TEXAS TECH UNIVERSITY
TTUISD™

MTHMOD 1A
Mathematical Models with Applications
First Semester
#PR-10235, BK-10236 (v.3.0)

To the Student:

After your registration is complete and your proctor has been approved, you may take the Credit by Examination for MTHMOD 1A.

WHAT TO BRING

- several sharpened No. 2 pencils
- graphing calculator
- extra sheets of scratch paper

ABOUT THE EXAM

The examination for the first semester of Mathematical Models with Applications consists of 40 questions, of which 35 are multiple choice and the rest are short answer. The exam is based on the Texas Essential Knowledge and Skills (TEKS) for this subject. The full list of TEKS is included in this document (it is also available online at the Texas Education Agency website, http://www.tea.state.tx.us/). The TEKS outline specific topics covered in the exam, as well as more general areas of knowledge and levels of critical thinking. Use the TEKS to focus your study in preparation for the exam.

The examination will take place under supervision, and the recommended time limit is three hours. You may not use any notes or books. A percentage score from the examination will be reported to the official at your school.

In preparation for the examination, review the TEKS for this subject. All TEKS are assessed. A list of review topics is included in this document to focus your studies. It is important to prepare adequately. Since questions are not taken from any one source, you can prepare by reviewing any of the state-adopted textbooks that are used at your school. A formula chart will be provided to you for use on your exam.

The practice exam included in this document will give you a model of the types of questions that will be asked on your examination. It is **not** a duplicate of the actual examination. It is provided to illustrate the format of the exam, not to serve as a complete review sheet.

Good luck on your examination!

CONCEPTS

The following is a list of concepts covered in Mathematical Models with Applications 1A and offers a view of topics that need to be studied, reviewed, and learned for this assessment.

Review of Algebraic Fundamentals

- Real numbers and mathematical operations
- Solving linear equations
- Percents
- Scientific notation

Fundamentals of Mathematical Modeling

- Mathematical models
- Formulas
- Ratio and proportion
- Word problems

Applications of Algebraic Modeling

- Models and patterns in plane and solid geometry
- Models and patterns in triangles
- Models and patterns in right triangles
- Right triangle trigonometry
- Models and patterns in art, architecture, and nature
- Models and patterns in music

Graphing

- Rectangular coordinate system

MTHMOD 1A Practice Exam

Multiple Choice. Choose the best answer for each question. Show your work on the exam or on scratch paper.

Check your answers with the answer key provided.

1. Evaluate the compound interest formula $I = P\left(1 + \frac{r}{n}\right)^{nt}$ using the given values of the variables. Round your answer to the nearest cent.

   P = \$1,115, r = 3.6%, n = 2, t = 7

   A. I = \$7,699.00
   B. I = \$1,431.34
   C. I = \$11,314.18
   D. I = \$316.34

2. Suppose p varies directly with the square root of q. If p = 36 when q = 16, find p when q = 4.

   A. 18
   B. 16
   C. 15
   D. 17

3. The current population of a certain small town is 4,700 and is declining at a rate of 1.7% per year. What will the population be in 10 years?

   A. 3,968
   B. 3,965
   C. 3,971
   D. 3,974

4. A ball is thrown upward from the top of a 140 ft. tall building. If the initial speed of the ball is 15 ft. per second, then the equation $h = 140 + 15t - 16t^2$ gives the height of the ball above the ground t seconds after it is thrown. After how many seconds will the ball hit the ground? (Round your answer to the nearest hundredth of a second.)

   A. 3.34 seconds
   B. 3.46 seconds
   C. 3.65seconds
   D. 3.77 seconds

5. A scale model is made of a plane whose length is 116 feet and whose wingspan is 94 feet. If the length of the model is 38 inches, find the wingspan of the scale model to the nearest tenth of an inch.

   A. 31.4 inches
   B. 31.2 inches
   C. 30.8 inches
   D. 31.0 inches

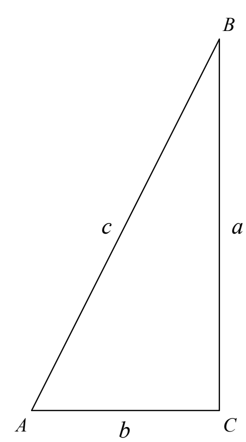
6. A golden rectangle is to be constructed such that the longest side is 13 inches long. How long is the other side? (Round your answer to the nearest tenth of an inch.)

   A. 8.0 inches
   B. 7.4 inches
   C. 7.2 inches
   D. 7.6 inches

7. An isosceles triangle has a base with length 17 inches and two congruent sides with lengths of 12 inches each. Find the height of the triangle. (Round your answer to the nearest tenth of an inch.)

