Uncertainty & Risk — { Decision Making / Policy

<u>Uncertainty</u> — A perception concept that represents a state of mind with respect to the outcome(s) of a particular action.

<u>Risk</u> — has to do with the degree of uncertainty in a given situation

* Webster's Dictionary definition — Risk can be defined as the chance of loss.

How to measure Risk —

1) As a measure of dispersion of possible outcomes (Expected mean/variance)

2) Probability of returns falling bellow some disaster level

3) Willingness to pay in order not to take a chance

** <u>Bottom line</u> — Risk is a piece of information about a frequency distribution that, together with expected value, serves as an imperfect substitute for the density function in prescribing or explaining choice under uncertainty.

MODELING DECISION-MAKING UNDER UNCERTAINTY COULD BE DIFFERENTIATED REGARDING:

1) IF THE MODELING APPROACH ASSUMES THAT THE D-M ACTS IN ACCORDANCE WITH HIS/HER PERSONAL PROBABILITIES (VALUATION/PERCEPTION OF OUTCOMES)

2) IF THE MODELING APPROACH SPECIFIES/FOLLOWS/ESTABLISHES A D-M PROCESS

3) LEARNING PLAYS A ROLE IN THE MODELING APPROACH

* RELEVANT APPROACHES FOLLOWED TO ADDRESS RISK RELATED D-M ISSUES

- BAYESIAN APPROACH(ES)
- EXPECTED UTILITY
- SAFETY FIRST
- CAUTIOUS OPTIMIZING
- PURE BEHAVIOURAL

CONCEPTS NEEDED to BE DISCUSSED:

1) Utility - UTILITY FUNCTION(S) AND THEIR MEASUREMENT

2) EXPECTED UTILITY THEOREM

3) RISK AVERSION

4) RISK PREFERENCES OF D-M

5) RISK PREMIUM(S)

6) CERTAINTY EQUIVALENCE

7) MODIFICATIONS to STANDARD π MAX THEORY OF THE FIRM AND ASSUMPTIONS

8) APPROACHES COMMONLY USED to ADDRESS VARIED types OF RISK AND/OR UNCERTAINTY ISSUES IN D-M OR POLICY FORMULATION

* Utility — A utility function is a device for assessing numerical utility values to consequences or outcomes in such a way so that the D-M should act to maximize subjective expected utility if he/she is to be consistent with his/her expressed preferences

* Bernoulli's principle states that an <u>extra</u> dollar is worth <u>more</u> to a poor man than to a rich man

* Von Newman and Morgenstern formulated what now is widely known as the <u>Expected utility theorem</u> in the 1940's.

The following axioms (assumptions) are <u>necessary</u> to fulfill the expected utility theorem:

1) Ordering and transitivity - implies that a given D-M is smart enough to rank risky prospects:

If $a_1 > a_2$ and $a_2 > a_3 \Rightarrow a_1 > a_3$

2) CONTINUITY – IF $a > b$ AND $b > c$, THEN THERE EXISTS SOME SUBJECTIVE LEVEL OF PROBABILITY P SUCH THAT A D-M WOULD BE INDIFFERENT BETWEEN a OR c WITH PROBABILITIES P AND $(1-P)$, RESPECTIVELY.

3) INDEPENDENCE – IF $a > b$ AND $P(a) = P(b)$, THEN

$$a \text{ AND } c > b \text{ AND } c$$

$\Rightarrow$ a & b ARE INDEPENDENT OF c

4) DESIRE FOR HIGH PROBABILITIES OF SUCESS – GIVEN TWO LOTTERIES WITH THE SAME PRICE, A D-M WILL CHOOSE THE ONE WITH THE HIGHEST PROBABILITY OF WINNING

5) COMPOUND PROBABILITIES – A COMPOUND LOTTERY, WHOSE OUTCOMES ARE LOTTERIES THEMSELVES, IS EQUALLY ATTRACTIVE TO A D-M THAN A SIMPLE LOTTERY THAT WOULD RESULT FROM MULTIPLYING THROUGH THE PAYOFF-PROBABILITY FACTORS.

Sketch of Baumol's Proof of Expected Utility Theorem

Expected Utility Theorem

Given the axioms regarding ordering and transitivity, continuity, independence, desire for high probability of success, and compound lotteries, a decision maker's preferences can be represented by a utility function by which the utility of a risky prospect can be evaluated as the expected value of the utility function in terms of the subjective probability distribution of outcomes.

Proof

1. Consider 2 risky prospects $a_1=[P:A,B]$ and $a_2=[P':A',B']$ where for example $[P:A,B]$ means a prospect of a payoff A with probability P and payoff B with probability 1-P.
2. By the continuity axiom, there exists a risky prospect $[P_a:E,D]$ such that $A\sim[P_a:E,D]$ assuming that $E>A>D$ where preference is denoted by > and indifference is denoted by ~. Similarly, there exists a prospect such that $B\sim[P_b:E,D]$. Then

$$[P:A,B] \sim \{P:[P_a:E,D],\ [P_b:E,D]\} \qquad (1)$$

The right hand side of (1) represents a lottery that instead of paying off A or B will pay off with another lottery ticket, $[P_a:E,D]$ or $[P_b:E,D]$.

