To the Student:

After your registration is complete and your proctor has been approved, you may take the Credit by Examination for PRE CALC 1A.

WHAT TO BRING

- several sharpened No. 2 pencils
- lined notebook paper
- a graphing calculator capable of performing exponential regression

ABOUT THE EXAM

The examination for the first semester of Precalculus consists of 15 problem-solving questions, and is based on the Texas Essential Knowledge and Skills (TEKS) for this subject. The full list of TEKS is included at the end of this document (it is also available online at the Texas Education Agency website, http://www.tea.state.tx.us/). The TEKS outline specific topics covered in the exam, as well as more general areas of knowledge and levels of critical thinking. Use the TEKS to focus your study in preparation for the exam.

The examination will take place under supervision, and the recommended time limit is three hours. You may not use any notes or books. You will need to bring the materials listed above with you to the testing site. A formula list (included in this document) will be provided with your exam. A percentage score from the examination will be reported to the official at your school.

By taking a Credit by Examination, you are attempting to demonstrate your mastery of a subject in an effort to receive credit for your current level of understanding. In preparation for the examination, review the TEKS for this subject. All TEKS are assessed. You would be well-advised to thoroughly review a state-adopted textbook by carefully reading each chapter and working through all of the included problems. Any textbook from the Texas Adoption list can be used for a review. It is important to prepare adequately.

There is also a sample examination included with this letter. The sample exam will give you a model of the types of questions that will be asked on your examination. It is not a duplicate of the actual examination. It is provided to illustrate the format of the exam, not to serve as a complete review sheet. A formula list is provided at the end of the exam.

Good luck on your examination!
1. Use the graph of each relationship to answer the following questions.

A.

Is the relationship a function? __________

Justify your answer. ____________________________________________________

Domain = __________

Range = __________

List the symmetries: ____________________________________________________
B.

Is the relationship a function? __________
Justify your answer. ______________________________________________________

Domain = __________
Range = __________
List the symmetries: ______________________________________________________

2. Use the relationship \{(0, 2), (3, 4), (5, 2), (8, 1)\} to answer the following.

A. Give the domain and range of the relationship.

   Domain = __________

   Range = __________

B. Is the relationship a function? __________  Justify your answer.
   _________________________________________________________________

C. Give the inverse of the relationship. {______________________________}

D. Give the domain and range of the inverse relationship.

   Domain = __________

   Range = __________

E. Is the inverse relationship a function? __________  Justify your answer.
   _________________________________________________________________
3. Given the functions $f(x) = x^2 - 3$ and $g(x) = \sqrt{x + 3}$, answer the following:

A. Find and simplify $(f + g)(x)$.

B. Find and simplify $f(4) - (f/g)(-2)$.

C. Find and simplify $g\left(b^4 - 3\right)$. 
D. Use the compositions \((f \circ g)(x)\) and \((g \circ f)(x)\) to verify that the functions are inverses of one another.

4. Use parent functions and transformations to provide the equations of the graphs below.

A.  

\[
\begin{align*}
y & \quad \text{equation: } y = \underline{\quad} \\
x & \quad \text{equation: } y = \underline{\quad}
\end{align*}
\]

B.  

\[
\begin{align*}
y & \quad \text{equation: } y = \underline{\quad} \\
x & \quad \text{equation: } y = \underline{\quad}
\end{align*}
\]

5. Write the standard form of the quadratic function \(f(x) = 2x^2 + 10x - 7\) and identify the vertex of the graph by completing the square.
6. Use the rational function \( f(x) = \frac{x - 3}{x^2 - 9} \) to answer the following.

A. Provide the information below. If there is none, please state so.

\[
\begin{align*}
\text{domain:} & \quad \underline{\text{__________________________________________}} \\
\text{range:} & \quad \underline{\text{__________________________________________}} \\
\text{x-intercepts:} & \quad \underline{\text{__________________________________________}} \\
\text{y-intercepts:} & \quad \underline{\text{__________________________________________}} \\
\text{vertical asymptotes:} & \quad \underline{\text{____________________________________________}} \\
\text{horizontal (end behavior) asymptotes:} & \quad \underline{\text{____________________________________________}} \\
\text{removable discontinuities (holes):} & \quad \underline{\text{____________________________________________}} \\
\end{align*}
\]

B. Graph the function using and labeling the information found above.
7. Refer to the sequences $A = 4, 1, -2, -5, -8, \ldots$ and $B = 9, 3, \frac{1}{3}, \frac{1}{9}, \ldots$ to answer the following.

