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On the Theory of the Competitive Firm Under Price Uncertainty

By Agnar Sandmo*

In recent years several contributions have been made to the theory of the firm under uncertainty, removing the assumption that the demand for the product is known with certainty at the time when the output decision is made. In most of these papers the assumption is made that the objective of the firm is to maximize expected profits.[1] This is hardly a very satisfactory assumption, since it completely rules out risk averse behavior, and so many elementary facts of economic life seem to indicate a prevalence of risk aversion.

The present paper is intended as a systematic study of the theory of the competitive firm under price uncertainty and risk aversion. We assume that the decision on the volume of output to be produced must be taken prior to the sales date, at which the market price becomes known. The firm's beliefs about the sales price can be summarized in a subjective probability distribution. However, since the firm is unable to influence this distribution, the basic assumption that the firm is a price taker is retained—in a probabilistic sense.[2]

It is perhaps most natural to interpret the model of the paper as being concerned with the short run. The firm makes its output decisions with sole regard for short-run profits and does not consider the relationship between this output policy and long-run policies for investment and finance. In a sense, it is a weakness of the model that it takes no account of this interrelatedness; but it may also be considered a strength, because a more complete model would make it necessary to draw up a much larger and more detailed list of assumptions about the economic environment of the firm than is needed for the present paper. The results presented here are thus compatible with several alternative sets of assumptions about investment opportunities, financial markets, and the structure of ownership. It is only essential to assume that short-run output decisions are dominated by a concern for short-run profits.

Occasionally, especially in Section III, we shall also find it convenient to use the model to analyze some long-run problems. It then becomes necessary to assume that these long-run elements have implicitly been accounted for. This is hardly satisfactory. Still, it is a useful simplification with long traditions in the theory of the firm.

We shall assume that the firm's attitude towards risk can be summarized by a von Neumann-Morgenstern utility function. This may be a strong assumption, because in many firms decisions are typically taken by a group of individuals, and group preferences may not always satisfy the

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[1] For some examples see the papers by Drèze and Gabszewicz, Kenneth Smith, Edward Zabel, and the book by Clement Tisdell.

[2] A similar approach is taken by Phoebus Dhrymes, Saul Hymans, John McCall, Bernt Stigum (1969a) and Hayne Leland (1969). Some interesting comments can also be found in Karl Borch (ch. 12, especially pp. 171–73).

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transitivity axiom required for the existence of a utility function. It is therefore possible that this approach implicitly assumes that the firm's reactions to changes in its environment are more predictable and stable than they really are. However, there are still many firms in which decisions are essentially made by one person, and there are presumably firms in which preferences are sufficiently similar within the group of decision makers to guarantee the existence of a group preference function. This provides justification for the approach taken in this paper.

I. Optimal Output under Uncertainty

We assume that the objective of the firm is to maximize the expected utility of profits. The utility function of the firm is a concave, continuous and differentiable function of profits, so that

$$U'(\pi) > 0, \quad U''(\pi) < 0 \tag{1}$$

Thus, the firm is assumed to be risk averse. It is well known that in order for a utility function to satisfy the von Neumann-Morgenstern axioms without giving rise to St. Petersburg phenomena, it must be bounded from above.[3] Strictly speaking, then, equation (1) holds only in the range below the upper bound of U.

The cost function of the firm is

$$F(x) = C(x) + B, \tag{2}$$

where x is output, $C(x)$ is the variable cost function, and B is "fixed cost." About the variable cost function we make the following general assumptions:

$$C(0) = 0, \quad C'(x) > 0 \tag{3}$$

The firm's profit function can now be defined as

$$\pi(x) = px - C(x) - B, \tag{4}$$

where p is the price of output, assumed to be a (subjectively) random variable with density function $f(p)$ and expected value $E[p]=\mu$. Naturally, p is restricted to be nonnegative. This means that, once x has been chosen, the firm's maximum loss is $(-C(x)-B)$. Clearly also, $\pi(0)=-B$.

