Applied Mathematical Modelling xxx (2013) xxx-xxx



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# Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

# Modeling the asymmetry in traffic flow (b): Macroscopic approach

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#### ARTICLE INFO

Article history: Received 5 November 2012 Received in revised form 2 April 2013 Accepted 22 April 2013 Available online xxxx

*Keywords:* Asymmetric car-following Continuum flow Anisotropic Weighted essentially no-oscillatory

## ABSTRACT

In [H. Xu, H. Liu, H. Gong, Modeling the asymmetry in traffic flow (a): microscopic approach, J. Appl. Math. Model. (submitted for publication)], the asymmetric characteristic of traffic flow has been studied from a microscopic approach through the modeling of carfollowing behavior. This paper further discusses the asymmetric traffic flow modeling at the macroscopic scale. The microscopic asymmetric full velocity difference model is extended to a continuum traffic flow model to study the anisotropic characteristic and diffusive influence under various traffic conditions. In order to accurately solve the mathematical problem, a weighted essentially no-oscillatory (WENO) approach is applied. The performance of the model is then demonstrated through thorough evaluation against select classic models and field data. The macroscopic model is the first of its kind that is directly developed from an asymmetric car-following approach. The results show that the model is able to present many complex traffic phenomena observed in the field such as shock waves, rarefaction waves, stop-and-go waves and local cluster effects at a better level of accuracy than most of the existing models.

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## 1. Introduction

Traffic flow theory has been investigated from two approaches. One is from a microscopic point of view and focused on car-following behavior [2–6]; the other is from a macroscopic approach that is focused on the relationship among speed, density, and volume in the time and space domain [7–12]. If technically sound, a car-following model can be fully developed into a macroscopic model which usually has a broader application.

In [1], the authors have developed a mathematical model based on the optimal velocity theory by Bando et al. [4] which directly models car-following from the physics laws of car motion. In the model, the asymmetric car-following property was considered in an asymmetric full velocity difference (AFVD) approach which uses two sensitivity coefficients to separately represent the intensity of acceleration and deceleration. The model was also validated against field data and the two sensitivity coefficients were calibrated as well. Whether this asymmetric car-following approach could be converted into a fully developed macroscopic model and how the continuum flow model would perform is the focus of the second part of the study.

Macroscopic models have been well studied since the work of Lighthill and Whitham [7] and Richards [12]. The LWR model follows a scalar hyperbolic conservation law and exhibits a wide range of phenomena such as traffic sound waves, shocks and rarefaction waves. One of the major deficiencies of the LWR model is that the speed is solely determined by

0307-904X/\$ - see front matter @ 2013 Elsevier Inc. All rights reserved. http://dx.doi.org/10.1016/j.apm.2013.04.039

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the equilibrium speed density relationship and no fluctuation of speed around the equilibrium values is allowed, thus, the model is not suitable for the description of non-equilibrium situations such as the stop-and-go traffic.

Many efforts have been devoted to improving the LWR model through developing high order models, which use dynamic equations for the speed to replace the equilibrium relationship. However, as pointed out by Daganzo [13], it was shown that nonequilibrium models had two families of characteristic along which traffic information is transmitted: one is slower and the other is faster than the speed of the traffic stream that carries them. The faster characteristic leads to a gas like behavior in which vehicle from behind can force vehicles in front to speed up, and the diffusion causes 'wrong way' travel [14].

As a fundamental principle of the traffic flow theory, vehicles are anisotropic which should respond only to the front vehicles. Testing and evaluation of this property of the new asymmetric car-following based continuum model is one of the focuses of this study. The authors chose several classic and recently developed non-equilibrium models to examine the properties of the proposed model. The next generation simulation data (NGSIM) was used in the evaluation process so that the performance of the selected models can be compared with each other against field data.

As the new continuum model involves complex higher order differential equations, the weighted essentially nooscillatory (WENO) method [15] is used to get the numerical solution. The WENO scheme is one of the most accurate methods in computational fluid dynamics for discontinuous situations with high order differential equations and non-oscillatory properties. A key idea of the WENO method is to use a convex combination of all candidate stencils to achieve high accuracy. In this research, the authors use the fifth order difference WENO scheme to perform the numerical calculation.

