MARKETING ON COTTON SPINNING QUALITIES

1989 Final Report

BUREAU OF BUSINESS RESEARCH
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Dr. Jerry Olson received his Ph.D. in Economics from the University of North Carolina at Chapel Hill in 1977 and began his career at the Research Triangle Institute. In 1979 he was recruited by the Bureau of Business Research, where he developed databases and statistical procedures for studying the Texas economy and conducted analyses of the state’s economy. Dr. Olson’s research has been published in numerous research reports and in two refereed journals, the *Journal of Econometrics* and *Technological Forecasting and Social Change*. In 1986 Dr. Olson left the Bureau to form Johnson, Olson, & Associates, a private consulting firm. His continued interest in cotton research resulted in the development of the model reported in this document.
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Introduction

The cost minimization model described in this paper demonstrates the feasibility of lowering materials cost by blending cotton of different properties. Designed for an establishment producing yarn from raw cotton, the model is a straightforward application of the economic theory of the firm. We used the theory as a means to develop a quantitative method for selecting and buying cotton fibers and to determine the value of different fiber properties in the production environment.

Despite some shortcomings, the model performed well, producing two useful results. We were able to establish a set of spinning values by observing changes in cost prompted by changes in fiber properties. The model also made it possible to estimate the value of fiber fineness and maturity tests performed on a Shirley Fineness and Maturity Tester (FMT); such information can further lower the cost of yarn.

The Model

We begin with a cost function for producing a specific kind of yarn from various grades of raw cotton available at specific prices, using specific production methods. Then the cost function is minimized by adjusting the portion of raw cotton used from each grade. Two minimizations have been performed. In the first, the producer is assumed to have measured the properties of the cotton on High Volume Instruments (HVI). In the second, we assume the producer also knows the fineness and maturity of the cotton as measured by the Shirley Fineness and Maturity Tester.

Total Cost Function

The cost of production is specified as:

\[ C = \sum_{i=1}^{n} w_i x_i + F \]  

where
i refers to the various grades of cotton that may be used,  
n=the number of grades of cotton that may be used,  
x_i=the number of pounds of cotton from grade i used in production,  
w_i=the price of a pound of cotton of grade i,  
F=costs that are not dependent on x_i, "fixed" costs.

Fixed Costs

In a yarn mill, fixed costs typically account for 33 to 40 percent of the overall costs of production. These costs vary primarily by spinning method, but they will also be affected by other factors, such as financial conditions and the price of land. For the purpose of this paper, we are not especially interested in fixed costs. We have assumed that the establishment is producing ring-spun 22s yarn, and all costs for labor, plant, and equipment are independent of the mix of raw cotton used as input.

Variable Costs

Variable costs depend on differing quantities used of the factors of production. In this prototype, the cost of cotton is the only variable cost in the model. Part of the reason for this simplifying assumption is that the data used for this model come from miniature spinning tests performed by USDA, rather than from an operational textile mill. These tests are performed under standardized procedures in which everything is held constant except the properties of the cotton fiber.

If the cost minimization procedure developed in this model were to be applied at a particular mill, a more complex cost function would have to be developed to describe the costs of that mill in detail. The bare bones specification of equation (1) is quite sufficient to demonstrate the principle of the cost minimization procedure.

Both labor costs and capital costs may vary as a result of changes in input mix because the amount of labor and capital used depends on the rate of production, and the rate of production depends on the properties of the input mix. We have not included capital and labor costs in the variable part of equation (1) because we lack data on the influence of mix on the rate of production. If we could obtain such data, we could incorporate this interaction into the model simply by respecifying equation (1) as needed.

Another cost of operation that varies with the mix of raw cotton is the cost of ends down. Understandably, spinners want to receive an "even running" supply of raw fiber so that their machinery will run smoothly. Ends down caused by poor raw material can add to overall costs by interfering
with capital and labor productivity. These costs were not included in our analysis because we had no data either on the relationship between ends down and fiber properties or on the cost of ends down.¹

Production Function

The usual specification for a production function is:

\[ Y = f(x_1, x_2, x_3, x_4, \ldots, x_n), \quad (2) \]

where

- \( f(\bullet) \) is a well-behaved function and
- \( Y \) is the number of pounds of yarn produced.

The specific production function used for this paper is:

\[ Y = \alpha \sum_{i=1}^{n} x_i \quad (2a) \]

where

- \( \alpha \) is a coefficient that depends on the fiber properties.

Both input and output are measured in the same (mass) units. Therefore, if there were no waste or other losses of fiber in production, the coefficient \( \alpha \) would equal one, because for every unit of mass of fiber input, the same mass of yarn would become output. However, because some fiber is lost as waste and some yarn will be rejected as below standard, the coefficient \( \alpha \) will be a function of the fiber properties of the input fiber blend.

In order to develop a functional form for the fixed coefficient, we introduce the following equations for waste and production below grade.

Waste

Waste is a function of the fiber properties of the blend being used in the manufacturing process. The properties of the blend are in turn functions of the percentage of cotton of each kind being used to make up the blend.

\[ \omega = h(p_1, p_2, p_3, \ldots, p_k), \quad (3) \]

where
\[ p_i = \text{property } i \text{ of the blend (i.e., fiber strength, length, micronaire, etc.),} \]
\[ k = \text{the number of properties, and} \]
\[ \omega = \text{the portion of the wasted input fiber, by weight.} \]

Furthermore, the properties of the blend are functions of the properties of the cottons being used to make up the blend and the portions of each used in the blend. Specifically,

\[ p_i = a(\rho_{i1}, \rho_{i2}, \rho_{i3}, \ldots, \rho_{in}, x_1 / \Sigma x_i, x_2 / \Sigma x_i, x_3 / \Sigma x_i, \ldots, x_n / \Sigma x_i), \quad i = 1 \text{ to } k, \quad (4) \]

where

- \( a(\cdot) \) is some kind of averaging function and
- \( \rho_{ij} = \text{the value of property } i \text{ in input cotton } j. \)

We have used arithmetically weighted averages as the specific functional form for equation (4). Earlier prototypes of this model used harmonic, rather than arithmetic means for blending micronaire, but because micronaire occupies such a narrow range, there was little practical difference between the two types of means.²

**Rejects**

We assume that the firm will try to produce yarn as close to the customer's specification as possible. However, because of random fluctuations in output quality, it must produce yarn of a slightly higher quality in order to keep rejections down. The goal of the cost minimization model is to adjust the mix of fiber such that the safety margin above the customer's specification is kept as small as possible. (While it is true that rejected yarn can probably be sold at a lower price, we assume for simplicity that rejected yarn is worthless.)