3. The expectation of the RHS is given by

$$EV = PP_aE + P(1-P_a)D + (1-P)P_bE + (1-P)(1-P_b)D$$

$$=\{PP_a + (1-P)P_b\}E + \{P(1-P_a) + (1-P)(1-P_b)\}D$$

$$=\{PP_a + (1-P)P_b\}E + \{P-PP_a + (1-P-P_b + PP_b)\}D$$

$$=\{PP_a + (1-P)P_b\}E + \{1-PP_a - (1-P)P_b\}D$$

Letting $r = PP_a + (1-P)P_b$ we have $EV = [r\ E + (1-r)D]$

4. Now generating a second prospect such that the decision maker is indifferent between it and a_2:

$$[P':A',B'] \sim \{P':[P_a':E,D],\ [P_b':E,D]\} \qquad (2)$$

Following the logic in (3), the expectation of the RHS is given by

$$EV' = [r'(E) + (1-r')D]$$

where $r' = P'\ P_a' + (1-P')P'_b$

Now since prospects (1) and (2) have identical payoffs E and D and E>D, prospect (1) will be preferred to prospect (2) if r>r' by assumption of desire for high probability of success.

5. Now introduce utility. The expected utility theorem states that the utility of a risky prospect is equal to the expectation of the utilities of its outcomes:

$$U[P:A,B] = P\ U(A) + (1-P)\ U(B) \qquad (3)$$

and from the equivalences stated above

$$U[P:A,B] = P\ U[P_a:E,D] + (1-P)\ U[P_b:E,D] \qquad (4)$$

Notice that by invoking (3), (4) becomes

$$U[P:A,B] = P\{P_a U(E) + (1-P_a)\ U(D)\} + (1-P)\{P_b\ U(E) + (1-P_b)U(D)\}$$

$$= \{PP_a + (1-P)P_b\}U(E) + \{P(1-P_a) + (1-P)(1-P_b)\}U(D)$$

$$= \{PP_a + (1-P)P_b\}U(E) + \{P-PP_a\} + (1-P-P_b + PP_b\}U(D)$$

$$= r\ U(E) + (1-r)\ U(D) \qquad (5)$$

Similarly, one can show that risky prospect [P':A',B'] has utility

$$U[P':A',B'] = r'\ U(E) + (1-r')\ U(D) \qquad (6)$$

7. By definition $U(E)>U(D)$. It follows that utility (5) will exceed utility (6) only if $r>r'$.

But we have already shown that a necessary and sufficient condition for prospect $[P:A,B]$ to be preferred to prospect $[P':A',B']$ is that $r>r'$.

Thus the more preferred prospect will always have the higher utility number and the prospect with the higher utility number will always be the preferred prospect.

QED

Key element: The utility of a risky prospect is its expected utility value.

Behavioral implication: A decision maker will choose the alternative which maximizes expected utility.

— IT IS IMPORTANT TO HIGHLIGHT THAT by THE TRANSFORMATION PROPERTY:

$$\text{If } E(U_{a_1}) > E(U_{a_2}) \Rightarrow a_1 > a_2$$

WHICH IMPLIES THAT UTILITY IS DEFINED UP TO A LINEAR TRANSFORMATION

* BASIC ASSUMPTION (CONSEQUENCE) → THE D-M'S UTILITY FUNCTION CAN BE USED TO REFLECT HIS/HER ATTITUDE REGARDING RISKY PROSPECT(S)

A <u>RISK AVERTER</u> IS A D-M WHO PREFERS THE EXPECTED VALUE OF A RISKY PROSPECT TO THE PROSPECT ITSELF:

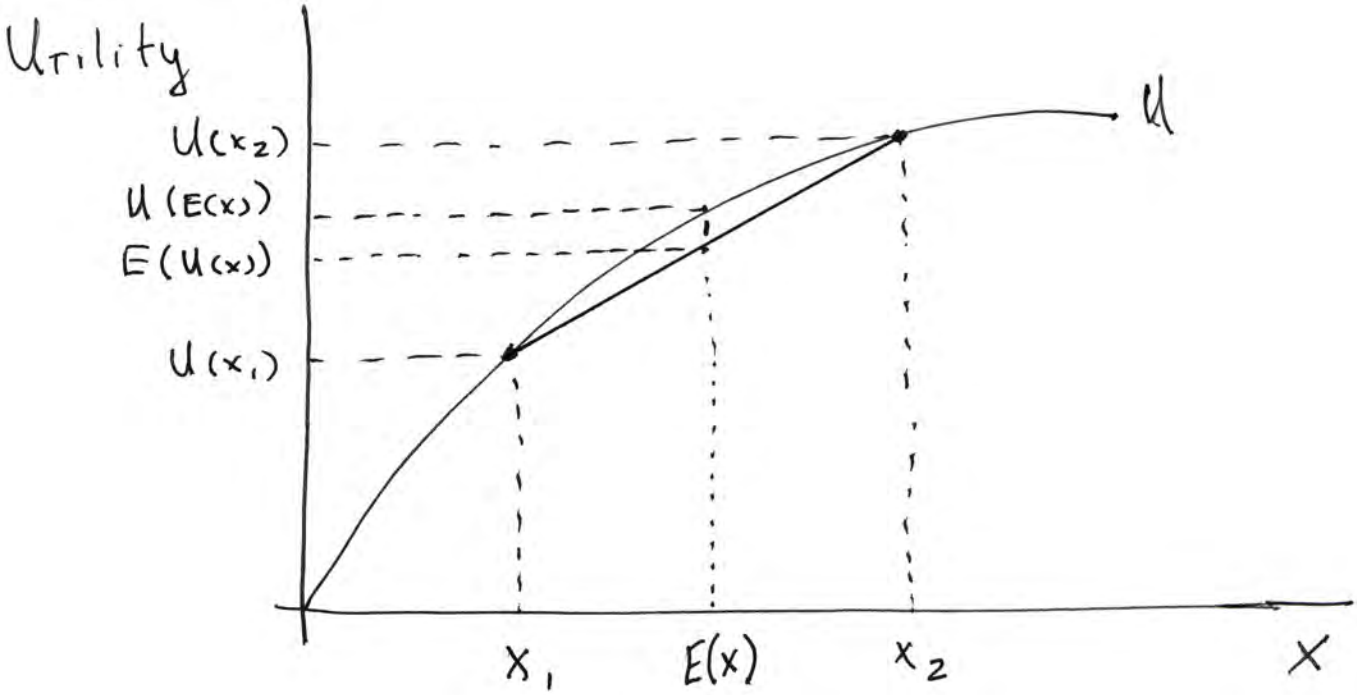
$$U\left[E(x)\right] > E\left[U(x)\right]$$

A <u>RISK SEEKER</u> (RISK LOVER) IS A D-M WHO PREFERS THE RISKY PROSPECT TO THE EXPECTED VALUE OF THE PROSPECT

$$E\left[U(x)\right] > U\left[E(x)\right]$$

A <u>RISK NEUTRAL</u> IS A D-M WHO WOULD BE INDIFFERENT BETWEEN THE RISKY PROSPECT AND THE EXPECTED VALUE OF THE PROSPECT

$$E\left[U(x)\right] \simeq U\left[E(x)\right]$$



So, if $U[E(x)] > E[U(x)] \Rightarrow$ Risk averter

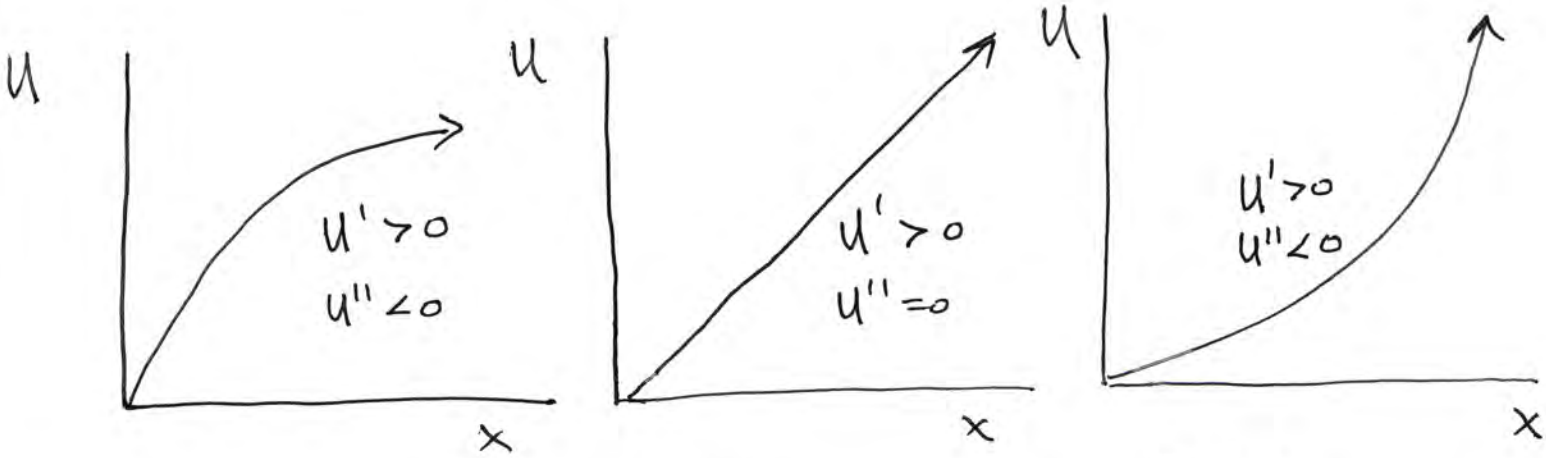
and a risk averter D-M has a <u>concave</u> utility function whose first derivative with respect to x, $\frac{\partial U}{\partial x} = U'$, is <u>positive</u> and the second derivative with respect to x, $\frac{\partial^2 U}{\partial^2 x} = U''$, is <u>negative</u>.

