	3
17	3
18	3
19	3
20	3

TABLE III

TIME DEPENDENT DEMAND FROM THE ORIGIN

Time	Demand
1	6
2	6
3	6
4	6
5	6
6	6
7	6
8	6
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	0
19	0
20	0

TABLE IV

SOLUTION 1 (WITHOUT PRESERVING MERGE PRIORITY RULES)

	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10
1	0.00 0.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 3.20	8.80 3.00	5.80 0.00	5.80 5.80
2	0.00 0.00	0.00 0.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 3.20	8.80 3.00	9.00 3.40	8.60 3.00	5.60 0.00
3	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 0.00	6.00 0.00	6.00 3.00
4	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 2.40	0.60 0.00	0.60 0.60
5	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 0.20	5.80 0.20	5.80 0.20	5.60 3.00	3.00 2.40	3.60 0.00
6	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 0.20	3.00 0.20	3.00 0.20	3.00 3.00	3.00 3.00	2.40 2.40
7	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.20 0.00	0.40 0.00	0.60 0.60	3.00 3.00	3.00 2.40
8	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00
9	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00

	t=11	t=12	t=13	t=14	t=15	t=16	t=17	t=18	t=19	t=20
1	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
2	11.4 3.00	8.40 0.00	8.40 5.40	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
3	3.00 3.00	3.00 3.00	0.00 0.00	3.00 3.00	3.00 0.00	3.00 0.00	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00
4	3.00 3.00	3.00 0.00	6.00 3.00	3.00 3.00	3.00 0.00	3.00 3.00	0.00 0.00	3.00 3.00	0.00 0.00	0.00 0.00
5	3.60 0.00	3.60 3.00	0.60 0.00	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
6	0.00 0.00	0.00 0.00	3.00 3.00	0.00 0.00	3.00 0.00	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
7	3.00 0.00	3.00 3.00	0.00 0.00	3.00 0.00	3.00 3.00	0.00 0.00	3.00 3.00	0.00 0.00	0.00 0.00	0.00 0.00
8	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	0.00 0.00
9	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00

TABLE V

SOLUTION 2 (PRESERVING MERGE PRIORITY RULES)

	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10
1	0.00 0.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 3.20	8.80 3.20	5.60 3.00	2.60 2.60
2	0.00 0.00	0.00 0.00	6.00 6.00	6.00 6.00	6.00 6.00	6.00 3.20	8.80 3.20	8.80 3.00	9.00 4.10	7.90 3.00
3	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 2.80	3.20 2.80	3.20 2.80	1.50 0.75
4	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 2.80	3.20 2.80	3.20 2.80	3.20 0.75	5.25 0.75
5	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 0.20	5.80 0.20	5.80 0.20	5.80 3.00	3.00 3.00	3.00 3.00
6	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 0.20	3.00 0.20	3.00 0.20	3.00 3.00	3.00 3.00	3.00 2.25
7	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.20 0.20	0.20 0.20	0.20 0.20	3.00 2.25	3.75 2.25
8	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00
9	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00

	t=11	t=12	t=13	t=14	t=15	t=16	t=17	t=18	t=19	t=20
1	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
2	7.50 3.00	4.50 2.25	2.25 2.25	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
3	0.75 0.75	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
4	5.25 0.75	5.25 0.75	4.50 0.75	3.75 0.75	3.00 0.75	2.25 0.75	1.50 0.75	0.75 0.75	0.00 0.00	0.00 0.00
5	3.00 2.25	3.25 2.25	3.75 2.25	3.75 2.25	1.50 1.50	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00
6	3.75 2.25	3.75 2.25	3.75 2.25	3.75 2.25	3.75 2.25	3.00 2.25	0.75 0.75	0.00 0.00	0.00 0.00	0.00 0.00
7	3.75 2.25	3.75 2.25	3.75 2.25	3.75 2.25	3.75 2.25	3.75 2.25	3.75 2.25	2.25 2.25	0.00 0.00	0.00 0.00
8	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	0.00 0.00
9	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00	3.00 3.00

IV. Conclusions

This paper presents a modeling component capable of capturing more realistically traffic dynamics in merge junctions. The model departs from many existing analytical models formulated in the past in that it does not assume system control over every part of the network. Our formulation enhances realism in modeling merge junctions by specifically treating the merge priority governed by the roadway geometry as a set of constraints. As a result, local control governed by the roadway geometry or other mechanism is separable from control aimed at achieving system optimization. We show in the paper that the solution obtained from our formulation is consistent with that from the nonlinear version of the merge model in the cell transmission model. The linear form of our formulation makes it possible to be integrated into an analytical SO-DTA model formulated as a linear programming problem. We demonstrate with a numerical example that the solution of the SO-DTA problem can be different with and without considering explicitly the different levels of control in a network. As a byproduct, the paper has also provided an alternative graphical representation for traffic dynamics at merge junctions described in the cell transmission model.

The discussion and formulation of the LP model for the SO-DTA problem in this paper is limited to a many-to-one network. More work will be required to extend the model to address the traffic flow issues on a network with multiple origins and multiple destinations.

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