*Rejection for Strength.* The customer specifies a minimum level of strength. Production above this level earns no reward, and production below this level results in the rejection of the yarn by the customer.

Of course, no firm would deliberately produce yarn below grade. The manufacturing process is affected by random variables that may not be observed until it is too late to save the yarn. Accordingly, we need a stochastic equation to model the production and rejection process. We assume the following form:

\[ S = s(p_1, p_2, p_3, \ldots, p_k) + \epsilon, \quad (5) \]
where
\( s(\bullet) \) is a function relating properties to strength,
\( S \) is yarn skein strength, and
\( \varepsilon \) is a random variable.

We assume the customer’s contract specifies that yarn below a certain strength is unacceptable. Thus:

\[ R_s = P(S < S^*), \]  \hspace{1cm} (6)

where
\( R_s \) is the percentage of manufactured yarn rejected for being below grade,
\( S^* \) is the strength specified by the customer, and
\( P(\text{a condition}) \) is the probability of a condition being true.

Note that \( P(S < S^*) \) is the same as \( P[\varepsilon < (S^* - \hat{S})] \), where \( \hat{S} = s(p_1, p_2, p_3, \ldots p_k) \). If the standard deviation of \( \varepsilon \), \( \delta_\varepsilon \) has been estimated and if \( \varepsilon \) is distributed approximately normal, then the probability of rejection for strength is approximately

\[ P \left[ z < \frac{(S^* - \hat{S})}{\delta_\varepsilon} \right] \] where \( z \) is the standard normal variate.

We had no data on strengths typically called for by customers, so our choice for \( S^* \) was arbitrary. In our sample data, the average yarn strength was 107.75, with a standard deviation of 13.25. We picked the customer’s \( S^* \) to be 96 pounds, resulting in rejection for about 20 percent of the bales in the sample.

**Rejection for Imperfections.** For the purpose of this paper, we define yarn imperfections as unacceptably thick or thin places measured by an Uster Tester II, model B. This instrument detects the nonuniform segments in the yarn by measuring changes in an electrostatic field through which the yarn is passed.

By the same reasoning used to derive equation (6), we assert the following equation for rejections due to excessive imperfections in the yarn.

\[ R_1 = P(I > I^*), \]  \hspace{1cm} (7)

where
R₁ is the portion of yarn rejected for imperfections,  
I is the realized number of imperfections in the manufactured yarn, and  
I* is the customer's specification for the maximum number of imperfections.

As with the strength rejection equation, we have no data on the number of imperfections likely to be tolerated by yarn buyers. In our sample data, the average number of imperfections was 150.7 per 1,000 yards, with a standard deviation of 84.5. We selected 300 imperfections per 1,000 yards for I*. This results in rejection for about 4 percent of the bales in the sample.

Unit Cost Equation

Combining equations 2a, 3, 6, and 7 and assuming that losses for strength are statistically independent of losses for imperfections gives us the following equation for yarn output:

\[
Y = (1-\omega)(1-R_S)(1-R_I) \sum_{i=1}^{n} x_i 
\]  

(2b)

Combining equations 1 and 2b, the materials cost per unit of output is:

\[
C/Y = \frac{\left[ \sum_{i=1}^{n} w_i x_i \right]}{(1-\omega)(1-R_S)(1-R_I) \sum_{i=1}^{n} x_i} 
\]

(8)

The unit cost function is homogeneous of degree zero in the x-variables. Thus, without loss of generality, we will specify production for the case \( \sum_{i=1}^{n} x_i = 1 \). When this restriction is imposed, the cost function concentrates to:

\[
C/Y = \frac{\left[ \sum_{i=1}^{n-1} w_i x_i + w_n \left[ 1 - \sum_{i=1}^{n-1} x_i \right] \right]}{(1-\omega)(1-R_S)(1-R_I)} 
\]

(8a)
This concentrated cost function has the advantage of having one less dimension than the original cost function.

The independent variables of the concentrated cost function are the w_i, the x_i, and through their inclusion in R_s and R_I, the p_ij, which are the properties of the cotton groups. However, we are assuming the firm is a price taker, so from their point of view, the w_i are fixed parameters. Similarly, the p_ij are fixed parameters because they are predetermined by the time the cotton is placed on the market. Thus, from the firm's point of view, the only controllable variables in the cost function are the x_i.

**Economic Optimization**

Models based on profit maximization and cost minimization can provide insight and testable hypotheses regarding many firm behaviors. In the following section, we will develop an empirically based model of cost minimization designed to determine changes in cost and fiber mix as a function of changes in input characteristics. These changes in cost can be interpreted as the value of the characteristics in production.

**The Data**

The empirical results of this paper are based on data from the cotton fiber and processing tests performed by the Cotton Division of the Agricultural Marketing Service, USDA, Clemson, South Carolina. The fiber and processing test data have been published in various forms since 1946. However, the machine-readable dataset used for this study covers annual data from 1981 through 1987. For each year, the USDA collected samples of cotton, tested the properties of the fiber, produced yarn from the fiber, and tested the properties of the yarn. The data contain fiber and yarn quality measurements for samples of each year's crop.

The methods of spinning used to transform the fiber to yarn varied by the location and staple length of the raw fiber. Short staple cottons were spun into 8s and 22s carded yarn using a twist multiplier of 4.4, with some using open-end spinning methods. Medium staple cottons were spun into 22s and 50s carded yarn with a twist multiplier of 4.0. Long and extra long fibers were spun into finer yarns, both carded and combed. This paper focuses on the 22s yarn spun from the medium-length fibers.
The data include two groups of variables, yarn and fiber. Among the yarn variables are strength, imperfections count, appearance grade, and elongation at break. The data on fiber properties include a variety of measures of length, length uniformity, strength, waste content, color, micronaire, fineness, and maturity.