The expected utility of profits can be written as

$$E[U(px - C(x) - B)],$$

where E is the expectations operator. Differentiating with respect to x, we obtain as necessary and sufficient conditions for a maximum:

$$E[U'(\pi)(p - C'(x))] = 0, \tag{5}$$

$$D = E[U''(\pi)(p - C'(x))^2 - U'(\pi)C''(x)] < 0 \tag{6}$$

It is interesting to note that in order for the second-order condition (6) to hold, it is not necessary to assume increasing marginal cost.

For the remainder of Section I and in Section II, we assume that (5) and (6) determine a non-zero, finite and unique solution to the maximization problem. The problems of existence and of corner solutions will be discussed in Section III.

One question which is naturally raised by the introduction of price uncertainty is this: how does the optimal output compare with the well-known competitive solution under certainty? Under certainty, the solution is characterized by equality between price and marginal cost. There is no obvious way of making such a comparison, but one possible and appealing specification of the problem is this: what is the optimal output under uncertainty as compared with the situation where the price is known to be equal to the expected value of the original distribution? Referring to the latter level of output as the certainty output, we shall now show that *under price uncertainty, output is smaller than the*

[3] See on this point Kenneth Arrow, who also argues that U must be bounded from below.

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certainty output. This is a generalization of a theorem of McCall, who proves a similar result for the case of a utility function with constant absolute risk aversion.

The first-order condition (5) can be written as

$$E[U'(\pi)p] = E[U'(\pi)C'(x)] \tag{7}$$

Subtract $E[U'(\pi)\mu]$ on each side of this equation. We then get

$$E[U'(\pi)(p-\mu)] = E[U'(\pi)(C'(x)-\mu)] \tag{8}$$

Since $E[\pi]=\mu x - C(x) - B$ (from the definition of profits), we have that $\pi = E[\pi] + (p-\mu)x$. Clearly

$$U'(\pi) \leqq U'(E[\pi]) \quad \text{if } p \geqq \mu \tag{9}$$

It follows immediately that

$$U'(\pi)(p-\mu) \leqq U'(E[\pi])(p-\mu) \tag{10}$$

This inequality holds for all p. For if $p \leqq \mu$, the inequality sign in (9) is reversed, but then multiplication by $(p-\mu)$ will still make $\leqq$ hold in (10). Taking expectations on both sides of (10) and noting that $U'(E[\pi])$ is a given number, we obtain

$$E[U'(\pi)(p-\mu)] \leqq U'(E[\pi])E[p-\mu]$$

But, here the right-hand side is equal to zero by definition, and so the left-hand side is negative. Then we know that the right-hand side of (8) is negative also. But this can be written as

$$E[U'(\pi)](C'(x)-\mu) \leqq 0,$$

and, since marginal utility is always positive, this implies

$$C'(x) \leqq \mu \tag{11}$$

That is, optimal output is characterized by marginal cost being less than the expected price. Now under certainty the only types of cost curves compatible with competitive assumptions are those for which the marginal cost curve is either everywhere increasing or else U-shaped. In those cases, (11) proves our statement above. Equation (11) is, of course, also valid for constant or decreasing marginal cost, but then the competitive output is not well defined.

This result is not the only conceivable answer to the question of the effect of uncertainty on the output decision. Following Jacques Drèze and Franco Modigliani, we may describe our result as concerned with the *overall* impact of uncertainty. However, one may also be interested in the question of the *marginal* impact; i.e., the effect of making a given distribution "slightly more risky." It is not obvious how this can be formalized; in the following we shall adopt a procedure used in Sandmo.