A. Is sequence $A$ arithmetic or geometric? _________________________________

   If the sequence is arithmetic, the common difference $d =$ ___________________

   If the sequence is geometric, the common ratio $r =$ _______________________

Is sequence $B$ arithmetic or geometric? _________________________________

   If the sequence is arithmetic, the common difference $d =$ ___________________

   If the sequence is geometric, the common ratio $r =$ _______________________

B. Give the $n$th term, $a_n$, of sequence $A$.

C. Give the sigma notation for the sum of the first six terms of series $A$.

D. Give the sum, $S_n$, of the first one hundred terms of the series $B.$

continued →
E. Find the limit as \( n \), the number of terms, approaches infinity for the sequence and series of \( B \). If they converge, give the value; if they diverge, state so.

8. Use the polynomial function \( f(x) = \frac{1}{3}x^3 - x^2 - 8x + 4 \) to answer the questions below.

A. Find the following, if they exist:

Relative minimum(s) = ____________________

Relative maximum(s) = ____________________

Intervals where \( f(x) \) is increasing = ____________________

Intervals where \( f(x) \) is decreasing = ____________________

B. Use the information above to provide a sketch of the function. Label the critical values.

\[
\begin{align*}
\text{continued} & \rightarrow
\end{align*}
\]
9. Find the difference quotient and simplify your answer.

\[ f(x) = x^2 - 3x + 1, \quad \frac{f(x + h) - f(x)}{h}, \quad h \neq 0 \]

10. Use the graph of the function below to answer the following questions.

Does \( f(0) \) exist? ________
If yes, \( f(0) = \) ________________.
From the left, \( \lim_{{x \to 0^-}} f(x) = \) ________________.
From the right, \( \lim_{{x \to 0^+}} f(x) = \) ________________.
Does the \( \lim_{{x \to 0}} f(x) \) exist? ________
If yes, \( \lim_{{x \to 0}} = \) ________________.
Does the \( \lim_{{x \to 0}} f(x) = f(0) \)? ________
Is the function continuous at \( f(0) \)? ________
If not, what type of discontinuity is it? ________________
11. Find the following limits algebraically.

A. \( \lim_{x \to -2} (5x^2 - 4x) \)

B. \( \lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8} \)

C. \( \lim_{x \to -1} \frac{1}{x + 2} - 1 \)

D. \( \lim_{x \to \infty} \frac{6x}{5x^2 - 3x} \)
E. \[ \lim_{x \to -\infty} \frac{3x + 8}{2x} \]

F. \[ \lim_{x \to -\infty} \frac{-7x^3}{x + 4} \]

12. Simplify the following expressions.

A. \[ \frac{\left( x^3 y^5 \right)^2}{\left( x^4 y^{-2} \right)^3} \]

B. \[ \sqrt[3]{32x^7 y^{10}} \]

continued →
C. \((3x^3y^4)^0\)

D. \(4^{\log_4 9}\)

E. \(\log(1000)^p\)

F. \(\ln 1\)

13. Solve the equations below for \(k\).

A. \(\log_3 (k + 8) + 2 \log_3 \sqrt{k} = 2\)
14. When initially observed, a container of bacteria under investigation had a total of 123 cells. In five days, the count has grown to 440. Assume the growth rate is constant and continuous.

A. What is the growth rate \( k \) of the cells?

B. How many cells would have been present two days before the initial observation?

C. How many days from the initial observation will it take for the cell count to reach 10,000?
15. The data below give hourly temperatures for an afternoon. Using them, perform a linear regression with a graphing calculator to find an equation which models the system.

<table>
<thead>
<tr>
<th>Hours after noon</th>
<th>4</th>
<th>5</th>
<th>1</th>
<th>3.5</th>
<th>2.7</th>
<th>3</th>
<th>4.1</th>
<th>2</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (F)</td>
<td>78</td>
<td>85</td>
<td>56</td>
<td>75</td>
<td>68</td>
<td>67</td>
<td>77</td>
<td>65</td>
<td>55</td>
</tr>
</tbody>
</table>

Perform a linear regression and give the values of \( y = ax + b \), \( a \), \( b \), and \( r \).

\( a = \) _________________
\( b = \) _________________
\( r = \) _________________  (correlation)

Use the equation to estimate the temperature at 2:15 pm.