In the sample data, fiber length is measured by two methods, classer's staple and Motion Control® HVI fiber length analyzer. The classer's estimate is the traditional method of length estimation. A subjective estimation, the test consists of manipulating a small sample of cotton between the thumb and index finger of both hands. The HVI length test measures the changes in airflow as a beard of cotton is drawn through an orifice. The measurement is expressed in inches as a mean length and an upper-half mean length. The mean length divided by the upper-half mean length is purported to be a measurement of the short fiber content or nonuniformity of the lengths of the fibers in the sample, higher values being samples with both higher uniformity of fiber length and lower short fiber content.

A Motion Control® HVI machine is also used to test fiber strength, as is a Stelometer. Both machines test the breaking load of a bundle of cotton fibers clamped between two jaws 1/8 inch apart. The measured breaking load is converted to grams per tex, units that take into account the mass of material broken.

Color, another fiber variable, is measured in two dimensions, with both measurements made by a Nickerson-Hunter Cotton Colorimeter. Rd, or reflectivity measures the amount of light reflected from the cotton (the opposite concept from grayness) and is expressed as a percentage. Yellowness is expressed as a Hunter's +b reading. Larger values indicate more yellow cotton.

Micronaire is a "measure of the resistance of a plug of cotton to airflow." A measured weight of cotton is placed in a chamber and the pressure drop is measured as air is forced through the sample. Although micronaire is commonly referred to as a fineness measurement alone, there exists a well documented, systematic relationship among micronaire, fineness, and maturity. This complicates the interpretation of any statistical analysis involving micronaire alone.

Fineness and maturity are estimated separately by the Shirley Fineness and Maturity Tester. Like micronaire, the test consists of measuring the resistance of the sample to airflow. The test is performed at two different
pressures and compressions, and the fineness and maturity estimates are mathematical functions of the two different readings.

While other data were available in the dataset, our research is primarily geared to tests that are practical in a production environment. A single HVI line can be capable of almost 1,500 tests per day, whereas some of the laboratory tests consume much more time and could not be introduced easily into the production environment.

In addition to yarn and fiber data, the cost minimization model requires a set of \( w_i \), \( i=1,2,3, \ldots, n \), the prices for the different grades of cotton. The logic of the model requires that the \( w_i \) correspond to the price that mills actually pay for delivered cotton. Of all published cotton prices available to the author, the landed mill prices published by USDA in *Weekly Cotton Market Review* were deemed the most appropriate measure of fiber costs at the mill.\(^8\) We arbitrarily chose the landed mill prices reported for December 19, 1986, to demonstrate the model.

Unfortunately, the landed mill price data cover only four grades and six staple lengths. We were forced to estimate prices for other combinations of grade and staple by using the relative prices from the 1987 loan sheet for Lubbock, Texas. This sheet contains base loan rates for 31 grades and 9 staple lengths.

The estimated prices were determined through a regression model. For each combination of grade and staple that is priced in the *Weekly Review*, we determined the corresponding loan value. The data are shown in table 1.

We performed a regression using the data in table 1, in which the landed mill price was the dependent variable and the Lubbock loan value was the explanatory variable. The results of this regression are as follows:

\[
\text{Landed mill price} = 0.2059 + 1.1401 \times \text{Lubbock loan value}
\]

\[
R^2 = 0.98 \quad F_{1,22} = 925.4
\]

where the number in parenthesis is the t-ratio for the null hypothesis of no relationship. Obviously, the fit of this regression was quite tight.

The estimated regression was used to estimate a landed mill price for all grade and staple combinations on the Lubbock loan sheet. These estimated prices were assigned to every bale in our sample data, based on its grade, staple, and micronaire.
Table 1
Landed Mill Price and Lubbock Loan Value

<table>
<thead>
<tr>
<th>Grade</th>
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<th>Lubbock loan value</th>
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Estimation of Regression Equations

Statistical Considerations

The regression equations reported below are typical of the accepted paradigm in this area of research. We are not especially concerned with the fit of the regressions; they are intended only to demonstrate how the cost minimization model can be made to rely on HVI properties.

The regressions are specific to the miniature yarn production methods used in our sample data. They are not purported to be useful in predicting yarn properties for an operational manufacturing establishment. In order to implement the cost minimization model for a specific establishment, it would be necessary to develop regression equations specific to the equipment and techniques used at that particular establishment. At one time, I thought it would be possible to develop a "universal" set of regression equations that could be applied in any production situation. However, the variety of methods used in practice preclude the possibility of a "universal" equation.

The usual statistical requirements for ordinary least squares (OLS) dictate that the matrix of regressors be constants, rather than random variables. However, one of the most often heard complaints against HVI measurements is that they are prone to measurement error. Error can be induced by failure to maintain standard temperature and humidity in cotton preparation rooms and by variability in calibration procedures and calibration cottons, among other things. Even if the instruments were operated under ideal conditions, with all possible sources of nonsample error eliminated, there would still be a substantial variability in the observed fiber property measurements. This error is partially the result of the lack of precision of the test (that is, random instrument error) as well as the variability of fibers within the bale of cotton.

In the presence of errors in the independent variables, ordinary least squares estimation of the parameters of a regression equation will result in biased regression coefficients. Because of this statistical problem with conventional estimation methods, it might be worthwhile to implement some kind of multiple-indicator, multiple-cause statistical configuration. This approach can handle multiple dependent variables.

The random error term for our equations is assumed to be normally distributed. However, some analysis of the regression residuals would determine if this assumption is a reasonable approximation. The assumption
is clearly only an approximation because the normal distribution has an infinite domain, whereas the dependent variables are constrained to finite domains. It would be worthwhile to study alternative specifications for this normal error term because the shape of the distribution of the error term has an effect on the optimization analysis.

Waste Equation

The waste regression equation used for this study is:

\[ MWPCT = \beta_0 + \beta_1 \text{HUNIF} + \beta_2 \text{MIKE} + \beta_3 \text{HLEN} + \beta_4 \text{PLUSB} + \beta_5 \text{REFL} + \beta_6 \text{HSTR} + \zeta \]  

(9)

where

- \( MWPCT \) is the total mill waste observed in the spinning test for this sample,
- \( \zeta \) is a random error term,
- \( \beta_0 \) is a constant term and a set of annual dummy variables,
- \( \beta_1 \) to \( \beta_6 \) are regression coefficients,
- \( \text{HUNIF} \) is the HVI uniformity measurement for the sample fiber,
- \( \text{MIKE} \) is the HVI micronaire reading for the sample fiber,
- \( \text{HLEN} \) is the HVI length reading for the sample fiber,
- \( \text{PLUSB} \) is the HVI yellowness (+b) reading for the sample fiber,
- \( \text{REFL} \) is the HVI reflectivity (whiteness) reading for the sample fiber, and
- \( \text{HSTR} \) is the HVI strength reading for the sample fiber.