The rest of the paper is structured as follows. The mathematical model is presented in Section 2, followed by the development of WENO and the numerical solutions. Data from the NGSIM dataset is then used to evaluate the performance of the selected classic models and the new approach.

## 2. The asymmetric anisotropic continuum flow model

The asymmetric property in traffic flow has received increasing attention in recent studies. For instance, in [16] Yeo and Skabardonis systematically analyzed the properties of asymmetric traffic flow and used the NGSIM data to examine their findings. In this section, we will start with verification of the existence of asymmetry in macroscopic flow models to lay a logical ground for the introduction of the new asymmetric anisotropic continuum flow model; then attention will be directed to the process of converting the asymmetric full velocity difference car-following approach to a continuum model.

In the LWR model, the relationship between the macroscopic density  $\rho$  with respects to space x and time t is

$$\frac{\partial \rho(\mathbf{x},t)}{\partial t} + q'_e(\rho)\frac{\partial \rho(\mathbf{x},t)}{\partial \mathbf{x}} = \mathbf{0},\tag{1}$$

where  $q'_e(\rho)$  is the flow-density relationship (the fundamental diagram). Pipes [18] derived the traffic stream acceleration in the form of

$$\frac{dv}{dt} = -\rho \left( v_e' \right)^2 \frac{\partial \rho}{\partial x}.$$
(2)

Substituting a specific flow-density relationship into the equation, the acceleration will become a function of the traffic density  $\rho$ .

For example, let us take the following speed-density relationship presented in Zhang [17],

$$\nu_e = \nu_f \left( 1 - \left( \frac{\rho}{\rho_j} \right)^n \right),\tag{3}$$

where *n* is a constant,  $v_e$  is the equilibrium speed,  $v_f$  is the free flow speed and  $\rho_j$  is the jam density. The relationship between acceleration and density can be obtained as follows:

$$\frac{d\nu}{dt} = -\rho \left(\nu_e'\right)^2 \frac{\partial\rho}{\partial x} = -\rho \left(-\frac{\nu_f n}{\rho_j} \left(\frac{\rho}{\rho_j}\right)^{n-1}\right)^2 \frac{\partial\rho}{\partial x} = -\frac{\nu_f^2 n^2}{\rho_j^{2n}} \rho^{2n-1} \frac{\partial\rho}{\partial x}$$
(4)

Consider the two scenarios as illustrated in Fig. 1. Fig. 1(a) has an upstream density of  $\rho_1$ , downstream density of  $\rho_2$ , with  $\rho_1 > \rho_2$ . Fig. 1(b) has an upstream density  $\rho_2$ , downstream density  $\rho_1$ . The wave front is at location  $x_0$ . Applying the scenario of Fig. 1(a) to Eq. (4), it indicates that traffic moving from the upstream with a high traffic density to the downstream with a low traffic density will be accelerating. On the other hand, the scenario showed in Fig. 1(b) depicts decelerating traffic.

Assuming that the change in density with respect to space is constant, since  $v_f$  and  $\rho_j$  are constant, the acceleration intensity and traffic density will have a relationship of  $\left|\frac{dv}{dt}\right| \sim -n^2 \rho^{2n-1} \frac{\partial \rho}{\partial x}$ . To simplify, we can make n = 1, then it will yield to the relationship of  $\left|\frac{dv}{dt}\right| \sim -\rho \frac{\partial \rho}{\partial x}$ . Suppose that  $\frac{\partial \rho}{\partial x}$  is constant in both of the scenario, in scenario (a), the driver is traveling in the accelerating scenario with the expectation that the traffic density will fall into a lower level, the acceleration intensity will fall. On the other hand, when the driver is traveling in scenario (b), the acceleration intensity will increase. In other words, the acceleration intensity is sensitive to traffic density.



Fig. 1. Illustration of accelerating and decelerating sections in a traffic stream.

## 2.1. The asymmetric full velocity car-following approach

The asymmetric full velocity car-following approach is developed on the basis of Bando et al.'s optimal velocity theory with additional terms to represent acceleration and deceleration. In earlier works prior to Bando et al.'s optimal velocity (OV) theory, car-following models were developed based on the idea that a driver adapts to the velocity of the leading vehicle with a delayed time. Models of this type take into account the time lag of the driver's response by introducing the time lag into the equations of motion.