BOTTOM LINE $U' > 0$ (POSITIVE)

IF $U'' < 0 \Rightarrow$ RISK AVERTER

IF $U'' = 0 \Rightarrow$ RISK NEUTRAL

IF $U'' > \Rightarrow$ RISK LOVER



RISK AVERTER

RISK NEUTRAL

RISK LOVER

A GOOD MEASURE OF RISK AVERSION IS THE ARROW-PRATT LOCAL RISK AVERSION COEFFICIENT

$$r(x) = -\frac{U''}{U'}$$

IF $r(x) > 0$ RISK AVERTER

$r(x) = 0$ RISK NEUTRAL

$r(x) < 0$ RISK LOVER

GIVEN $r(x) = -\frac{u''}{u'}$, THE UTILITY FUNCTION COULD BE WRITTEN IN TERMS OF $r(x)$

$$r(x) = -\frac{u''}{u'}$$

$$r(x) = -\frac{d\left[\ln u'(x)\right]}{dx}$$

$$-r(x) = \frac{d\left[\ln u'(x)\right]}{dx} \quad (\int \text{ IN BOTH SIDES})$$

$$-\int r(x)\,dx = \ln u'(x) + c \quad (e \text{ IN BOTH SIDES})$$

$$e^{-\int r(x)\,dx} = u'(x) + e^{c} \quad (\int \text{ IN BOTH SIDES})$$

$$\int e^{-\int r(x)\,dx} = u(x) + c$$

$$\int e^{-\int r(x)\,dx} - c = u(x)$$

OR

$$\boxed{u(x) = \int e^{-\int r(x)\,dx} - c} \quad \text{QED.}$$

IN SOME CASES WE CAN / COULD / SHOULD USE A MEASURE OF "RELATIVE" RISK AVERSION

WHERE E(x) OR MEAN OF x COULD BE USED FOR x

$$\boxed{r^*(x) = x\,r(x) = -x\frac{u''}{u'}}$$

<u>CERTAINTY EQUIVALENCE</u> IS DEFINED AS THE AMOUNT OF MONEY, $\hat{x}$, SUCH THAT THE D-M IS INDIFFERENT BETWEEN RECIEVING $\hat{x}$ WITH CERTAINTY OR INCURRING THE RISKY PROSPECT, i.e.

$$U(\hat{x}) = E\left[U(x)\right]$$

FOR EXAMPLE, CONSIDER THE LOTTERY

$$X\left[0.5 : -100, 1{,}000\right]$$

IN THIS CASE THE EMV (EXPECTED MONETARY VALUE OR $E(x)$ IS

$$E(x) = p(-100) + (1-p)(1{,}000)$$

$$E(x) = 0.5(-100) + 0.5(1{,}000)$$

$$E(x) = -50 + 500$$

$$\boxed{E(x) = \$450}$$

IF: CE OR

$\hat{x} < E(x) \rightarrow$ RISK AVERSE

$\hat{x} > E(x) \rightarrow$ RISK LOVER

$\hat{x} = E(x) \rightarrow$ RISK NEUTRAL

LET'S SAY THAT CE OR $\hat{x} = 300$,

SO IN THIS CASE:

$$\hat{x} < E(x)$$

$(300 < 450)$ AND THIS WOULD INDICATE THAT THE DM IS RISK AVERSE.

<u>RISK PREMIUM</u> IS DEFINED AS THE AMOUNT OF MONEY A DM IS WILLING TO GIVE UP IN ORDER FOR HIM/HER NOT TO TAKE THE RISK OF TAKING THE RISKY PROSPECT.