\( MWPCT \) is the sum of card and picker waste observed in the spinning tests. Other things being equal, higher levels of short, weak fibers will induce larger amounts of waste. Accordingly, we should include \( \text{HSTR}, \text{HUNIF}, \) and \( \text{HLEN} \) in the regression. Inasmuch as \( \text{MIKE} \) measures both maturity and fineness, it also belongs in the equation. Excessively fine fibers and immature fibers should both contribute to total waste. The color variables are included in the equation because they might indicate the presence of foreign material contamination that could affect mill waste.

Strength Equation

The strength equation is:

\[ YS = \gamma_0 + \gamma_1 \text{HSTR} + \gamma_2 \text{HLEN} + \gamma_3 \text{MIKE} + \gamma_4 \text{HUNIF} + \xi \]  

(10)
where
\[ YS \text{ is the skein strength observed for the sample,} \]
\[ \xi \text{ is a random error term,} \]
\[ \gamma_0 \text{ is a constant term and a set of annual dummy variables,} \]
\[ \gamma_1 \text{ to } \gamma_n \text{ are regression coefficients, and the other variables have been introduced earlier.} \]

Fiber strength enters the yarn equation because, other things being equal, stronger fibers will give stronger yarn. Length, uniformity, and micronaire are included in the equation because longer or finer fibers will contact a greater number of other fibers and will have greater cohesive forces than shorter, coarser fibers. The degree of cohesion between fibers affects both the breaking strength of the yarn and whether the yarn will break through slippage or fiber rupture. In this equation, micronaire is acting primarily as a measure of the fineness, rather than the maturity, of the fiber.

There is some suggestion that inasmuch as MIKE is positively related to the coarseness of the fiber, \( 1/\text{MIKE} \) would be proportional to the number of cotton fibers in a given diameter cross section.\(^{10}\) Because yarn strength is proportional to the number of cotton fibers in the yarn, \( 1/\text{MIKE} \) should produce better fit than MIKE in equation 10. We found that substituting \( 1/\text{MIKE} \) or \( \text{MIKE}^2 \) for MIKE or using them in addition to MIKE had little effect on fit.

**Imperfections Equation**

The imperfections equation is:

\[ \text{IMPS} = \phi_0 + \phi_1 \text{HUNIF} + \phi_2 \text{MIKE} + \phi_3 \text{HLEN} + \phi_4 \text{PLUSB} + \phi_5 \text{REFL} + \phi_6 \text{HSTR} + \eta \]

(11)

where
\[ \text{IMPS} \text{ is the number of imperfections detected in 1,000 yards of yarn in the spinning test for this sample,} \]
\[ \eta \text{ is a random error term,} \]
\[ \phi_0 \text{ is a constant term and a set of annual dummy variables,} \]
\[ \phi_1 \text{ to } \phi_6 \text{ are regression coefficients, and the other variables have been introduced earlier.} \]

The variables in this equation represent all HVI data likely to be collected on a sample. We included HUNIF because we would expect greater
uniformity of fiber lengths to contribute to greater uniformity in the yarn. We included MIKE as a measure of fiber fineness because we would expect finer fibers to have a higher tendency to become entangled. MIKE also represents the maturity of the fibers. We would expect the less mature fibers to be more likely to induce imperfections. Based on these considerations, high MIKE cottons can be expected to produce fewer imperfections. Yellowness is symptomatic of immaturity, so we would expect the coefficient of the yellowness variable to be positive. Longer fibers will have a greater likelihood of entanglement; therefore we would expect the sign of coefficient for fiber length to be positive. The reflectivity variable was included because it might indicate the presence of trash particles or surface deterioration that could cause yarn imperfections. Strength was included because we found it to be significant and we have no a priori reason to exclude it.

The specification of regression equations is as much an art as a science. However, penalties for including a variable that does not belong differ from those for excluding a variable that does belong. If a particular variable truly belongs in a regression equation, excluding it will bias the coefficients for all regressors that are correlated to the wrongly omitted regressor. The exclusion will also bias the variance estimators and all test statistics based on the coefficients and variance estimators. On the other hand, including a regressor that does not belong in the regression will not bias the other coefficients, even if the wrongly included regressor is correlated with the other regressors. The expected value of the estimated coefficient for the wrongly included regressor is zero. The main effect of wrongly including a regressor is that the statistical efficiency of the regression is reduced, costing us one "degree of freedom." With 752 observations in our data, the loss of one "degree of freedom" is a small price to insure ourselves against bias.

Results of Estimation

The theoretical equations described in the last subsection were fitted by ordinary least squares to the data. The subset of observations used included those that satisfied all the following conditions: (1) observation contained valid data for all the variables; (2) observation was from group 2 of sample (medium staple cotton); and (3) observation related to fiber sample being spun into 22s carded ring-spun yarn.

The estimation produced the following results:

Waste Equation

The waste equation, with $\beta_0$ adjusted for 1986 is:
MWPCT = 33.47811 - 0.10825 HUNIF - 0.35805 MIKE
    (-2.92)   (-4.14)

-4.99502 HLEN - 0.04572 PLUSB
    (-5.34)   (-0.73)

-0.13532 REFL - 0.02473 HSTR
    (-13.24)  (-1.402)

R² = 0.428    F₁₁,₇₄₀ = 50.5

Number of observations = 752

The figures in parentheses under the regression coefficients are t-ratios for the null hypothesis that the corresponding coefficient is zero. The F statistic is for the null hypothesis that all coefficients are zero.

The relatively low R² statistic for the waste equation shows that there remains much to be desired in predicting mill waste from measured cotton properties. However, even with the low R², the F-statistic affirms that the regression as a whole is significant at the 0.01 level. Furthermore, the large t-ratios attached to most of the coefficients affirm that there are some significant relations in our sample data. The looseness of fit should be attributed to omitted variables or a large error variance, rather than to the failings of the variables included in the equation.

