To the Student:

After your registration is complete and your proctor has been approved, you may take the Credit by Examination for PRE CALC 1B.

WHAT TO BRING

- several sharpened No. 2 pencils
- notebook paper
- a graphing calculator capable of performing exponential regression

ABOUT THE EXAM

The examination for the second semester of Precalculus consists of 14 problem-solving questions, and is based on the Texas Essential Knowledge and Skills (TEKS) for this subject. The full list of TEKS is included at the end of this document (it is also available online at the Texas Education Agency website, http://www.tea.state.tx.us/). The TEKS outline specific topics covered in the exam, as well as more general areas of knowledge and levels of critical thinking. Use the TEKS to focus your study in preparation for the exam.

The examination will take place under supervision, and the recommended time limit is three hours. You may not use any notes or books. You will need to bring the materials listed above with you to the testing site. A formula list (included at the end of this document) will be provided with your exam. A percentage score from the examination will be reported to the official at your school.

By taking a Credit by Examination, you are attempting to demonstrate your mastery of a subject in an effort to receive credit for your current level of understanding. In preparation for the examination, review the TEKS for this subject. All TEKS are assessed. You would be well-advised to thoroughly review a state-adopted textbook by carefully reading each chapter and working through all of the included problems. Any textbook from the Texas Adoption list can be used for a review. It is important to prepare adequately.

Good luck on your examination!
PRE CALC 1B Sample Exam

Take this exam without reference to any books or notes, exactly as you will during the actual test. You will need a graphing calculator capable of performing exponential regression. Refer to the list of formulas at the end of the exam. Check your answers with the answer key provided.

1. Find the area of the triangle with sides $a = 5$, $b = 7.8$, and $c = 11$.

   Area __________

2. Solve the triangle given the angle $C = 68^\circ$, $b = 7$, and $c = 10$. (There may be no solution, one solution, or two solutions.)

   $A = \underline{}$

   $A' = \underline{}$

   $B = \underline{}$

   $B' = \underline{}$

   $a = \underline{}$

   $a' = \underline{}$

   continued →
3. Write the following conic sections in standard form and graph them. After you identify the conic section, give the following information:

- If a circle: coordinates of center, and the radius
- If a parabola: coordinates of the vertex and focus, and equations for the directrix and axis of symmetry
- If an ellipse: coordinates of the center, major and minor endpoints, and foci
- If a hyperbola: coordinates of the center, vertices and foci, and equations of the asymptotes

A. \(4x^2 + y - 8x = 0\)

\[\begin{array}{c}
\text{y-axis} \\
\hline
\text{x-axis} \\
\end{array}\]

B. \(x^2 - 4y^2 - 6x - 8y - 11 = 0\)

\[\begin{array}{c}
\text{y-axis} \\
\hline
\text{x-axis} \\
\end{array}\]
4. A. Identify the graph of the equation. Then give the angle of rotation $\theta$.

$$2x^2 - 4xy + y^2 + 3 = 0$$

Type __________

$\theta = ________$

B. Graph the conic with the rotation identified in 4A.

5. Graph the trigonometric functions below and provide the requested information. You must show two complete periods of the function and label the graph.

A. $y = \csc (x - 90^\circ)$

Amplitude __________
Period __________
Phase shift __________
B. \( y = -3 \sin (2x - \pi) + 2 \)

Amplitude __________
Period __________
Phase shift __________

6. A tractor tire has a diameter of 6 feet and is revolving at a rate of 45 rpm. At \( t = 0 \), a certain point is at height 0.

A. Write an equation with phase shift 0 to describe the height of the point above the ground after \( t \) seconds.

B. How high is the point above the ground after 28 seconds?

7. Verify the trig identities below.

A. \( \frac{\sin 2A}{1 - \cos 2A} = \cot A \)
B. \((1 + \cot^2 x)(1 - \cos 2x) = 2\)

8. Solve the trig equations below for \(0 \leq x \leq 2\pi\).

A. \(\sin 2x \sec x + 2 \cos x = 0\)

B. \(\cos 2x = 5\sin^2 x - \cos^2 x\)

9. Use the half/double angle or sum/difference identities to solve the following. You must show your work.

A. \(\cos 15^\circ\)
B. \( \sin 105^\circ \)

10. Evaluate the following expressions, given that \( \cos \alpha = \frac{4}{5} \), \( \alpha \) is in the fourth quadrant, \( \tan \beta = -\frac{4}{3} \), and \( \beta \) is in the second quadrant.