IF $RP > 0 \rightarrow$ WOULD INDICATE RISK AVERSION

$RP = 0 \rightarrow$ WOULD INDICATE RISK NEUTRALITY

$RP < 0 \rightarrow$ WOULD INDICATE RISK PREFERENCE

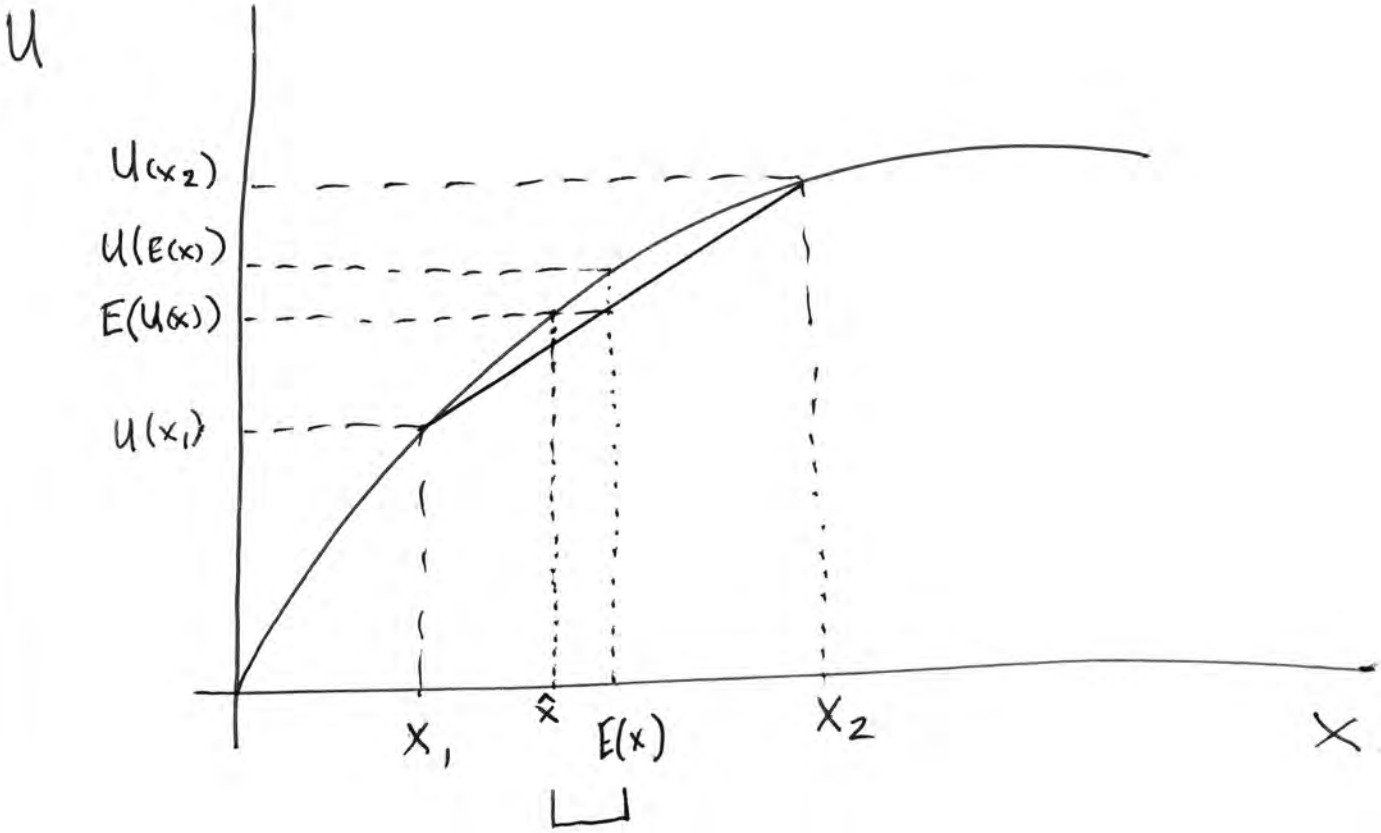
RP IS DEFINED AS: $RP = E(x) - \hat{x}$

IN THE EXAMPLE ABOVE:

$$RP = 450 - 300 = 150$$

i.e. THE D-M IS WILLING TO GIVE UP \$150 IN ORDER FOR HIM/HER NOT TO TAKE THE RISKY PROSPECT.

GRAPHICALLY – FOR A RISK AVERSE D-M



THE DIFFERENCE BETWEEN E(x) AND $\hat{x}$ IS DEFINED AS THE RISK PREMIUM

$$RP = E(x) - \hat{x}$$

AND IN THIS CASE RP is (+) OR > 0

ILLUSTRATION REGARDING THE CALCULATION OF THE RISK AVERSION COEFFICIENT, CERTAINTY EQUIVALENCE, AND RISK PREMIUM

CONSIDER $U(x) = -e^{-cx}$, WHERE $c > 0$

AND LET $c = 0.2$ AND THE RISKY PROSPECT BE $[0.3 : 20, 30]$.

* WE KNOW THAT THE CERTAINTY EQUIVALENCE MUST SATISFY: $U(\hat{x}) = E[U(x)]$

$$-e^{-0.2\hat{x}} = 0.3\left(-e^{-0.2(20)}\right) + 0.7\left(-e^{-0.2(30)}\right)$$

$$e^{-0.2\hat{x}} = 0.3\,e^{-4} + 0.7\,e^{-6}$$

TAKING logs IN BOTH SIDES GIVES

$$-0.2\hat{x} = \ln\left(0.3\,e^{-4} + 0.7\,e^{-6}\right)$$

$$-0.2\hat{x} = \ln(.0072)$$

$$-0.2\hat{x} = -4.934$$

$$\hat{x} = 24.66 \leftarrow \text{CERTAINTY EQUIVALENCE}$$

THE EXPECTED VALUE OF THE RISKY PROSPECT IS:

$$E(x) = 0.3(20) + 0.7(30) = 27$$

THEREFORE THE RISK PREMIUM IS:

$$RP = E(x) - \hat{x} = 27 - 24.66 = \$2.34$$

RISK AVERSION COEFFICIENT IS DEFINED AS: $r(x) = -\dfrac{u''}{u'}$

$\dfrac{\partial U(x)}{\partial x} = c\,e^{-cx}$ AND $\dfrac{\partial^2 U(x)}{\partial^2 x} = -c^2 e^{-cx}$ SO $r(x) = c = 0.2$