Let us define a small increase in risk as a "stretching" of the probability distribution around a constant mean. This requires the introduction of two shift parameters, one multiplicative and one additive. Thus, let us write price as

$$\gamma p + \theta,$$

where γ is the multiplicative shift parameter and θ is the additive one. An increase of γ alone (from the point $\gamma=1$, $\theta=0$) will "blow up" all values of p; it will therefore increase the mean as well as the variance. To restore the mean we have to reduce θ simultaneously, so that

$$dE[\gamma p + \theta] = 0, \quad \text{or} \quad \mu d\gamma + d\theta = 0, \quad \text{i.e.,}$$

$$\frac{d\theta}{d\gamma} = -\mu \tag{12}$$

We can now write the profit function as $\pi(x) = (\gamma p + \theta)x - C(x) - B$ and differentiate with respect to γ, taking account of (12). The result is then

$$\frac{\partial x}{\partial \gamma} = -x \cdot \frac{1}{D} E[U''(\pi)(p-\mu)(p-C'(x))] - \frac{1}{D} E[U'(\pi)(p-\mu)] \tag{13}$$

Of these two terms, the last one is clearly

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negative (from the proof above and from the second-order condition). However, the sign of the first term is in general indeterminate, so that at the present level of generality it does not seem possible to make a precise statement about the marginal impact of uncertainty.

There is one special case in which we would expect the marginal impact of uncertainty to become identical to the overall impact. That is in the case where we start from the certainty of $p=\mu$ and replace this certain price by a probability distribution with all outcomes concentrated in the neighborhood of μ. This is not too easily handled, since our stretching procedure breaks down in that case. However, we can get around this difficulty by noting that, when price is known to be equal to μ, we must have $C'(x)=\mu$. Then the first term in (13) becomes

$$-x \cdot \frac{1}{D} E[U''(\pi)(p-\mu)^2],$$

which is certainly negative. Thus, both terms in (13) are negative, and their signs depend only on the assumption of risk aversion. The connection with the overall impact of uncertainty is thereby established.

II. The Comparative Statics of the Firm

Simply assuming the existence of risk aversion is a very weak restriction on the firm's attitudes to risk. Further restrictions on the utility function may be introduced by means of the Arrow-Pratt risk aversion functions:

$$\text{Absolute risk aversion: } R_A(\pi) = -\frac{U''(\pi)}{U'(\pi)}$$

$$\text{Relative risk aversion: } R_R(\pi) = -\frac{U''(\pi)\pi}{U'(\pi)}$$

It seems reasonable to assume that $R_A(\pi)$ is a decreasing function of π. This would reflect the hypothesis that as a decision maker becomes wealthier (in terms of income, profit etc.), his risk premium for any risky prospect, defined as the difference between the mathematical expectation of the return from the prospect and its certainty equivalent, should decrease, or at least not increase. If $R_R(\pi)$ is increasing, this means that the elasticity of the risk premium with respect to π is less than one in absolute value. Arrow argues that there are good theoretical and empirical reasons for making this assumption, but the evidence for it does not seem conclusive, and we shall not commit ourselves to a specific hypothesis as to the form of $R_R(\pi)$.[4]

One of the basic results in the theory of the firm under certainty is that fixed costs do not matter in the sense that once a strictly positive output level has been chosen, this output is unaffected by an infinitesimal increase in fixed costs. This is not so under uncertainty. Differentiating in (5) with respect to B, we obtain

$$\frac{\partial x}{\partial B} = \frac{1}{D} E[U''(\pi)(p - C'(x))] \tag{14}$$

Decreasing absolute risk aversion is a necessary and sufficient condition for $\partial x/\partial B$ to be negative. The proof of this is as follows: Let $\bar{\pi}$ be the level of profits when $p=C'(x)$. Then, since $R_A(\pi)$ is decreasing[5]

[4] Some remarks on the empirical evidence can be found in the article by Joseph Stiglitz. For derivations of the risk aversion functions the reader is referred to the contributions of Arrow and John Pratt. Hypotheses about the risk aversion functions have been applied to portfolio theory by Arrow, to insurance purchasing and to taxation and risk-taking by Jan Mossin (1968a, b), and to the analysis of saving decisions by Sandmo. Several other examples of application could easily be given.

[5] This must be interpreted with care. We are interested in the properties of the risk aversion function at the optimum position, i.e., for the output level $x=x^*$ which is the solution to (5). For this given output level, (15) is certainly true. It is important to note that this *local* relationship is independent of the *global* lack of any one-to-one relationship between the algebraic signs of profits and marginal revenue.