The coefficients of the regression are either of the expected sign, or they are insignificant.

In-sample prediction was much closer in some trial regressions in which Shirley Waste Analyzer data was included as an independent variable. However, because the Shirley waste test is not part of the usual HVI line, the data were excluded from consideration in the waste equation.

*Strength Equation*

The strength equation for 1986 is:
YS = -388.95 +3.364479 HSTR +42.82683 HLEN
\[ (23.99) \quad (5.62) \]
-8.28736 MIKE +4.937419 HUNIF
\[ (-11.52) \quad (15.81) \]
\[ R^2 = 0.707 \quad F_{9,742} = 199.3 \]
Number of observations = 752
\[ \hat{\sigma} = 7.203 \]

The fit of the strength equation was much better than that of the waste equation, but it is still somewhat of a disappointment. The observed relationships between the variables were not a disappointment, however. The large t-ratios for all of the independent variables speak for themselves.

**Imperfections Equation**

The imperfections equation for 1986 is:

IM = -632.5366 +1.750969 HUNIF +236.0875 HLEN
\[ (0.536) \quad (2.87) \]
+9.730171 HSTR -21.4987 MIKE
\[ (6.27) \quad (-2.82) \]
+42.46406 PLUSB -1.42888 REFL
\[ (7.74) \quad (-1.59) \]
\[ R^2 = 0.225 \quad F_{11,740} = 19.6 \]
Number of observations = 752
\[ \hat{\sigma} = 74.927 \]
Like the waste equation, the imperfections equation was characterized by poor overall fit, but significant t-ratios for some of the variables. Despite the poor fit, the regression is significant at the 0.01 confidence level, as shown by the F statistic.

We expected the coefficient of length uniformity to be negative because we would expect a more uniform feed of raw material to result in a more uniform product. However, the sign of the coefficient is positive and, luckily, also insignificant.

The Model in Operation

Minimize Cost for 1986 Crop Year

Our economic model and our regression equations can now be combined into an operating model of a textile mill. We have referred to the existence of n "types" of cotton, as if it required no explanation. In fact, because the instrument-measured attributes of cotton are measured on continuous scales, the process used to establish limits for type boundaries may affect the analysis in nontrivial ways. Fortunately, industry practice has begun to standardize the method of establishing operational boundaries for cotton types. Practical experience with Cotton Incorporated's EFS method of processing laydown mixes suggests that no more than three groups of fibers need be differentiated along three dimensions: strength, length, and micronaire.11 Larger numbers either of properties or of groups for each property will cause the grouping system to be excessively complex. Fewer groups may still give adequate freedom for differentiation, but the risk of aggregating too different a configuration of individual fiber types into the same groups arises. Determining the best grouping method for a particular mill remains something of an art.

For the purpose of this paper, we set length and strength groups as shown in tables 2 and 3. We did not establish micronaire groupings because we wished to keep the number of categories of cotton from becoming too large.
Table 2
Length Groupings Used for Optimization

<table>
<thead>
<tr>
<th>Length group</th>
<th>HVI length (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-1.03</td>
</tr>
<tr>
<td>2</td>
<td>1.04-1.06</td>
</tr>
<tr>
<td>3</td>
<td>1.07-1.09</td>
</tr>
<tr>
<td>4</td>
<td>1.10-∞</td>
</tr>
</tbody>
</table>

Table 3
Strength Groupings Used for Optimization

<table>
<thead>
<tr>
<th>Strength group</th>
<th>HVI strength (g/tex)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-24</td>
</tr>
<tr>
<td>2</td>
<td>25-27</td>
</tr>
<tr>
<td>3</td>
<td>28-30</td>
</tr>
<tr>
<td>4</td>
<td>31-35</td>
</tr>
<tr>
<td>5</td>
<td>36-∞</td>
</tr>
</tbody>
</table>

Next, we averaged prices and fiber attributes for all bales in the 1986 sample for each length and strength subgroup. The results are reported in table 4.
Table 4
Attributes of Fiber Groups Used in Cost Minimization

<table>
<thead>
<tr>
<th>Length and strength group</th>
<th>Shirley fineness (FFIN)</th>
<th>Shirley maturity (FMAT)</th>
<th>HVI micro-maturity (MIKE)</th>
<th>HVI uniformity (HUNIF)</th>
<th>HVI length (HLEN)</th>
<th>HVI plus B yellowness (REFL)</th>
<th>HVI strength (HSTR)</th>
<th>Estimated price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1L2S</td>
<td>168.85</td>
<td>0.8237</td>
<td>3.89</td>
<td>80.10</td>
<td>1.019</td>
<td>9.07</td>
<td>72.7</td>
<td>24.94</td>
</tr>
<tr>
<td>2L1S</td>
<td>174.22</td>
<td>0.8751</td>
<td>4.10</td>
<td>80.57</td>
<td>1.047</td>
<td>8.77</td>
<td>72.6</td>
<td>23.54</td>
</tr>
<tr>
<td>2L2S</td>
<td>167.97</td>
<td>0.8592</td>
<td>3.93</td>
<td>80.33</td>
<td>1.050</td>
<td>8.86</td>
<td>72.4</td>
<td>24.97</td>
</tr>
<tr>
<td>2L3S</td>
<td>159.75</td>
<td>0.8362</td>
<td>3.60</td>
<td>79.33</td>
<td>1.052</td>
<td>9.20</td>
<td>73.3</td>
<td>28.40</td>
</tr>
<tr>
<td>3L2S</td>
<td>184.47</td>
<td>0.9621</td>
<td>4.56</td>
<td>81.04</td>
<td>1.080</td>
<td>8.57</td>
<td>73.6</td>
<td>25.09</td>
</tr>
<tr>
<td>3L3S</td>
<td>176.98</td>
<td>0.9915</td>
<td>4.45</td>
<td>81.00</td>
<td>1.085</td>
<td>8.38</td>
<td>77.8</td>
<td>28.40</td>
</tr>
<tr>
<td>4L2S</td>
<td>183.34</td>
<td>0.9438</td>
<td>4.51</td>
<td>81.22</td>
<td>1.123</td>
<td>8.30</td>
<td>75.1</td>
<td>25.65</td>
</tr>
<tr>
<td>4L3S</td>
<td>176.75</td>
<td>0.9666</td>
<td>4.49</td>
<td>81.33</td>
<td>1.122</td>
<td>8.32</td>
<td>77.9</td>
<td>28.67</td>
</tr>
<tr>
<td>4L4S</td>
<td>167.48</td>
<td>0.9941</td>
<td>4.39</td>
<td>81.33</td>
<td>1.113</td>
<td>8.33</td>
<td>80.4</td>
<td>31.24</td>
</tr>
</tbody>
</table>

Note: The notation "nLmS" means the nth length group and the mth strength group. Some groups (for example 3L4S) are missing because they represented less than 6 bales in the sample.