   A. 7.9 inches
   B. 8.5 inches
   C. 8.8 inches
   D. 8.2 inches

8. For the triangle shown below, find a if $\angle A = 55^o$ and $b = 15$. (Round your answer to the nearest tenth.)



   A. 21.4
   B. 20.8
   C. 21.2
   D. 21.6

9. If you start with a note with a frequency of 620 Hz, what frequency is two octaves lower than this frequency?

   A. 2480 Hz
   B. 310 Hz
   C. 155 Hz
   D. 1240 Hz

10. If a certain piece of music is written in $\frac{3}{8}$ time, how many eighth notes are required for one measure of music?

   A. 6
   B. 4
   C. 12
   D. 3

11. If a golden rectangle has a width of 18 feet, what is its length? (Round your answer to the nearest tenth of a foot.)

   A. 10.7 feet
   B. 11.1 feet
   C. 10.5 feet
   D. 10.9 feet

12. Use the exponential growth model to calculate the amount of money you will have in the bank after 12 years if you deposit $31,000 into an account that pays 2.6% interest compounded continuously.

   A. $42,513.63
   B. $42,640.04
   C. $42,235.18
   D. $42,350.80

Short Answer. For each question, show your work on the answer sheet, and circle your answer.

13. The height h of a ball after it is thrown vertically upward is given in feet by the function $h(t) = 4 + 6t - 16t^2$.

   A. Determine the time when the ball is at its peak.

   B. Determine the maximum height of the ball.

MTHMOD 1A Practice Exam Answer Key

1. B
2. A
3. B
4. B
5. C
6. A
7. B
8. A
9. C
10. D
11. B
12. D
13. A. 0.2 seconds

    B. 4.6 feet

Mathematical Models with Applications Formula Charts

GEOMETRY FORMULAS		
Circumference of a circle:		$C = 2\pi r = \pi d$
Area formulas:	square	$A = s^2$
	rectangle	$A = lw$
	triangle	$A = 0.5bh$
	circle	$A = \pi r^2$
Hero's formula for the area of a triangle: $A = \sqrt{s(s-a)(s-b)(s-c)}$ $\quad s = \frac{a+b+c}{2}$		
Pythagorean theorem: In a right triangle, $a^2 + b^2 = c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse.		
Cylinder:	volume	$V = \pi r^2 h$
	lateral surface area	$L.A. = 2\pi rh$
	total surface area	$S.A. = 2\pi rh + 2\pi r^2$
Sphere:	volume	$V = \frac{4}{3}\pi r^3$
	total surface area	$S.A. = 4\pi r^2$
Cone:	volume	$V = \frac{1}{3}\pi r^2 h$
	lateral surface area	$L.A. = \pi r\sqrt{r^2 + h^2}$
	total surface area	$S.A. = \pi r^2 + \pi r\sqrt{r^2 + h^2}$
Trigonometric functions:		$\sin = \frac{opp}{hyp}$
		$\cos = \frac{adj}{hyp}$
		$\tan = \frac{opp}{adj}$
Sine wave for musical pitch: $Y = A\sin(Bt)$ where A = amplitude, B = frequency, and t = time		
Frequencies: Ratio between the frequencies of any 2 successive pitches = 1.05946		

BUSINESS FORMULAS

Percent increase/decrease $= \dfrac{\text{new value} - \text{original value}}{\text{original value}} \times 100$

Simple interest: $I = Prt$

Maturity value: $M = P + I$

Interest compounded quarterly: $M = P\left(1 + \dfrac{r}{4}\right)^{4t}$, where r = annual rate,

t = number of years

Interest compounded monthly: $M = P\left(1 + \dfrac{r}{12}\right)^{12t}$, where r = annual rate,

t = number of years

Interest compounded daily: $M = P\left(1 + \dfrac{r}{365}\right)^{365t}$, where r = annual rate,

t = number of years

Straight-line depreciation: $\dfrac{\text{original value} - \text{residual value}}{\text{number of years}}$

Fixed-rate mortgage monthly payment formula:

where A = amount borrowed

t = number of years

r = annual rate

$$P = A\left[\frac{\frac{r}{12}\left(1 + \frac{r}{12}\right)^{12t}}{\left(1 + \frac{r}{12}\right)^{12t} - 1}\right]$$

continued →

ALGEBRA FORMULAS

Direct variation: $y = kx$

Inverse variation: $y = \frac{k}{x}$

Joint variation: $y = kxz$

Slope of a line: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Linear equation: $Ax + By = C$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Quadratic formula: If $ax^2 + bx + c = 0$, $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exponential growth: $y = Ae^{rn}$