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$$(15)\quad R_A(\pi) \leqq R_A(\bar{\pi}) \quad \text{for } p - C'(x) \geqq 0$$

Substituting from the definition of $R_A(\pi)$, we obtain

$$(16)\quad -\frac{U''(\pi)}{U'(\pi)} \leqq R_A(\bar{\pi}) \quad \text{for } p - C'(x) \geqq 0$$

(Note that $R_A(\bar{\pi})$ is a given number and not a random variable.) We know of course that

$$(17)\quad -U'(\pi)(p - C'(x)) \leqq 0 \text{ for } p - C'(x) \geqq 0,$$

since marginal utility is positive. Now multiply (16) by the left-hand side of (17). We then get

$$U''(\pi)(p - C'(x)) \geqq -R_A(\bar{\pi})U'(\pi)(p - C'(x))$$

This holds for all p. For if $p \leqq C'(x)$, the inequality in (16) is reversed, but so is that in (17). Now taking expected values we obtain

$$E[U''(\pi)(p - C'(x))] \geqq -R_A(\bar{\pi})E[U'(\pi)(p - C'(x))]$$

But by the first-order condition (5), the right-hand side is equal to zero, and the left-hand side is accordingly positive. But then the derivative (14) is negative and our proposition is proved.

Is this conclusion in itself intuitively plausible? This question may perhaps best be judged by considering whether a lump sum tax or a lump sum subsidy would be the most appropriate policy measure for making the firm increase its output. Economic intuition seems strongly to suggest the latter alternative, which is exactly what our result implies.

We turn now to an examination of the firm's supply function. Since the price is seen by the firm as a random variable, it does not make sense to speak about the effect of an "increase in price." It seems natural, however, to discuss the closely related problem of an increase in the mathematical expectation of the price with higher central moments constant. We can do this in the following way: Let us write price as $p+\theta$, where θ is again an additive shift parameter. Increasing θ is equivalent to moving the probability distribution to the right without changing its shape. Differentiating (5) with respect to θ and evaluating the derivative at $\theta=0$ we obtain

$$\frac{\partial x}{\partial \theta} = -x \cdot \frac{1}{D} E[U''(p - C'(x))] - \frac{1}{D} E[U'(\pi)],$$

or, substituting from (14),

$$(18)\quad \frac{\partial x}{\partial \theta} = -x\frac{\partial x}{\partial B} - \frac{1}{D} E[U'(\pi)]$$

This expression is similar to the Slutsky equation familiar from demand analysis. It says that the firm's response to an increase in expected price can be decomposed into two separate effects, one of which is analogous to a decrease in fixed costs, and the other one is a pure substitution effect. Of the latter effect we can immediately say that it is positive. As for the sign of the former effect we can draw on our previous result to conclude that *decreasing absolute risk aversion is a sufficient condition for $\partial x/\partial \theta$ to be positive*, i.e., for an upward-sloping supply curve. Again the implication of decreasing absolute risk aversion seems intuitively plausible. It implies, e.g., that in order to increase output the government should consider a per unit subsidy, rather than a per unit tax, as the appropriate policy measure.[6]

Another well-established result in the theory of the firm is that a change in a proportional rate of profit taxation will have no effect on the level of output. A priori there is no reason to expect this result to hold under uncertainty.

[6] The interested reader who wishes to see an example where the possibility of a downward-sloping supply curve does occur may consider the simple case of a quadratic utility function and constant marginal cost, where the supply curve bends backward for expected price sufficiently high.

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With price uncertainty the question of loss offset provisions becomes important. If there is no loss offset, the profit function of the firm becomes

$$\pi(x) = \begin{cases} px - C(x) - B & \text{for } p \leqq \dfrac{C(x) + B}{x} \\ (px - C(x) - B)(1 - t) & \text{for } p > \dfrac{C(x) + B}{x} \end{cases}$$

On the other hand, if there is full loss offset, the profit function can be written as

$$\pi(x) = (px - C(x) - B)(1 - t) \quad \text{for all } p$$

It is not easy to decide which of these two assumptions is the more interesting and realistic one. Full loss offset presupposes that the firm or its owner(s) has other income from which any loss can be deducted. In fact, tax laws in many countries do provide for loss offset, either against other income or against future profits, so that there may be reasons for concentrating attention on this case.[7]

With full loss offset expected utility is

$$E[U((px - C(x) - B)(1 - t))],$$

and the first-order condition becomes

$$E[U'(\pi)(p - C'(x))] = 0, \tag{19}$$

as before, since the multiplicative factor $(1-t)$ can be factored out.

