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Calculus and Math Reference Sheet

I. Some rules of differentiation

1. $\frac{d[f(x)+g(x)]}{dx} = f'(x)+g'(x)$ ADDITION RULE

2. $\frac{d[f(x)\cdot g(x)]}{dx} = f(x)\cdot g'(x)+f'(x)\cdot g(x)$ PRODUCT RULE

3. $\frac{d[g\{f(x)\}]}{dx} = g'[f(x)]\cdot f'(x)$, CHAIN RULE

4. $\frac{d\left(\frac{1}{g(x)}\right)}{dx} = \frac{-g'(x)}{[g(x)]^2}$, RECIPROCAL RULE or ff. CHAIN RULE

$$\frac{d[1/g(x)]}{dx} = \frac{d[g(x)]^{-1}}{dx} = -g(x)^{-2}\ g'(x) = \frac{-g'(x)}{[g(x)]^2}$$

5. $\frac{d\left[x^n\right]}{dx} = nx^{(n-1)}$ EXPONENT RULE

6. $\frac{d[g(x)/q(x)]}{dx} = \frac{g'(x)\cdot q(x)-g(x)q'(x)}{[q(x)]^2}$ QUOTIENT RULE

Or using "product" and "chain" rules to get same result.

above $= \frac{d[g(x)\cdot\{q(x)\}^{-1}]}{dx}$

$$= \frac{g(x)\cdot(-q'(x))}{\{q(x)\}^2} + \frac{g'(x)}{q(x)} = \frac{-g(x)q'(x)+g'(x)q(x)}{\{q(x)\}^2}$$

7. $\frac{d\left[b^x\right]}{dx} = b^x(\ln b)$, where $b>0$ and "ln" denotes natural logarithm POWER RULE

8. $\frac{d\left[e^x\right]}{dx} = e^x$ EXPONENTIAL RULE

9. $\frac{d[\ln x]}{dx} = \frac{1}{x}$, for $x>0$ NATURAL LOG, RULE

10. $\frac{d\left[b^{f(x)}\right]}{dx} = b^{f(x)} \cdot (\ln b) \cdot f'(x)$ USING RULES 3 AND 7

11. $\frac{d\left[e^{f(x)}\right]}{dx} = e^{f(x)} \frac{df(x)}{dx}$ USING RULES 3 AND 8

12. $\frac{d\left[\log_b x\right]}{dx} = \frac{1}{x \ln b}$, where $b>0$ and $\log_b$ denotes logarithm of base b.

Note symmetry of power functions' and logarithmic functions' derivatives---

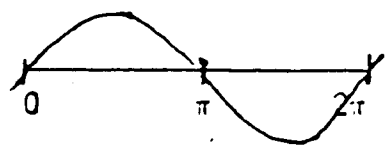
$\frac{d\left[e^x\right]}{dx} = e^x$ $\frac{d\left[b^x\right]}{dx} = b^x(\ln b)$

$\frac{d\left[\ln x\right]}{dx} = \frac{1}{x}$ $\frac{d\left[\log_b x\right]}{dx} = \frac{1}{x(\ln b)}$

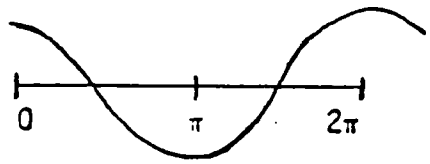
13. $\frac{d[\cos x]}{dx} = -\sin x$, $\frac{d[\sin x]}{dx} = \cos x$

can you discern these from slopes of functions?

sin



cos



II. Integration rules

Rules of integration are based on "Fundamental Theorem of Calculus", i.e.

If $G'(u) = g(u)$, Then

$\int_a^b g(u)\, du = G(b) - G(a) \equiv$ definite integral

$G(u)$ is called the anti-derivative or "indefinite" integral of $g(u)$; also can be written as $\int g(u)\, du$, i.e. the definite integral without limits.

III. Plus a little algebra (quadratic formula)

If $(ax^2 + bx + c = 0)$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

IV. Total Differential

Given the function: $w = f(x, y, z)$,

then the total differential, dw, of the function is:

$$dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial z} dz$$

where: dx, dy and dz are small changes in x, y and z.