NOTICE IN THE ABOVE EXAMPLE THAT THE RISK AVERSION COEFFICIENT IS CONSTANT AND EQUAL TO "c" OR 0.2. IN SOME CASES THIS MIGHT NOT BE THE CASE. THAT IS, IF THE RISK AVERSION COEFFICIENT DECREASES AS WEALTH INCREASES, D-M ARE SAID TO DISPLAY DECREASING ABSOLUTE RISK AVERSION (DARA). THIS IMPLIES THAT THE RISK PREMIUM FOR A LOTTERY DECREASES AS THE WEALTH OF THE D-M INCREASES. SIMILARLY, IF THE RISK AVERSION COEFFICIENT DOES NOT CHANGE REGARDLESS OF THE LEVEL OF WEALTH OF THE D-M, THEN THIS D-M IS SAID TO DISPLAY CONSTANT ABSOLUTE RISK AVERSION (CARA). IMPLYING THAT RISK PREMIUMS WOULD NOT CHANGE AS THE LEVEL OF WEALTH CHANGES. FINALLY, AN INCREASING ABSOLUTE RISK AVERSION (IARA) WOULD INDICATE THAT THE RISK PREMIUM INCREASES FOR THE SAME LOTTERY WITH INCREASES IN WEALTH. GENERALLY SPEAKING, WE WOULD EXPECT THE RISK PREMIUM TO DECREASE AS THE LEVEL OF WEALTH OF THE D-M INCREASES. HINT .... BERNOULLI'S PRINCIPLE.

THE NEXT PAGE PRESENTS SOME SPECIFICS ASSOCIATED WITH VARIOUS TYPES OF UTILITY FUNCTIONS FUNCTIONAL FORMS AND THEIR CORRESPONDING RISK AVERSION COEFFICIENTS ALONG WITH RESTRICTIONS ON PARAMETERS AND RISK RANGES.

Table 1—Alternative utility functions and the coefficients of risk aversion

Type of risk aversion	Functional form	Restriction	Coefficient of risk aversion	Risk average range[1]
Increasing	$U(x) = a + bx + cx^2$	$a, b > 0$ $c < 0$	$\frac{-2c}{b + 2cx}$	$\frac{b}{2c} \geq x \geq 0$
Constant[2]	$U(x) = k - \Theta e^{-\lambda x}$	—	λ	all x
Decreasing	$U(x) = a + b \log(x + d)$	—	$\frac{1}{x + d}$	$x \geq -d$
Decreasing	$U(x) = ax - be^{-cx}$	$a, b, c > 0$	$\frac{bc^2e^{-cx}}{a + bce^{-cx}}$	all x
Decreasing	$\log U(x) = a + b \log x$	—	$\frac{1 - b}{x}$	$b < 1$
Decreasing	$U(x) = (x + b)^c$	$0 < c < 1$	$\frac{-(c - 1)}{x + b}$	$x \geq -b$
Decreasing	$U(x) = (x + b)^{-c}$	$c > 0$	$\frac{c + 1}{x + b}$	$x \geq -b$
Decreasing	$U(x) = x + c \log (x + b)$	$c > 0$	$\frac{c}{(x + b)(x + c + b)}$	$x > -b$
Decreasing	$U(x) = -e^{-ax} - be^{-cx}$	$a, b, c > 0$	$\frac{a^2e^{-ax} + bc^2e^{-cx}}{ae^{-ax} + bce^{-cx}}$	all x
Decreasing	$U(x) = -e^{-ax} + bx$	$a, b > 0$	$\frac{a^2e^{-ax}}{ae^{-ax} + b}$	all x
Decreasing	$u(x) = (\beta x^{-\rho} + \alpha)^{-\frac{1}{\rho}}$	$-1 < \rho < \infty$	$(1 + \rho)\left[\frac{1}{x} + \frac{\beta}{(\beta x^{-\rho} + \alpha) \cdot x^{(1 + \rho)}}\right]$	all x

[1] Wherever the value of x goes beyond the risk aversion range, the properties of the utility function in terms of risk aversion may be changed and the utility function probably needs to be reverified. [2] See (6) for an example of this type of utility function