The optimal velocity model, on the other hand, is based on the idea that vehicles adapt to a distance-dependent optimal velocity. The OV model accounts the effect of time lag through the second order differential equations based on the equation of motion in physics. Therefore, the time lag in the OV model is not the delay from driver's response, but the delay of car motion which has its root just in the dynamic equation itself. The OV model takes the form of:

$$\frac{d\nu_n(t+\tau)}{dt} = \kappa[V(\Delta x_n(t)) - \nu_n(t)].$$
(5)

Despite its simplicity and few parameters, the OV model describes many properties of real traffic flows (e.g., stop-and-go traffic at signalized intersections) and has become one of the most referenced car-following studies. Notable works along this line include the work of Helbing and Tilch and Jiang et al. [5,6]. The AFVD model developed by the authors is also an extension of Bando's work which introduces two higher order terms and two sensitivity coefficients to accurately model the asymmetry in car-following, it takes the following form:

$$\frac{d\nu_n(t)}{dt} = \kappa [V(\Delta x_n(t)) - \nu_n(t)] + \lambda_1 H(-\Delta \nu_n(t)) \Delta \nu_n(t) + \lambda_2 H(\Delta \nu_n(t)) \Delta \nu_n,$$
(6)

where *H* is the Heaviside function,  $\lambda_1$  and  $\lambda_2$  are sensitivity coefficients,  $\kappa[V(\Delta x_n(t)) - v_n(t)]$  is the interaction term that comes from the OV model, and  $\lambda_1 H(-\Delta v_n(t)) \Delta v_n(t)$  and  $\lambda_2 H(\Delta v_n(t)) \Delta v_n(t)$  are called velocity differences terms. Details of the model and calibration of the two coefficients could be found in [1].

## 2.2. Converting the asymmetric car-following approach into a continuum flow model

The works in [1] have laid a technical ground for the development of the continuum flow models that take into account the asymmetric characteristic of vehicles. In order to develop the corresponding macroscopic continuum model, we need to transform the discrete variables of individual vehicles into the continuous flow variables [19], which are shown below:

$$\begin{aligned}
\nu_{n+1}(t) &\to u(x,t), \quad \nu_n(t) \to u(x+\Delta,t), \quad V(\Delta x) \to u_e(k(x,t)) \\
\kappa &\to 1/T, \quad \lambda_1 \to 1/\tau_1, \quad \lambda_2 \to 1/\tau_2
\end{aligned} \tag{7}$$

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where

q: traffic flow volume,

k: traffic flow density in the time and space regime,

*u*: traffic flow space mean speed,  $\partial_r$ ,  $\partial_y$ : the partial derivative with respect to time and space, respectively,

 $V_e(k)$ : the equilibrium speed,

 $Q_e(k)$ : the flow-density relationship,

T: the relaxation time,

 $\tau$ : the time needed for the backward propagated disturbance to travel a distance of  $\Delta$ .

The derivation is as follows:

$$\frac{du(x,t)}{dt} = \frac{1}{T} [u_e(k(x,t)) - u(x,t)] + \frac{1}{\tau_1} (u(x+\Delta,t) - u(x,t)) H(u(x+\Delta,t) - u(x,t)) + \frac{1}{\tau_2} (u(x+\Delta,t) - u(x,t)) H(u(x+\Delta,t) - u(x,t)) H(u(x+\Delta,t) - u(x,t)) + \frac{1}{\tau_2} (u(x+\Delta,t) - u(x,t)) H(u(x+\Delta,t) - u(x,t)) H(u(x+\Delta,t) - u(x,t)) + \frac{1}{\tau_2} (u(x+\Delta,t) - u(x$$

$$\frac{\partial u(x,t)}{\partial t} + u \frac{\partial u(x,t)}{\partial x} = \frac{u_e(k(x,t)) - u(x,t)}{T} + \frac{\Delta}{\tau_1} \frac{\partial u(x,t)}{\partial x} H\left(\frac{\partial u(x,t)}{\partial x}\right) + \frac{\Delta}{\tau_2} \frac{\partial u(x,t)}{\partial x} H\left(-\frac{\partial u(x,t)}{\partial x}\right),$$
$$\frac{\partial u(x,t)}{\partial t} + u \frac{\partial u(x,t)}{\partial x} = \frac{u_e(k(x,t)) - u(x,t)}{T} + c_1 \frac{\partial u(x,t)}{\partial x} H\left(\frac{\partial u(x,t)}{\partial x}\right) + c_2 \frac{\partial u(x,t)}{\partial x} H\left(-\frac{\partial u(x,t)}{\partial x}\right).$$