A. \( \cot \alpha \)

B. \( \sin (\alpha - \beta) \)

C. \( \cos 2\beta \)

continued \( \rightarrow \)
11. Given the vectors \( \mathbf{v} = (-2, 4, 0) \), \( \mathbf{w} = (5, -1, 6) \), and \( \mathbf{u} = (3, 2, -3) \), evaluate or solve the following problems.

A. Find \( 3\mathbf{v} - (2\mathbf{w} + \mathbf{u}) \), give the magnitude of the resultant vector.

B. Find the dot product, \( \mathbf{v} \cdot \mathbf{w} \), and state whether the vectors are perpendicular to one another. Justify your answer.

12. Use vectors to solve the following problem.

A ship sails 40° east of north for 11 miles. The ship then changes course to 28° south of east. If the ship travels for another 7 miles on its new course, how far is the ship from its starting point?
13. Juanita puts the shot for her high-school track team. She releases the shot 5 feet above the ground with an initial velocity of 28 ft/sec at an angle of 35° to the horizontal.

A. Give a set of parametric equations that model the flight of the shot.

B. How far does the shot travel horizontally?

C. What was the maximum height the shot reached while on its flight?

14. Given the point (5, –1) and the vector \( \mathbf{v} = (-2, 4) \) answer the following.

A. Give the parametric equations of the line passing through the given point and parallel to the vector \( \mathbf{v} \).
B. Graph the line represented in problem 14A. Indicate the path of a particle traveling along the line from time $t = -1$ to time $t = 2$. 
### PRE CALC 1B Formula List

The following formulas will be included on a formula sheet with the exam. It is the student’s responsibility to know when and how to apply the formulas. Any formulas not included should be memorized by the student.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \sin(u \pm v) = \sin u \cos v \pm \cos u \sin v )</td>
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<tr>
<td>( \cos(u \pm v) = \cos u \cos v \mp \sin u \sin v )</td>
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<tr>
<td>( \tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} )</td>
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<tr>
<td>( \sin 2u = 2 \sin u \cos u )</td>
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<tr>
<td>( \cos 2u = \cos^2 u - \sin^2 u )</td>
<td></td>
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<tr>
<td>( \tan 2u = \frac{2 \tan u}{1 - \tan^2 u} )</td>
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<tr>
<td>( \sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}} )</td>
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<tr>
<td>( \sin u + \sin v = 2 \sin \left( \frac{u + v}{2} \right) \cos \left( \frac{u - v}{2} \right) )</td>
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<tr>
<td>( \sin u - \sin v = 2 \cos \left( \frac{u + v}{2} \right) \sin \left( \frac{u - v}{2} \right) )</td>
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<td></td>
</tr>
<tr>
<td>( \cos u - \cos v = -2 \sin \left( \frac{u + v}{2} \right) \sin \left( \frac{u - v}{2} \right) )</td>
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</table>
1. \[ s = \frac{1}{2} (5 + 7.8 + 11) = 11.9 \]

\[ k = \sqrt{11.9(11.9 - 5)(11.9 - 7.8)(11.9 - 11)} \]

\[ k = \sqrt{302.99} = 17.41 \]

Area = 17.41

2. \[ \sin 68° = \frac{h}{7} \]

\[ 6.49 = h \]

\[ \therefore \text{only one solution} \]

\[ B = 40.47° \]

\[ \frac{\sin 68°}{10} = \frac{\sin B}{7} \]

\[ \sin B = \frac{7 \sin 68°}{10} \]

\[ B = 40.47° \]

\[ 68° + 40.47° + A = 180° \]

\[ A = 71.53° \]

\[ \frac{\sin 71.53°}{a} = \frac{\sin 68°}{10} \]

\[ a = \frac{10 \sin 71.53°}{\sin 68°} = 10.23 \]
3. A. \[ 4x^2 + y - 8x = 0 \]

\[ 4\left(x^2 - 2x + 1\right) = -y + 4 \]

\[ (x - 1)^2 = -\frac{1}{4}(y - 4) \]

Vertex: (1, 4) \[ p = -\frac{1}{16} \]

Directrix: \[ y = 4 + \frac{1}{16} \]

Focus: \[ \left(1, \frac{3}{16} + \frac{15}{16}\right) \]

Axis of symmetry: \[ x = 1 \]

B. \[ x^2 - 4y^2 - 6x - 8y - 11 = 0 \]

\[ \left(x^2 - 6x + 9\right) - 4\left(y^2 + 2y + 1\right) = 11 + 9 - 4 \]

\[ \frac{(x - 3)^2}{16} - \frac{(y + 1)^2}{4} = 1 \]