In order to achieve the minimization of the cost function, it is necessary to restrict and bind $x_i$ to the feasible space. This restriction was achieved by subjecting the $x_i$ to a monotonic transformation (the logistic function) with an infinite domain and a range between 0 and 1. Further transformations were also used to map the n restricted shares to n-1 unrestricted share parameters. The transformed cost function was well behaved, and minimization proceeded smoothly from a variety of starting points to what appeared to be the global minimum.

**The Two-Dimensional Case**

In order to demonstrate the results graphically, we will present a simple two-dimensional case. Compact enough to replicate on a microcomputer spreadsheet, the two-dimensional case involves a choice between fiber in length group 4 and strength group 2 (4L2S) versus fiber in length group 4 and strength group 3 (4L3S). If we were to spin yarn from samples from these two groups, the results would be as shown in table 5.
The table shows that the more expensive, stronger fiber yields the stronger yarn. However, the stronger fiber also costs more and gives the greater number of imperfections. Figure 1 shows the result of spinning yarn using fiber blended from these two groups. Losses from waste rise linearly as the portion of 4L2S fiber in the blend is increased. Losses from strength rejection also rise, but the effect is not linear—the slope of the curve depends on the slope of the cumulative normal distribution function. Losses due to imperfections fall as the 4L2S cotton is increased. The minimum loss of fiber occurs at about a half-and-half blend of the two fibers.

Figure 2 shows the price of the input fibers as a function of the portion of 4L2S fiber in the blend. Because the 4L2S fiber is cheaper than the other fiber, it is economical to increase its portion in the blend beyond the minimum fiber loss point. Minimum cost per unit of output (66.35 cents/pound) is achieved when the portion of weaker fiber in the blend is about 77.5 percent.

The minimum cost solution is characterized by tradeoffs between the price of the inputs and their performance in spinning. As more of the cheaper component is used in the blend, the cost per pound of blend falls, but as the cheaper fiber begins to cause quality losses, the cost per pound of output rises.

Table 5

<table>
<thead>
<tr>
<th>Length and predicted strength group</th>
<th>Percentage predicted skein waste (pounds)</th>
<th>Predicted strength for imperfections (per 1000 yards of yarn)</th>
<th>Percentage predicted imperfections</th>
<th>Percentage rejected for imperfections</th>
<th>Percentage total fiber loss</th>
<th>Materials input price (cents per pound)</th>
<th>Materials cost per pound of yarn (cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4L2S</td>
<td>6.64</td>
<td>108.4</td>
<td>4.35</td>
<td>176.6</td>
<td>3.68</td>
<td>13.99</td>
<td>57.47</td>
</tr>
<tr>
<td>4L3S</td>
<td>6.16</td>
<td>118.8</td>
<td>0.08</td>
<td>199.1</td>
<td>7.17</td>
<td>12.97</td>
<td>60.26</td>
</tr>
</tbody>
</table>

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Figure 1
Blending Yarn from 4L2S and 4L3S Cotton,
Percentage Loss Due to Waste and Rejection

Figure 2
Blending Yarn from 4L2S and 4L3S Cotton,
Materials Costs--Input and Output
The Multidimensional Case

The two-dimensional case can be graphed, but actual fiber distributions such as in table 4 require more than three dimensions and thus cannot be easily demonstrated in graphic media. To find the ultimate minimum cost using the nine cotton groups in table 4 requires an optimization solution in eight (n-1=8) dimensions.

In the early work on this project, we discovered that the optimal solution is usually characterized by fiber groups that are not used in the optimum mix. We believe this is true, but we have not developed rigorous proof that if there are \( m \) opportunities for fiber loss (waste, strength, and imperfections, for example) there will be, at most, \( m \) non-zero fiber groups in the optimal solution. This result is analogous to the linear programming theorem that limits the number of variables in the solution to the number of constraints. Ours is not exactly the same kind of problem because this model is nonlinear and does not actually have constraints. Unlike constraints, our rejection and waste equations are always binding.

With five of the cotton groups excluded, our optimal solution quickly converged from eight dimensions to three. At optimum, the three nonzero \( x_i \) were:

\[
\begin{align*}
  x_{1L2S} &= 0.2079 \\
  x_{2L3S} &= 0.4956 \\
  x_{4L2S} &= 0.2965
\end{align*}
\]

At this minimum cost point, the materials cost of a pound of yarn is 64.744 cents per pound. Any change in the portions will induce an increase in cost over this amount.

Spinning Value of Properties

Having achieved this equilibrium, we can subject the optimal solution to some sensitivity analysis to determine the spinning values for the fiber properties. The value of a fiber property in spinning is its effect on cost. The partial derivative of the unit cost function, with respect to a fiber property, \( \frac{\partial(C/Y)}{\partial p_i} \), is a measure of the change in cost induced by a unit change in the property. In order to get a measure of a fiber property value relative to its own dispersion, we are also interested in the quantity:
where
\[ \sigma_p_i \] is the sample standard deviation of fiber property i in the 1986 sample.

Table 6 shows the value of cotton properties computed in this manner.

We can use the results in table 6 to rank the importance of the fiber properties in terms of their contribution to production. A large negative \( \psi \) means that in the neighborhood of optimum, increasing the level of the attribute will reduce cost. Similarly, increasing the level of properties with large positive \( \psi \) will increase cost. In this example, we have the seemingly peculiar result that length is the most undesirable attribute in the neighborhood of optimum, the reason being the large positive coefficient for length in the imperfections equation.