The conservation equation is:

$$\partial_t k + \partial_x (ku) = 0.$$

And the vector form of the AFVD flow model can be rewritten as

$$\frac{\partial U}{\partial t} + [A] \frac{\partial U}{\partial x} = E,$$

$$U = \begin{bmatrix} k \\ u \end{bmatrix}, \quad A = \begin{bmatrix} u & k \\ 0 & u - c \end{bmatrix}, \quad E = \begin{bmatrix} g \\ (u_e - u)/T \end{bmatrix}$$

$$\begin{cases}
c = c_1, & \text{if } \frac{\partial u}{\partial x} \ge 0 \\
c = c_2, & \text{if } \frac{\partial u}{\partial x} < 0.
\end{cases}$$
(9)

(8)

## 3. The weighted essentially non-oscillatory scheme

The WENO scheme is currently the most accurate method in computational fluid dynamics for discontinuous situations with high order differential equations and non-oscillatory properties. The key idea of WENO scheme is to use a convex combination of all candidate stencils to achieve the high accuracy. We will use the fifth order difference WENO scheme [15] to conduct the numerical calculation. The third order total variation diminishing (TVD) Runge–Kutta method is used for time evolution, which has the following form:

$$u^{(1)} = u^{n} + \Delta t L(u^{n})$$

$$u^{(2)} = \frac{3}{4} u^{n} + \frac{1}{4} u^{(1)} + \frac{1}{4} \Delta t L(u^{(1)})$$

$$u^{n+1} = \frac{1}{3} u^{n} + \frac{2}{3} u^{(2)} + \frac{2}{3} \Delta t L(u^{(2)})$$
(10)

The order of WENO scheme is based on the order of ENO (Essentially Non-Oscillatory) scheme [20]. For example, if the order of ENO scheme is r, the order of WENO will be 2r - 1. So if one wants to get the fifth order WENO scheme, the third order ENO scheme will be used in calculation.

The third order ENO scheme has the following forms:

$$\hat{f}_{j+\frac{1}{2}}^{(1)} = \frac{1}{3}f_{j-2} - \frac{7}{6}f_{j-1} + \frac{11}{6}f_j, \quad \hat{f}_{j+\frac{1}{2}}^{(2)} = -\frac{1}{6}f_{j-1} + \frac{5}{6}f_j + \frac{1}{3}f_{j+1}, \quad \hat{f}_{j+\frac{1}{2}}^{(3)} = \frac{1}{3}f_j + \frac{5}{6}f_{j+1} - \frac{1}{6}f_{j+2}, \quad (11)$$

Then the weights can be obtained as

$$\tilde{\omega}_j = \frac{\gamma_l}{\left(\varepsilon + \beta_l\right)^2}, \quad j = 1, 2, 3 \tag{12}$$

with  $\gamma_1 = \frac{1}{10}, \gamma_2 = \frac{3}{5}, \gamma_3 = \frac{3}{10}$ .

 $\beta_l$  is called the smoothness indicators, have the following form from [11,16].

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$$\beta_{1} = \frac{13}{12} (f_{j-2} - 2f_{j-1} + f_{j})^{2} + \frac{1}{4} (f_{j-2} - 4f_{j-1} + 3f_{j})^{2}, \quad \beta_{2} = \frac{13}{12} (f_{j-1} - 2f_{j} + f_{j+1})^{2} + \frac{1}{4} (f_{j-1} - f_{j+1})^{2}, \quad \beta_{3} = \frac{13}{12} (f_{j} - 2f_{j+1} + f_{j+2})^{2} + \frac{1}{4} (3f_{j} - 4f_{j+1} + f_{j+2})^{2}, \quad (13)$$

Furthermore,  $\varepsilon$  is a very small number like  $10^{-6}$  which avoids the denominator to be zero.