ANOTHER EXAMPLE – QUADRATIC UTILITY FUNCTION

$$U(x) = a + bx + cx^2$$

$a > 0$
$b > 0$
$c < 0$

$a = 2 \quad b = 1.5 \quad c = -0.05$

$$U(x) = 2 + 1.5x - 0.05x^2$$

$[0.3 : 20, 30]$

$$U(\hat{x}) = E(U(x))$$

$$2 + 1.5\hat{x} - 0.05\hat{x}^2 = 0.3\left[2 + 1.5(20) - 0.05(20)^2\right] + 0.7\left[2 + 1.5(30) - 0.05(30)^2\right]$$

$$2 + 1.5\hat{x} - 0.05\hat{x}^2 = 0.3[2 + 30 - 20] + 0.7[2 + 45 - 45]$$

$$2 + 1.5\hat{x} - 0.05\hat{x}^2 = 3.60 + 1.40$$

$$2 + 1.5\hat{x} - 0.05\hat{x}^2 = 5$$

$$\boxed{-3 + 1.5\hat{x} - 0.05\hat{x}^2 = 0}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

WHERE $a = -0.05$ (QUADRATIC TERM)

$b = 1.5$ (LINEAR TERM

$c = -3$ (CONSTANT TERM)

$$* \quad \frac{-1.5 \pm \sqrt{(1.5)^2 - 4(-0.05)(-3)}}{2(-0.05)}$$

$$* \quad \frac{-1.5 \pm \sqrt{2.25 - 0.6}}{-0.10}$$

1.285

$$\frac{-1.5 \pm \sqrt{1.65}}{-0.10} \quad *$$

$$\ominus \rightarrow \frac{-1.5 - 1.285}{-.10} = 27.845$$

$$+ \rightarrow \frac{-1.5 + 1.285}{-.10} = 2.15$$

So $\hat{x}$ CAN be <u>EITHER</u> \$27.845 <u>OR</u> \$2.15

CERTAINTY EQUIVALENCE

?

Let's calculate $E(x)$ ....

$$E(x) = 0.3\,(20) + 0.7\,(30) = \underline{\underline{27}}$$

if $\hat{x} = 27.845$

$$RP = E(x) - \hat{x}$$

$$-0.845 = 27 - 27.845$$

$$\Rightarrow RP < 0 \Rightarrow \text{Risk lover}$$

if $\hat{x} = 2.15$

$$RP = E(x) - \hat{x}$$

$$24.85 = 27 - 2.15$$

$$\Rightarrow RP > 0 \Rightarrow \text{Risk averse}$$

..... What about the Risk Aversion coefficient .... it would help!

$$r(x) = \frac{-u''}{u'}$$

So given $u(x) = a + bx + cx^2$

$$\frac{\partial u(x)}{\partial x} = u' = b + 2cx$$

$$\frac{\partial^2 u(x)}{\partial x^2} = u'' = 2c$$

So $\boxed{r(x) = -\frac{2c}{b+2cx}}$

Values from the utility function $\left\{ a = 2, \; b = 1.5, \; c = -0.05 \right.$

$$r(x) = -\frac{2(-0.05)}{1.5 + 2(-0.05)x}$$

$$r(x) = -\frac{(-0.10)}{1.5 - 0.10x}$$

$$\boxed{r(x) = \frac{0.10}{1.5 - 0.10x}}$$

if $x = 15$ $r(x) = 0.10$

if $x > 15$ $r(x) < 0$

* if $x < 15$ $r(x) > 0$

THEREFORE, THE $\hat{X}$ (CERTAINTY EQUIVALENT) RELEVANT ROOT IS \$2.15 AND THE RELEVANT RP IS \$24.85

---- IN WORDS:

* A RISKAVERSE DECISION MAKER POSSESSING A UTILITY FUNCTION OF THE QUADRATIC TYPE GIVEN BY $U(x) = 2 + 1.5X - 0.05X^2$ WHO FACES A RISKY PROSPECT WHICH IS GIVEN BY $[0.3 : 20, 30]$ WOULD BE WILLING TO PAY OR GIVE UP \$24.85 IN ORDER NOT TO TAKE THE RISKY PROSPECT ---- i.e. He/SHE RATHER HAVE \$2.15 FOR SURE/CERTAIN AND WALK AWAY!

Average Risk Aversion and Normal Distributions

Where the utility function is characterized by constant absolute risk aversion and $g(y)$ is normally distributed with mean $E(y)$ and variance σ^2, the utility of the certainty equivalent income y_{CE} set equal to expected utility can be written as:

$$U(y_{CE}) = -\int (2\pi\sigma^2)^{\frac{-1}{2}} \exp\left\{\frac{-[y - E(y)]^2}{\sigma^2} - \lambda y\right\} dy \tag{3.14}$$

Freund, and Hildreth before him, solved for y_{CE} in the above expression and found:

$$y_{CE} = E(y) - \frac{\lambda\sigma^2}{2} \tag{3.15}$$

Rearranging and remembering that $E(y) - y_{CE}$ is the risk premium yields:

$$\pi = \frac{\lambda\sigma^2}{2} \tag{3.16}$$

which is Pratt's local approximation. Because λ is constant in this case and no moments above the second exist, the expression is an exact equality and no longer just a local measure. Another implication is that, for decision makers with constant absolute risk aversion, at least locally, variance can be traded for expected returns at the rate of $\lambda/2$ without affecting their well-being or their certainty equivalent income. This result will prove useful later on.