Center: (3, -1)

\[ a = 4, \ b = 2, \ c = \sqrt{20} \]

Foci: \[ (3 \pm \sqrt{20}, -1) \]

Vertices: (3 ± 4, -1)

Asymptotes: \( y + 1 = \pm \frac{1}{2}(x - 3) \)

4. A. \( A = 2, \ B = -4, \ C = 1 \)

\[ B^2 - 4AC = 16 - 4(2)(1) = 8 \quad \text{(and} \ 8 > 0) \]

\[ \tan 2\theta = \frac{-4}{2 - 1} \]

\[ 2\theta = \tan^{-1}(-4) \]

\[ 2\theta \approx -76 \]

\[ \theta \approx -38 \]

Type: Hyperbola
B. Use quadratic formula and graphing calculator.

\[ y^2 - (4x)y + (3 + 2x^2) = 0 \]

\[ y = \frac{4x \pm \sqrt{16x^2 - 4(1)(3 + 2x^2)}}{2} \]

\[ y = 2x \pm \sqrt{4x^2 - (3 + 2x^2)} = 2x \pm \sqrt{2x^2 - 3} \]

5. A.

Amplitude = N/A
Period = 360°
Phase Shift = 90° right

continued →
B. Amplitude = 3
Period = 180° or π
Phase Shift = π/2 right

6 A.

\[ y = -3 \cos \left( \frac{3\pi}{2} x \right) + 3 \]
or

\[ h = -3 \cos \left( \frac{3\pi}{2} t \right) + 3 \]

\[ t = 0 \]
\[ h = -3 \cos 0 + 3 = -3 + 3 = 0 \]

Period = \( \frac{1}{45} \) of a minute

\[ \frac{1}{45} \times 60 \text{ sec} = \frac{4}{3} \text{ period} \]

B. \( h = -3 \cos \left( \frac{3\pi}{2} \times 28 \right) + 3 \approx 0 \text{ feet} \)

7. A. \[
\frac{\sin 2A}{1 - \cos 2A} = \cot A
\]

\[ = \frac{2 \sin A \cos A}{1 - (1 - 2 \sin^2 A)} \]
\[ = \frac{2 \sin A \cos A}{2 \sin^2 A} \]
\[ = \frac{\cos A}{\sin A} \]
\[ = \cot A \]
B. \((1 + \cot^2 x)(1 - \cos 2x) = 2\)
\((\csc^2 x)(1 - (1 - 2\sin^2 x)) = 2\)
\((\csc^2 x)(2\sin^2 x) = 2\)
\(\left(\frac{1}{\sin^2 x}\right)\left(\frac{2\sin^2 x}{1}\right) = 2\)

8. A. \(\sin 2x \sec x + 2 \cos x = 0\)
\(2 \sin x \cos x \left(\frac{1}{\cos x}\right) + 2 \cos x = 0\)
\(2 \sin x + 2 \cos x = 0\)
\(\sin x + \cos x = 0\)
\(\sin x = -\cos x\)
\(x = \frac{3\pi}{4}, \frac{7\pi}{4}\)

B. \(\cos 2x = 5 \sin^2 x - \cos^2 x\)
\(2 \cos^2 x - 1 = 5(1 - \cos^2 x) - \cos^2 x\)
\(2 \cos^2 x - 1 = 5 - 5 \cos^2 x - \cos^2 x\)
\(8 \cos^2 x = 6\)
\(\cos^2 x = \frac{3}{4}\)
\(\cos x = \pm \frac{\sqrt{3}}{2}\)
\(x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\)

9. A. \(\cos 15^\circ = \cos \frac{30^\circ}{2} = \sqrt{\frac{1 + \cos 30^\circ}{2}} = \sqrt{\frac{1 + \sqrt{3}/2}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}} = \sqrt{\frac{2 + \sqrt{3}}{2}}\)

B. \(\sin 105^\circ = \sin (150^\circ - 45^\circ)\)
\(= (\sin 150^\circ \cos 45^\circ) - (\cos 150^\circ \sin 45^\circ)\)
\(= \frac{1}{2} \left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{2}}\right)\)
\(= \frac{1 + \sqrt{3}}{2\sqrt{2}}\)
10. A. \[ \cot \alpha = \frac{\text{adj}}{\text{opp}} = -\frac{4}{3} \]

B. \[ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \]
\[= \left( -\frac{3}{5} \right) \left( -\frac{3}{5} \right) - \left( \frac{4}{5} \right) \left( \frac{4}{5} \right) \]
\[= \frac{9}{25} - \frac{16}{25} = -\frac{7}{25} \]