 $\sum_{j=1}^{3} \omega_j = 1$  is required and  $\omega_j \ge 0$  for stability and consistency. So it will yield to the following revised weights form:

$$\omega_i = \frac{\omega_i}{\sum_{j=1}^3 \tilde{\omega}_j}, \quad i = 1, 2, 3 \tag{14}$$

Therefore, the fifth order WENO flux form is as follows,

$$\hat{f}_{j+\frac{1}{2}} = \omega_1 \hat{f}_{j+\frac{1}{2}}^{(1)} + \omega_2 \hat{f}_{j+\frac{1}{2}}^{(2)} + \omega_3 \hat{f}_{j+\frac{1}{2}}^{(3)}.$$
(15)

To avoid the sign changing of f', the flux splitting will use the Lax–Friedrichs form, which is given by

$$f^{\pm} = \frac{1}{2}(f(u) \pm \alpha u), \tag{16}$$

with  $\alpha = \max |f'(u)|$ , which is v for the ARZ model as previously discussed since v is the larger one of the Eigen values. In order to apply the WENO scheme, the flux formation of AFVD flow model is essential for calculation.

$$\partial_t U + \partial_x F(U) = \mathbf{0}, \tag{17}$$
with  $U = \begin{pmatrix} k \\ u \end{pmatrix}$ , and  $F(U) = \begin{pmatrix} ku \\ \frac{u^2}{2} - c_0 u \end{pmatrix}$ .

The Eigen values,  $\lambda$  of the *A* matrix are found as  $\lambda_1(U) = u$ ,  $\lambda_2(U) = u - c_0$ . Then we have the following Eigen values associated with the flux form.

$$\lambda_1(U) = u - c_0 \leqslant u$$
  

$$\lambda_2(U) = u$$
(18)

From the Eigen values above, we can see the greater one is equal to the flow speed *u*. It follows that the characteristic speeds are always less than or equal to the macroscopic flow velocity *u*. In other words, the new continuum model does not have any characteristic speed greater than the macroscopic flow velocity. This property is significant because vehicles are anisotropic.

#### 4. Evaluation with field data

This section evaluates the performance of the new AFVD model with three classic models using the NGSIM data. The three models are the ARZ model [21,22], the LWR model, and the PW model [18]. The WENO scheme described in Section 3 is applied to solve the Riemann problem. The test site is divided into five consecutive segments, by providing the same initial condition to the models, traffic densities at downstream segments ware estimated by the models, which are then compared with the real data.

## 4.1. Description of the dataset

The NGSIM data from the US 101 during the time of 7:50–8:05 am was used. The test data comes from the five lanes of the highlighted section in Fig. 2 with a total length of 500 feet. For illustrative purpose, Fig. 3 shows a plot depicting the flow density relationship of the data from this section.

#### 4.2. Comparison with selected classic models

The total time of the experiment is 900 s with 10 s increment from time 0 and a total of 81 time cells are used in the valuation process. In addition, the 500 feet section was divided into five segments, noted as sections 1 through 5. The initial conditions were set up based on the real data at the upstream sections (in our case, section 2), for each time step (5 s), the Riemann problem solver developed according to the WENO scheme calculates the estimated densities at every downstream section. The densities of the real data are calculated based on the method described in [23], comparison is made between the estimated densities and the densities from real data. For illustrative purpose, Table 1 shows the part of the estimated densities from 10 s to 100 s at section 3 along with the densities from real data.

Traffic densities over the full time period at section 3 are plotted in Fig. 4. As the ARZ model has high deviation at some time points, it is eliminated from the plot.

From the paired *t*-tests in Table 2, the PW model has the best fitness to the real data in general while the AFVD model has the best performance in the anisotropic type of models.

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Fig. 2. The schematic of the study area from US 101.



Fig. 3. Flow density relationship of the US 101 data during 7:50-8:05am.

## 5. Numerical tests

To further demonstrate the property of the AFVD flow model, in this section, the authors use WENO scheme to compare the numerical solutions of the Riemann problems between the AFVD flow model and the LWR model. In the following,  $u_u$  and  $u_d$  represent upstream and downstream velocities, while  $k_u$  and  $k_d$  are upstream and downstream densities, respectively.

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Table 1						
Partial results of	the estimated	densities a	t section 3	along with	the real	data.