One of the most interesting findings in the table is that, relative to the optimum, higher micronaire contributes to higher cost. This effect is the net result of three countervailing forces. Table 7 shows the effect of increasing the micronaire of the blend from its optimum value, 3.93, to 4.03. Waste is little affected, but rejection for strength has increased, and rejection for imperfections has decreased. The size of these changes depends on the coefficients for micronaire in the yarn property equations and the degree to which each property is a factor in the number of rejects. It requires more improvement in a smaller factor to produce an equal effect with a larger factor.

The net effect of the change in micronaire is that the final rejection rate has increased by about two-tenths of a percent. This rise in rejections induces a proportional rise in the cost per unit. The 0.134 cent per pound increase in cost can be interpreted as the spinning value for the tenth of a unit of micronaire. That is, if 3.93 micronaire is taken as a base, then the discount for 4.03 micronaire cotton should be 0.134 cent per pound.

It is surprising that higher micronaire in the blend induces a higher cost in manufacturing inasmuch as the current marketing system places a discount on low micronaire fiber. Further analysis, however, reveals that the result is only local. Viewing a larger variation in micronaire gives the result shown in figure 3, where the discounts implied by the cost minimization model are compared to average market discounts for micronaire as of
December 19, 1986. It is gratifying to observe that the theoretical and actual discounts have roughly the same shape, considering the limitations of our model.

Note that in figure 3, the maximum point of the discount curve is near the optimum micronaire, but not exactly equal to it. The premium and discount in the graph are plotted relative to the properties optimum fiber blend. That is, the plot always crosses the x-axis at the $p_i$ of the optimal blend. Depending on its shape, the curve may cross at other points as well.

The model-implied micronaire discounts are a manifestation of changes in total fiber loss induced by the change in micronaire. Figure 4 details the losses. At the very lowest micronaire ranges, imperfections are the main source of rejection, and at the highest strength, rejection becomes important.

The other fiber properties also have spinning values based on similar analysis. Figure 5 shows the spinning value for length; figure 6, for yellowness; and so forth. These figures are analogous to figure 3, relating to the discount for micronaire. Here, however, there are no established discounts related directly to these other properties. In each case, there would also be a figure corresponding to figure 4, relating to the losses of fiber. However, in the interest of space, these figures were omitted.

### Table 6

<table>
<thead>
<tr>
<th>Fiber property</th>
<th>$\frac{\partial(C/Y)}{\partial p_i}$</th>
<th>$p_i$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micronaire</td>
<td>0.6632</td>
<td>0.5156</td>
<td>1.2862</td>
</tr>
<tr>
<td>Uniformity</td>
<td>-2.679</td>
<td>0.9771</td>
<td>-2.742</td>
</tr>
<tr>
<td>Length</td>
<td>15.09</td>
<td>0.0366</td>
<td>412.22</td>
</tr>
<tr>
<td>Yellowness</td>
<td>7.878</td>
<td>0.684</td>
<td>11.518</td>
</tr>
<tr>
<td>Whiteness</td>
<td>-0.36</td>
<td>3.8493</td>
<td>-0.094</td>
</tr>
<tr>
<td>Strength</td>
<td>-0.201</td>
<td>2.2188</td>
<td>-0.091</td>
</tr>
</tbody>
</table>
### Table 7
Effect of Changing Micronaire of Blend from 3.93 to 4.03

<table>
<thead>
<tr>
<th>Yarn property</th>
<th>Micronaire =3.93</th>
<th>Micronaire =4.03</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waste (percentage)</td>
<td>7.03</td>
<td>7.00</td>
<td>-0.03</td>
</tr>
<tr>
<td>Strength rejection (percentage)</td>
<td>2.79</td>
<td>3.62</td>
<td>+0.83</td>
</tr>
<tr>
<td>Imperfection rejection (percentage)</td>
<td>11.26</td>
<td>10.72</td>
<td>-0.54</td>
</tr>
<tr>
<td>Total rejection (percentage)</td>
<td>19.81</td>
<td>19.98</td>
<td>+0.17</td>
</tr>
<tr>
<td>Cost per unit (cents per pound)</td>
<td>64.744</td>
<td>64.878</td>
<td>+0.134</td>
</tr>
</tbody>
</table>

#### Figure 3
Comparison of Spinning Value and Market Discount for Micronaire
Figure 4
Fiber Loss as a Function of Micronaire

Figure 5
Spinning Value for Length
Figure 6
Spinning Value for Yellowness (PLUSB)

Figure 7
Spinning Value for HVI Strength
Figure 8
Spinning Value for Uniformity

Figure 9
Spinning Value for Whiteness (REFL)
While it is interesting to compare the shapes and magnitudes of the foregoing discount plots to get an idea of the relative importance of the various fiber properties, the reader is cautioned against inferring too much from them. Strictly speaking, the discounts are valid only in the case of 22s yarn being manufactured to the specifications described above. The curves could change substantially if any of the specifications are changed. For example, if the imperfections rejection boundary ($I^*$) were set higher, the imperfections curve in figure 4 would be lower, the point of minimum cost would move to the left, and the slope of the low-end micronaire discount plot would become much flatter. Furthermore, the curves present only production materials cost savings, rather than contribution to profit. It is entirely possible that in a multiproduct model, expensive fiber that is not cost-effective for making 22s yarn would generate larger profits making finer yarn, and therefore, the spinning value of the fiber properties would be different than that depicted above.

Cost Minimization with Shirley FMT Data

Having put the model through its paces to develop spinning values for the fiber properties, we now demonstrate its application to the problem of determining the value of testing cotton fiber with the Shirley Fineness and Maturity Tester (FMT). The method employed in estimating the value of the test compares the optimum cost achieved in the last section to the optimum cost given the additional information from the FMT. We will show that the FMT predictive equations for the yarn properties are somewhat better than those based on the smaller set of usual HVI properties. This increase in accuracy permits the mill to set the blend to produce yarn with expected properties somewhat closer to $I^*$ and $S^*$, with a concomitant reduction in raw materials cost.

The following predictive equations are exactly analogous to the equations presented in earlier sections, except that FFIN and FMAT have been substituted for micronaire.