C. \[ \cos 2\beta = 2\cos^2 \beta - 1 \]
\[= 2 \left( -\frac{3}{5} \right)^2 - 1 \]
\[= \frac{18}{25} - 1 = -\frac{7}{25} \]

D. \[ \sin \frac{\alpha}{2} \]
\[= \sqrt{\frac{1 - \cos \alpha}{2}} \]
\[= \sqrt{\frac{1 - \frac{4}{5}}{2}} \]
\[= \sqrt{\frac{1}{10}} \]

11. A. \((-6, 12, 0) - [(10, -2, 12) + (3, 2, -3)]\)
\[= (-6, 12, 0) - (13, 0, 9) \]
\[= (-19, 12, -9) \]

Magnitude \[= \sqrt{(-19)^2 + (12)^2 + (-9)^2} \approx 24.21 \]

B. \[ \mathbf{v} \cdot \mathbf{w} = -2 \cdot 5 + 4 \cdot -1 + 0 \cdot 6 \]
\[= -10 - 4 \]
\[= -14 \neq 0, \text{ so they are not perpendicular.} \]
12. \[ r^2 = 11^2 + 7^2 - 2(11)(7)\cos 102^\circ = 202.02 \]
\[ r = 14.21 \text{ miles} \]

13. A. \[ y = 28t \sin 35^\circ - 16t^2 + 5 \]
\[ x = 28t \cos 35^\circ \]

B. \[ y = 0 \]
\[ 16t^2 - 16.06t - 5 = 0 \]
\[ t \approx 1.25 \]
Use quadratic equation of trace.
\[ x = 28 \times 1.25 \cos 35^\circ \approx 28.67 \text{ feet} \]
Using Tstep of 0.01, approximation is close enough to exact solution.

C. \[ t \approx 0.5 \]
\[ y = 28(0.5) \sin 35^\circ - 16(0.5)^2 + 5 \]
\[ y = 9.03 \text{ feet} \]

It took 1 second to reach 5 feet again, so max is at \( \frac{1}{2} \) second.
14. A. \((x - 5, y + 1) = t(-2, 4)\)
   \[x - 5 = -2t\]
   \[x = -2t + 5\]
   \[y + 1 = 4t\]
   \[y = 4t - 1\]

B.

<table>
<thead>
<tr>
<th>(t)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>7</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>7</td>
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</table>
§111.35. Precalculus (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998, and at that time shall supersede §75.63(bb) of this title (relating to Mathematics). Students can be awarded one-half to one credit for successful

(b) Introduction.

(1) In Precalculus, students continue to build on the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent math

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications an

(c) Knowledge and skills.

(P.1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic

(A) describe parent functions symbolically and graphically, including \( f(x) = x^n \), \( f(x) = \frac{1}{n} x \), \( f(x) = \log_a x \), \( f(x) = \frac{1}{x} \), \( f(x) = e^x \), \( f(x) = |x| \), \( f(x) = ax \), \( f(x) = \sin x \), \( f(x) = \arcsin x \), etc.;

(B) determine the domain and range of functions using graphs, tables, and symbols;

(C) describe symmetry of graphs of even and odd functions;

(D) recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc.), points on the graph of a function, and the symbolic representation of a function; and

(E) investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically.

(P.2) The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. The student is expected to:

(A) apply basic transformations, including \( a \cdot f(x) \), \( f(x) + d \), \( f(x - c) \), \( f(b \cdot x) \), and compositions with absolute value functions, including \( |f(x)| \) and \( f(|x|) \), to the parent functions;

(B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; and

(C) investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.

(P.3) The student uses functions and their properties, tools and technology, to model and solve meaningful problems. The student is expected to:

(A) investigate properties of trigonometric and polynomial functions;

(B) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

(C) use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient);

(D) use properties of functions to analyze and solve problems and make predictions; and

(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.

(P.4) The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. The student is expected to:

(A) represent patterns using arithmetic and geometric sequences and series;

(B) use arithmetic, geometric, and other sequences and series to solve real-life problems;

(C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and

(D) apply sequences and series to solve problems including sums and binomial expansion.
(P.5) The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations. The student is expected to:

(A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets;
(B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound;
(C) convert between parametric and rectangular forms of functions and equations to graph them; and
(D) use parametric functions to simulate problems involving motion.

(P.6) The student uses vectors to model physical situations. The student is expected to:

(A) use the concept of vectors to model situations defined by magnitude and direction; and
(B) analyze and solve vector problems generated by real-life situations.

Source: The provisions of this §111.35 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1931.