Differentiating in (19) with respect to t yields

$$\frac{\partial x}{\partial t} = \frac{1}{1 - t} \cdot \frac{1}{D} \cdot E[U''(\pi)\pi(p - C'(x))] \tag{20}$$

It can be shown that *increasing the tax rate will increase, leave constant or reduce output according as relative risk aversion is increasing, constant, or decreasing.*

If $R_R(\pi)$ is increasing, we must have that

$$-\frac{U''(\pi)\pi}{U'(\pi)} \geqq R_R(\bar{\pi}) \quad \text{for } p - C'(x) \geqq 0 \tag{21}$$

Multiplying this by $-U'(\pi)(p - C'(x))$ yields

$$U''(\pi)\pi(p - C'(x)) \leqq -R_R(\bar{\pi})U'(\pi)(p - C'(x)),$$

and by the argument used in the proof above, this inequality holds for all p. Taking expectations, the right-hand side vanishes, and we have that

$$E[U''(\pi)\pi(p - C'(x))] \leqq 0$$

From this it follows that $\partial x/\partial t$ is positive in the case of increasing relative risk aversion. The proof of the rest of the statement follows immediately.

III. Profits, Entry, and Returns to Scale

It is well known that under certainty increasing marginal cost is necessary for the existence of a competitive optimum for the firm. This is not so under uncertainty, as we shall now demonstrate.[8]

Consider first the case where marginal cost is constant. Then concavity and boundedness of U as a function of π is sufficient to show that there exists a finite $x = x^*$ which gives a maximum of U. The case $C''(x) > 0$ is equally simple, because increasing marginal cost only reinforces the concavity of U as a function of x. It follows also that the case of a U-shaped marginal cost curve is only slightly more complicated for then U will be concave in x in the region for which $C'(x) \geqq \min C'(x)$.

Note also that in the case of decreasing MC followed by constant MC the above

[7] This argument is not entirely satisfactory, however. If "other income" of "future profits" are at least partially determined by the firm's own actions, they should presumably be integrated into the model.

[8] For a rigorous discussion of the existence of optimal policies under uncertainty the reader is referred to Leland (1970).

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argument remains valid; there will be a determinate optimal level of output for the firm. The troublesome case is where MC is everywhere decreasing and boundedness of the utility function no longer guarantees the existence of an optimal policy. However, it remains true that decreasing MC is not a sufficient condition for the nonexistence of an optimal output level; thus a market *may* be competitive even under this assumption.

So far, we have assumed the existence of an interior maximum for the firm; i.e., we have assumed that the optimal level of output is strictly positive. But we know from received theory that even if the condition "price=marginal cost" determines a local maximum of profits, the maximum need not, even if it is a unique interior maximum, give us the global maximum. The reason is simply that the interior maximum may result in negative profits, so that the best policy is to produce nothing at all. In other words, production will take place at a positive level if, and only if, the best positive production level results in nonnegative profit.

Let x^* be the output level which is the solution to (5) and satisfies (6). Then x^* will also give a global utility maximum, provided that

$$E[U(px^* - C(x^*) - B)] \geqq U(-B) \tag{22}$$

It will be recalled that $-B$ is the level of profit for $x=0$.[9]

Developing the left-hand side of (22) in a Taylor series around the point $p=\mu$ we obtain, neglecting higher-order terms,

$$E[U(\mu x^* - C(x^*) - B) + U'(\mu x^* - C(x^*) - B)x^*(p-\mu) + \tfrac{1}{2}U''(\mu x^* - C(x^*) - B)x^{*2}(p-\mu)^2] \geqq U(-B)$$