- Power function: $f(x) = cx^k$
- Exponential function: $f(x) = b^x$

Cramer's rule: To find the solution of a system of equations $ax + by = c$, $dx + ey = f$:

$$x = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

continued →

PROBABILITY FORMULAS

Empirical probability:

$$P(E) = \frac{\text{number of times event } E \text{ has occurred}}{\text{total number of times the experiement has been performed}}$$

Theoretical probability:

$$P(E) = \frac{\text{number of ways event } E \text{ can occur}}{\text{total number of possible outcomes}}$$

Odds in favor: event occurs : event does not occur

Odds against: event does not occur : event occurs

Permutations: $nPr = \frac{n!}{(n-r)!}$

Combinations: $nCr = \frac{n!}{r!(n-r)!}$

STATISTICS FORMULAS

Mean average: $\overline{x} = \frac{\Sigma x}{n}$

Range: R = highest number – lowest number

Sample standard deviation: $s = \sqrt{\frac{\Sigma\left(x-\overline{x}\right)^2}{n-1}}$

z-scores: $z = \frac{x-\overline{x}}{s}$

Texas Essential Knowledge and Skills

MTHMOD 1A – Mathematical Models with Applications, First Semester

§111.43. Implementation of Texas Essential Knowledge and Skills for Mathematical Models with Applications (One Half Credit), Beginning with School Year 2011-2012.

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I. This course must be taken before receiving credit for Algebra II.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) Mathematical Models with Applications is designed to build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. This mathematics course provides a path for students to succeed in Algebra II and prepares them for various post-secondary choices. Students learn to apply mathematics through experiences in personal finance, science, engineering, fine arts, and social sciences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, model information, solve problems, and communicate solutions. Students will select from tools such as physical objects; manipulatives; technology, including graphing calculators, data collection devices, and computers; and paper and pencil and from methods such as algebraic techniques, geometric reasoning, patterns, and mental math to solve problems.

(4) In Mathematical Models with Applications, students will use a mathematical modeling cycle to analyze problems, understand problems better, and improve decisions. A basic mathematical modeling cycle is summarized in this paragraph. The student will:

(A) represent

(i) identify the variables in the problem and select those that represent essential features; and

(ii) formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describe the relationships between the variables;

(B) compute: analyze and perform operations on the relationships between the variables to draw conclusions;

(C) interpret: interpret the results of the mathematics in terms of the original problem;

(D) revise: confirm the conclusions by comparing the conclusions with the problem and revising as necessary; and

(E) report: report on the conclusions and the reasoning behind the conclusions.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Mathematical modeling in personal finance. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. The student is expected to:

(A) use rates and linear functions to solve problems involving personal finance and budgeting, including compensations and deductions;

(B) solve problems involving personal taxes; and

(C) analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees.

(3) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. The student is expected to:

(A) use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases;

(B) analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option;

(C) use technology to create amortization models to investigate home financing and compare buying a home to renting a home; and

(D) use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.

(4) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:

(A) analyze and compare coverage options and rates in insurance;

(B) investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans; and

(C) analyze types of savings options involving simple and compound interest and compare relative advantages of these options.

(5) Mathematical modeling in science and engineering. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. The student is expected to:

(A) use proportionality and inverse variation to describe physical laws such as Hook's Law, Newton's Second Law of Motion, and Boyle's Law;

(B) use exponential models available through technology to model growth and decay in areas, including radioactive decay; and

(C) use quadratic functions to model motion.

(6) Mathematical modeling in science and engineering. The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. The student is expected to:

(A) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture;

(B) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields;

(C) use the Pythagorean Theorem and special right-triangle relationships to calculate distances; and

(D) use trigonometric ratios to calculate distances and angle measures as applied to fields.

(7) Mathematical modeling in fine arts. The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. The student is expected to:

(A) use trigonometric ratios and functions available through technology to model periodic behavior in art and music;

(B) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography;

(C) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music; and

(D) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields.

(8) Mathematical modeling in social sciences. The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. The student is expected to:

(A) determine the number of ways an event may occur using combinations, permutations, and the Fundamental Counting Principle;

(B) compare theoretical to empirical probability; and

(C) use experiments to determine the reasonableness of a theoretical model such as binomial or geometric.

(9) Mathematical modeling in social sciences. The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. The student is expected to:

(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem-and-leaf plots, and box and whisker plots, to draw conclusions from the data and determine the strengths and weaknesses of conclusions;

(B) analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions;

(C) distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies;

(D) use data from a sample to estimate population mean or population proportion;

(E) analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions; and

(F) use regression methods available through technology to model linear and exponential functions, interpret correlations, and make predictions.

(10) Mathematical modeling in social sciences. The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. The student is expected to:

(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions; and

(B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation.

Source: The provisions of this §111.43 adopted to be effective September 10, 2012, 37 TexReg 7109.