$\therefore$ FROM before $RP = E(x) - \hat{x}$

OR $\hat{x} = E(x) - RP$

So RP is equivalent to $\pi = \frac{\lambda}{2}\sigma^2$ Above

WHERE: λ IS THE RISK AVERSION COEFFICIENT $\left(-\frac{u''}{u'}\right)$

APPENDIX 3A

Pratt has derived an approximate relationship between $R(y)$, the risk premium, and the variability of the choice. Recall that there exists a risk premium π such that the utility of the certain income $U[y - \pi]$ is equally preferable to the expected utility of any lottery $EU(y + z)$ where z is a random variable with mean zero and variance σ^2. Thus, $E(y + z) = y$. So, we can write:

$$EU(y + z) = U(y - \pi) \tag{3A.1}$$

From Pratt we can solve for π by taking the Taylor series expansion about y of both sides of the expression above. This we write as:

$$U(y - \pi) = U(y) - \pi U'(y) + 0(\sigma^2) \tag{3A.2}$$ ✓

and

$$EU(y + z) = E\left[U(y) + zU'(y) + \frac{U''(y)}{2}z^2 + \cdots\right] \tag{3A.3}$$

The term $0(\sigma^2)$ indicates a term small enough to be ignored without significantly changing the results. Taking the expectations operator inside the bracket in Eq. (3A.3) produces the result:

$$EU(y + z) = U(y) + \frac{U''(y)}{2}\sigma^2 + \cdots \tag{3A.4}$$ ✓

Next we equate the two Taylor expansions and write:

$$U(y) - \pi U'(y) + 0(\sigma^2) = U(y) + \frac{U''(y)}{2}\sigma^2 + \cdots \tag{3A.5}$$

Canceling out $U(y)$ from both sides and ignoring all terms beyond the second term in both expressions (since they must be small) we obtain an approximate equality relationship which we write as:

$$\pi = \frac{-U''(y)\sigma^2}{2U'(y)} \tag{3A.6}$$

Recognizing the ratio $-U''(y)/U'(y)$ as $R(y)$, we write:

$$\pi = \frac{R(y)\sigma^2}{2} \tag{3A.7}$$

R(y) is the Risk Aversion coefficient

OR λ in the previous page.

* Typo on Pg. 41 Robison & Barry's Book!

ORGANIZING MODELS BY SOURCE OF RISK AND RISK RESPONSES

The two most important categories for classifying firms under uncertainty are (1) the sources of risk and (2) the firm's risk responses. These two categories often are closely related. In some cases a direct relationship exists between the source of risk and the possible risk responses; in other cases the relationship is less evident. Establishing and measuring these relationships are important objectives of risk analysis.

We begin this section by identifying potential sources of uncertainty. In reference to the expected utility model in Eq. (5.9), the firm's sources of risk include:

1. $p*$, an uncertain price for its final product
2. p, an uncertain price for its intermediate product
3. $\hat{p}$, an uncertain future price for its stored intermediate product
4. r, the storage cost per unit of q
5. $p_i \quad (i = 1, \ldots, n)$, the price of its inputs $x_1, \ldots, x_n$
6. $\hat{r}$, the price of borrowed funds used by the firm, and implicitly the return on unused borrowed funds or credit
7. The input-output relationship between q^* and q described by h
8. The input-output relationship between q and $x_1, \ldots, x_n$ described by f
9. The input-output relationship between $D + C$ and E described by the parameter ς
10. The availability of $x_1, \ldots, x_n$

Other sources of risk are attributed to the reliability and availability of information about the probability density functions for random events and about the utility function of the decision maker. We resolve risky decisions by maximizing single-valued utility functions over probability functions, both of which are assumed to be measured without errors. In reality, however, error-free measurements are rare. Thus the incorporation of new information is another dimension of firm-level models.

Standing in relationship to the sources of risk are the potential responses to risk. These include:

1. Adjusting input levels $x_1, \ldots, x_n$ and output q
2. Holding reserves $s_1 q$
3. Holding credit reserves C
4. Holding reserves of inputs $x_1, \ldots, x_n$
5. Integrating vertically to produce $q*$ *
6. Gathering information
7. Postponing decisions
8. Forward-contracting
9. Hedging
10. Diversifying enterprises, that is, integrating horizontally to produce products in addition to q, $\hat{q}$, and $q*$ *
11. Acquiring risk-reducing inputs
12. Investing in production processes with a flat average cost curve
13. Buying flexible inputs
14. Buying insurance
15. Specializing
16. Adjusting financial leverage
17. Diversifying operations spatially
18. Spreading transactions over time
19. Participating in public programs designed to reduce risk
20. Utilizing share leasing of resources

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