Waste Equation

$$MWPCT = 33.697 - 0.11256 \text{ HUNIF} + 0.00242 \text{ FFIN} - 3.052 \text{ FMAT} - 5.2763 \text{ HLEN} - 0.03066 \text{ PLUSB}$$

(9b)
As expected, this waste equation fits slightly better than its earlier counterpart. The \( R^2 \) has increased by 2.4 percent, and the estimated standard deviation has decreased by 0.01.

**Strength Equation**

\[
YS = -360.25 + 3.066 \text{HSTR} + 44.196 \text{HLEN} \\
-0.301 \text{FFIN} + 4.465 \text{FMAT} \\
+4.817 \text{HUNIF}
\]

\[
(22.27) \quad (6.12) \quad (-14.65) \quad (1.01) \quad (16.27)
\]

\( R^2 = 0.736 \)

Number of observations = 752

\( \hat{\sigma} = 6.852 \)

The strength equation also fits better than its earlier counterpart. The \( R^2 \) has gone up by 3 percent, and the estimated standard deviation has fallen by 0.351. FFIN has the expected negative sign, and FMAT has the expected positive sign.

**Imperfections Equation**

\[
IM = -578.16 + 2.8411 \text{HUNIF} + 249.057 \text{HLEN}
\]

\[
(0.88) \quad (3.07)
\]
The imperfections equation fits better than its earlier counterpart. The $R^2$ has gone up by 9 percent, and the estimated standard deviation has fallen by 0.765.

Having estimated these three new equations, we use them to replace the micronaire-oriented equations in the model and solve again for the optimum fiber mix. The nonzero coefficients are:

$$x_{1L2S} = 0.0748$$
$$x_{2L3S} = 0.5819$$
$$x_{4L2S} = 0.3433$$

At this point, optimum cost is 64.620 cents per pound, compared to 64.744 from the micronaire-only formulation. The cost has fallen by 0.124 cent per pound, simply by using FMT test data. The input opportunities and prices faced in this optimization were identical to those in the earlier model. In fact, the same three fiber groups were included in the final optimization; only the mix has changed.

The savings of 0.124 cent per pound translates to a savings of $0.62 for a 500-pound bale. If it costs less than 62 cents per bale to perform the FMT test, the mill can reduce materials cost by performing the test and adjusting its mix accordingly.
Summary and Directions for Future Research

We have discussed various inadequacies of this model. This section summarizes these inadequacies in a list of enhancements that must be made in order to add realism to this prototype model.

Profit Maximization

The spinning values above are the values of the properties in reducing materials cost. A profit maximization model would include other costs, such as labor, and this could change the results in various ways. For example, the loss of yarn to imperfections and strength is assumed in this model to carry zero costs, but in fact, labor costs are expended in the rejected yarn. Including labor cost in the optimization would tend to reduce rejection.

In addition, a profit maximization model would permit analysis of the contribution to profit of the fiber properties, rather than just their effect on cost. This is especially important when considering multiple products with different prices.

Multiple Products and Different Spinning Methods

Inasmuch as the data used were quite specific to 22s ring-spun yarn, it would be desirable to generalize about other products, especially open-end spun yarn. Fiber that is not useful for one application might prove useful for another.

Estimation Methods

We used ordinary least squares to estimate the regressions, but the model violates the assumptions needed to make OLS an efficient estimator, namely: (1) error in the independent variables; (2) correlation between the error terms of the variables; and (3) nonnormal disturbances. Given these violations of the ideal conditions, it may be worthwhile to try an estimation procedure that counteracts statistical bias contributed by errors in the variables.12

The nonnormal disturbances do not necessarily hurt the efficiency of the estimator, but they do affect the optimization. We expect to find that the distributions for the error terms will be asymmetric, such as a chi-squared
distribution or Pareto distribution. If this is the case, then even with the same \( I^* \) and \( S^* \), rejection rates for strength and imperfections could change substantially.

Future Directions

Future generations of this model can be made to produce further useful results. For example, cotton buyers could use the model to help them formulate buying decisions. A desktop version of this program could be developed to give buyers an immediate estimate of a cotton's value for spinning in their establishment or to tell them what quantity to buy of several competing offers. Such a program could be integrated with existing inventory control and laydown mix software, so that current inventories as well as offers to purchase could be considered in the optimization process.

The economic optimization approach to determining spinning values could also prove useful in developing future pricing structures for cotton fiber as it relates to production values, rather than market prices. Current market prices contain built-in institutional distortions, thus any newly designed pricing rules that depend on these past prices will perpetuate the distortion. In particular, models in which the value of fiber properties is estimated by a regression of cotton price against fiber property data will be biased by systematic market distortions.\(^\text{13}\)

On the other hand, the approach demonstrated in this paper is free of such distortions. If this model were generalized along the lines described above, the resulting graphs corresponding to figures 3 through 9 could be implemented as a set of premium and discount figures for cotton properties. This change would have two important consequences. First, the pricing system would correspond to actual spinning values and would therefore become more efficient. (That is, institutional distortions would be reduced or eliminated.) Second, cotton growers would be rewarded for producing cottons with desirable spinning values, and over time, they would respond to the price mechanism by producing more spinnable cotton.
Notes

1. For discussion on the importance of production rate and ends down and their dependence on fiber properties, see Bragg, "Use of Optimization Concepts in Establishing Fiber Yarn Relationships," p. 128.


3. The Uster Tester II model was adjusted as described in USDA, "Summary of Cotton Fiber and Processing Test Results," p. 94.

4. The assumption of statistical independence should be relaxed in future work because the statistical efficiency of the parameter estimators could be improved by using a generalized least squares estimator incorporating a non-diagonal covariance matrix for the error terms of the yarn-properties equations.

5. For additional documentation of this data, see USDA, "Summary of Cotton Fiber and Processing Test Results."

6. Ibid., p. 83.


9. There are many excellent articles available on the fitting of equations such as these. See, for example, Cotton Incorporated, "Fiber-Yarn Forecasting Research Directions"; El Mogahzy, "Selecting Cotton Fiber Properties"; Olson, "Marketing on Cotton Spinning Qualities"; and Suh, "Methods for Improving Statistical Prediction."


13. This regression approach has been used by Hudson and Williams, "Relationship of Fiber Test Data"; Horak and Harris, "Cotton Pricing According to the Fiber Properties"; and more recently by Hembree, Ethridge, and Neeper, "Market Values of Fiber Properties."
References


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