\( \text{temperature} = \) __________
The following formulas will be included with the exam. All others must be memorized. Note that formulas are not labeled, so you will need to know when each is applicable.

\[ a_n = dh + c \]
\[ a_n = a_1 + (n-1)d \]
\[ a_n = a_1 r^{n-1} \]
\[ S_n = \frac{n}{2} (a_1 + a_n) \]
\[ S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right) \]
\[ S = \frac{a_1}{1 - r} \]
PRE CALC 1A Sample Exam Answer Key

1. A. Is the relationship a function? no
   Justify your answer: fails the vertical line test
   Domain: $-3 \leq x \leq 5$
   Range: $-2 \leq y \leq 2$
   Symmetries: $x$-axis ($y = 0$), $x = 1$, point (1, 0)

   B. Is the relationship a function? yes
   Justify your answer: passes the vertical line test
   Domain: all reals
   Range: $y \geq 2$
   Symmetries: $y$-axis ($x = 0$)

2. A. Domain: $\{0, 3, 5, 8\}$
   Range: $\{2, 4, 1\}$

   B. Is the relationship a function? no
   Justify your answer: $x = 2$ of the domain goes to both $y = 0$ and $y = 5$.

   C. Inverse: $\{(2, 0), (4, 3), (2, 5), (1, 8)\}$

   D. Domain: $\{2, 4, 1\}$
      Range: $\{0, 3, 5, 8\}$

   E. Is the relationship a function? yes
   Justify your answer: Each $x$ of domain goes to only one $y$.

3. A. $x^2 - 3 + \sqrt{x+3}$
   
   B. $16 - 3 - \frac{4-3}{-2+3} = 16 - 3 - \frac{1}{1} = 13 - 1 = 12$

   C. $\sqrt{b^4 - 3 + 3} = \sqrt{b^4} = b^2$

   D. $(f \circ g)(x) = \left(\sqrt{x+3}\right)^2 - 3 = x + 3 - 3 = x$

   $(g \circ f)(x) = \sqrt{x^2 - 3 + 3} = \sqrt{x^2} = x$

4. A. $y = -(x-2)^2 + 3$
   B. $y = -(x+1)^3$
5. \( f(x) = 2x^2 + 10x - 7 \)
   \[ f'(x) = 2(x^2 + 5x) - 7 \]
   \[ f'(x) = 2\left(x^2 + 5x + \frac{25}{4}\right) - 7 - 2\left(\frac{25}{4}\right) \]
   \[ f'(x) = 2\left(x + \frac{5}{2}\right)^2 - \frac{39}{2} \]
   Vertex \( \left(-\frac{5}{2}, -\frac{39}{2}\right) \)

6. A. domain: all reals, \( x \neq \pm 3 \)
   range: all reals, \( y \neq 0 \)
   x-intercepts: none
   y-intercepts: \( \frac{1}{3} \)
   vertical asymptotes: \( x = -3 \)
   horizontal asymptotes: \( y = 0 \)
   holes: \( x = 3 \)

B.

7. A. sequence \( A \): arithmetic
   common difference: \( d = -3 \)
   sequence \( B \): geometric
   common ratio: \( r = \frac{1}{3} \)

B. \( a_n = a_1 + (n - 1)d \)
   \[ a_n = 4 + (n - 1) - 3 = -3n + 7 \]
C. \[ \sum_{k=1}^{6} -3k + 7 = -3(1) + 7 - 3(2) + 7 - 3(3) + 7 - 3(4) + 7 - 3(5) + 7 - 3(6) + 7 = -21 \]

D. \[ S_n = \frac{a_1 - a_n r^n}{1-r} = \frac{9 - 9 \left( \frac{1}{3} \right)^{100}}{1 - \frac{1}{3}} \approx \frac{9}{2} \approx 13.5 \]

E. Series \[ S_n = \frac{a_1}{1-r} = \frac{9}{1 - \frac{1}{3}} = 13.5 \]

Sequence \[ \lim_{x \to \infty} \frac{9}{3^n} = 0 \]

8. A. relative minimum: \( x = 4 \) or \( (4, -22.667) \)
    relative maximum: \( x = -2 \) or \( (-2, 13.333) \)
    intervals where \( f(x) \) is increasing: \( (-\infty, -2) \) and \( (4, \infty) \)
    intervals where \( f(x) \) is decreasing: \( (-2, 4) \)

B. **drawing not to scale**

---

continued →
9. \( f(x) = x^2 - 3x + 1, \quad \frac{f(x+h) - f(x)}{h}, \quad h \neq 0 \)

\[
\begin{align*}
    f(x) &= x^2 - 3x + 1 \\
    f(x + h) &= (x + h)^2 - 3(x + h) + 1 \\
    \frac{f(x+h) - f(x)}{h} &= \frac{[(x + h)^2 - 3(x + h) + 1] - [x^2 - 3x + 1]}{h} \\
    &= \frac{x^2 + 2hx + h^2 - 3x - 3h + 1 - x^2 + 3x - 1}{h} \\
    &= \frac{2hx + h^2 - 3h}{h} \\
    &= 2x + h - 3
\end{align*}
\]

10. Does \( f(0) \) exist? yes

\( f(0) = -2 \)