The second term on the left-hand side is zero by definition. Rearranging the remaining terms and dividing through by $U'(\mu x^* - C(x^*) - B)$ so as to make the expressions invariant under linear transformations of the utility function, we then get

$$\frac{U(\mu x^* - C(x^*) - B) - U(-B)}{U'(\mu x^* - C(x^*) - B)} \geqq -\frac{1}{2}\frac{U''(\mu x^* - C(x^*) - B)}{U'(\mu x^* - C(x^*) - B)} x^{*2}E[p-\mu]^2 \tag{23}$$

Both sides of this inequality have the dimension of money. The factors on the right-hand side are the risk aversion function, evaluated at the expected level of profit for $x=x^*$, and the variance of sales, $x^{*2}E[p-\mu]^2$. Since both these factors are positive, the left-hand side must also be positive, and with a strictly increasing utility function this implies that

$$\mu x^* - C(x^*) - B > -B,$$

or

$$\mu > \frac{C(x^*)}{x^*}, \tag{24}$$

i.e., at the optimum *expected price must be larger than average cost, so that the firm requires positive expected profit in order to choose a positive output level.* It should be stressed that "positive" here means "strictly positive." If expected profit for $x=x^*$ were zero, (23) would not be satisfied, and the output level of zero would be chosen. We conclude, therefore, that competitive equilibrium under price uncertainty and risk aversion requires the existence of positive profits.[10]

It is interesting to study the role of risk aversion in the long-run equilibrium posi-

[9] The argument here could equally well be carried out under the "long-run" assumption that $B=0$.

[10] As in any partial equilibrium analysis this statement is somewhat incomplete. Implicit in it is the assumption that by not producing anything the owners of firms can make a sure return by employing their resources elsewhere in the economy. If this return is strictly positive, "normal profits" should be included among the firms' costs.

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tion.[11] We assume therefore, to make the discussion simpler, that firms have identical cost functions and identical probability beliefs. Looking at (23) it is easy to see that a ("almost") risk-neutral firm will require only a nonnegative profit to enter the industry; in other words, as long as any positive level of expected profit remains, risk-neutral firms will enter. It is also clear from (23) that firms with "very high" risk aversion will not enter the industry at all, or they will be marginal firms in the sense that a very small decrease in expected price will make them leave the market. The risk neutral firms will of course set marginal cost equal to expected price (assuming U-shaped cost curves), while the risk-averse firms in the industry will choose output levels for which marginal cost is less than expected price. In general, the distribution of output and expected profit among firms will vary with their degree of risk aversion. Expected profit will be highest for those firms which come very close to being risk neutral and have the highest output in the industry. This observation confirms a view which has long traditions in economic theory, viz. to regard profit as a reward to risk-bearing.

Let us now turn to the case where marginal cost is constant or decreasing. We have shown that this case is not inconsistent with competitive assumptions. However, if one or a few firms are much less risk averse than the others, they may choose very high output levels and thereby lower expected price so much that the others will leave the industry. An uneven distribution of risk aversion may therefore be a source of oligopolistic concentration in its own right.

IV. Concluding Remarks

There are many ways in which this analysis can be extended and generalized. We have had nothing to say on the subject of the multiproduct firm, which is of particular interest under uncertainty, since the firm is able to spread its risks by output diversification.[12] Neither have we had anything to say about the role of inventories under demand uncertainty. Finally, investment and financing decisions can hardly be given adequate treatment in the present framework.

It would also be interesting to place the competitive firm facing price uncertainty in a general equilibrium framework. This would require a different type of analysis from that of Debreu, in which there exists a complete set of markets for contingent commodities and the firm bears no risk at all. An alternative approach is contained in a recent paper by Stigum (1969b), in which firms do bear risks and entrepreneurs display risk averse behavior. Evidently, alternative models can be constructed with different assumptions about ownership and market opportunities: the theory of the firm developed in the present paper presumably will fit into some, but not all, of these models.

[11] For the following discussion, which is essentially long-run, it is appropriate to assume $B=0$; in the long run all costs are variable costs.

[12] This problem has been studied by Dhrymes for the special case of a quadratic utility function.

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