Time	LWR density 3	ARZ density 3	PW density 3	AFVD density 3	Real data density 3
10	58.99108	67.05186	58.94286	57.03545	57.024
20	73.88554	66.53553	64.83506	70.61872	62.304
30	3.56082	0.42525	21.45853	0.69139	100.32
40	3.56082	0.42525	46.57996	0.69608	152.592
50	49.66478	0.0267	62.44706	80.52602	112.992
60	92.5977	87.11365	74.50705	88.49712	88.176
70	65.3039	61.77961	68.75449	61.90468	76.56
80	43.26827	42.24163	53.79586	41.42998	45.408
90	0.56391	0.00365	8.20062	0.03314	61.248
100	47.59239	52.26821	38.53137	49.35314	48.048



Fig. 4. Estimated and real traffic densities.

**Table 2**The *t*-test of the estimated densities and the density from real data.

Data	Ν	Mean	STD	P-value
LWR	81	68.07573	28.86592	0.10200
ARZ	81	70.29043	57.55066	0.93793
PW	81	68.22392	19.4494	0.02348
AFVD	81	67.70254	25.9239	0.02667
NGSIM	81	75.9603	27.2309	

The equilibrium speed-density relationship for the numerical tests is expressed as

$$u_e = u_f \left[ 1 - \exp\left( 1 - \exp\left( \frac{c_m}{u_f} \left( \frac{k_m}{k} - 1 \right) \right) \right) \right],\tag{19}$$

and free-flow speed  $u_f = 30m/s$ , maximum density  $k_m = 0.2veh/m$ , kinetic wave speed  $c_1 = 11m/s$ ,  $c_2 = 20m/s$  under the jam density.

The total length of the simulation road is 950 m which is divided into 100 segments. The time interval is 1 s and total simulation time is T = 10s. Two cases are designed with different initial conditions and the initial condition of case one is set as  $k_u = 0.04$  veh/m and  $k_d = 0.18$  veh/m. The results are shown in Fig. 5.

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Fig. 5a. The velocity plot of the AFVD flow model (case one).



Fig. 5b. The velocity plot of the LWR model (case one).

From the numerical results from the AFVD flow model and the LWR model, it can be seen that the major traffic activity which is the shock wave acts almost identical. Nonetheless, the AFVD flow model shows subtle wondering in the front of the shock wave which better represents the real traffic condition of the selected dataset which has several stop-and-go sessions

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Fig. 6a. The density plot of the AFVD flow model (case two).



Fig. 6b. The density plot of the LWR model (case two).

as shown in Fig. 3. To further depict its property, another test was conducted with the initial condition of  $k_u = 0.18$  veh/m and  $k_d = 0.04$  veh/m, the results are shown in Fig. 6.

The rarefaction wave solution is given by case two with a different initial condition. Similarly, the numerical results do not show much difference in the jam region between the AFVD flow model and the LWR model. However, there is oscillation

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on the low density end for the AFVD flow model. Two major activities can be found from the plots. One is the stop-and-go activity. Usually the traffic flow does not turn to free flow phase suddenly, therefore, it is more realistic for the stop-and-go phase connects the free flow phase and congestion phase. The second important phenomenon is that the density oscillation is not amplified over time. This phenomenon is also easy to explain. For the rarefaction wave, the traffic flow basically propagates from the congestion phase to the free flow phase. The drivers are less likely to have strong deceleration activities. It is different with the shock solution. The numerical tests under various initial conditions further demonstrated the importance of considering asymmetric properties in the continuum flow model.

## 6. Conclusion

We presented a continuum flow model which was derived from an asymmetric car-following approach. A weighted essentially non-oscillatory scheme was developed to accurately solve the high order differential equations. Three classic models were selected to examine the performance of the new model against the field data. Through comparisons, the advantage of the model was demonstrated. Derived directly from a car-following approach, it does not have the problem commonly found in high-order models, in which there exists a characteristic speed that is greater than the macroscopic flow velocity. Using the real data, the experimental analysis demonstrates that the asymmetric car-following based flow model outperforms most of the selected classic models. The numerical tests show that the model is able to simulate complex traffic phenomena observed in the field such as shock waves, rarefaction waves, stop-and-go waves and local cluster effects.

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