From the left, \( \lim_{x \to 0} f(x) = 1 \)

From the right, \( \lim_{x \to 0} f(x) = 1 \)

Does the \( \lim_{x \to 0} f(x) \) exist? yes

\( \lim_{x \to 0} = 1 \)

Does the \( \lim_{x \to 0} f(x) = f(0) \)? no

Is the function continuous at \( f(0) \)? no

Discontinuity = Point

11. A. \( \lim_{x \to -2} (5x^2 - 4x) = \lim_{x \to -2} (5(2)^2 - 4(2)) = (5(4) - 8) = 12 \)

B. \( \lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8} = \lim_{x \to 2} \frac{2^2 - 4}{2^3 - 8} = 0 \), which is indeterminate, so try more algebra:

\[
\begin{align*}
    \lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8} &= \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x^2 + 2x + 4)} \\
    &= \lim_{x \to 2} \frac{x+2}{x^2 + 2x + 4} \\
    &= \frac{2 + 2}{2^2 + 2(2) + 4} \\
    &= \frac{4}{12} \\
    &= \frac{1}{3}
\end{align*}
\]

C. \( \lim_{x \to -1} \frac{1}{x+1} \quad 1 \quad \frac{1}{x+1} = \frac{1 - x - 2}{x + 2} = \frac{-(x+1)}{x+1} = -1 \)
D. \( \lim_{{x \to \infty}} \frac{6x}{5x^2 - 3x} = 0 \) because the degree in the denominator is larger.

E. \( \lim_{{x \to -\infty}} \frac{3x + 8}{2x} = \frac{3}{2} \) because the degrees of the numerator and denominator are equal.

F. \( \lim_{{x \to -\infty}} \frac{-7x^3}{x + 4} \approx -7x^2 \). So the limit is \(-\infty\), because the function values decrease without bound as the value of \(x\) decreases. Or you can say the limit does not exist because there is no unique value the function approaches as \(x\) decreases without bound.

12. A. \( x^{21}y^4 \)

B. \( (2y^2x)\left(\frac{5}{x^2}\right) \)

C. \( \lim_{{x \to -1}} \frac{1}{x + 2} - 1 \)

D. 3

E. 3p

F. 0

13. A. \( k = 1 \)

B. \( k = -1.24 \)

14. A. \( k = 0.255 \)

B. \( y \approx 73.86 \) cells (74)

C. \( t = 17.25 \) days (17)

15. \( a = 7.018 \)

\( b = 49.202 \)

\( r = 0.989 \)

Temperature = 64.99
§111.35. Precalculus (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998, and at that time shall supersede §75.63(bb) of this title (relating to Mathematics). Students can be awarded one-half to one credit for successful

(b) Introduction.

(1) In Precalculus, students continue to build on the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent math

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications an

(c) Knowledge and skills.

(P.1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic

(A) describe parent functions symbolically and graphically, including \( f(x) = x^n \), \( f(x) = \frac{1}{n} x \), \( f(x) = \log_a x \), \( f(x) = \frac{1}{x} \), \( f(x) = e^x \), \( f(x) = |x| \), \( f(x) = a^x \),

(B) determine the domain and range of functions using graphs, tables, and symbols;

(C) describe symmetry of graphs of even and odd functions;

(D) recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc.), points on the graph of a function, and the symbolic representation of a function; and

(E) investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically.

(P.2) The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. The student is expected to:

(A) apply basic transformations, including \( a \cdot f(x) \), \( f(x) + d \), \( f(x - c) \), \( f(b \cdot x) \), and compositions with absolute value functions, including \( |f(x)| \), and \( f(|x|) \), to the parent functions;

(B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; and

(C) investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.

(P.3) The student uses functions and their properties, tools and technology, to model and solve meaningful problems. The student is expected to:

(A) investigate properties of trigonometric and polynomial functions;

(B) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

(C) use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient);

(D) use properties of functions to analyze and solve problems and make predictions; and

(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.

(P.4) The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. The student is expected to:

(A) represent patterns using arithmetic and geometric sequences and series;

(B) use arithmetic, geometric, and other sequences and series to solve real-life problems;

(C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and

(D) apply sequences and series to solve problems including sums and binomial expansion.
(P.5) The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations. The student is expected to:

(A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets;
(B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound;
(C) convert between parametric and rectangular forms of functions and equations to graph them; and
(D) use parametric functions to simulate problems involving motion.

(P.6) The student uses vectors to model physical situations. The student is expected to:

(A) use the concept of vectors to model situations defined by magnitude and direction; and
(B) analyze and solve vector problems generated by real-life situations.

Source: The provisions of this §111.35 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1931.