On the feasibility of using thermal gradients for active control of interlaminar stresses in laminated composites

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Abstract

In this paper, a method is proposed to actively control interlaminar stresses near the free edges of laminated composites by through-thickness thermal gradients. Theoretical solutions are given for optimal steady-state through-thickness temperature distributions under uniaxial loading that are required to eliminate or reduce the interlaminar stresses below a prescribed level. The optimal solutions are obtained by minimizing appropriate performance indices that are functions of the far-field properties, with respect to the through-thickness temperature differences. In the second part, an experimental investigation is conducted on a glass/epoxy cross-ply laminate with embedded piezoelectric sensors and a thermal heater. Through the experiment, the feasibility of the thermal control of interlaminar stresses is demonstrated.

1. Introduction

In the past, various passive control strategies have been suggested to control the interlaminar stresses and avoid delamination in laminated composites. Pagano and Pipes [1] have suggested the use of a specific stacking sequence to eliminate the interlaminar tension. This technique can be used in the neighborhood of delamination-prone areas. Pagano and Lackman [2] have shown that the onset of delamination can be prevented by serrating the fiber edges and filling in voids with the matrix materials. Mignery et al. [3] have suggested sewing or wrapping the laminate in delamination prone areas. Chan et al. [4] used a thin layer of tough adhesive at the interface of the fiber/matrix to delay the onset of delamination. However, because of their passive nature, all of these techniques are often ineffective and may not be compatible with other design goals.

On the other hand, investigations on thermally induced stresses in composite laminates have revealed that edge effects induced by a uniform temperature change can result in interlaminar...
stresses of significant amounts [5–7]. This suggests that thermal loading, when applied with controlled magnitudes and spatial gradients, has the potential to compensate for the interlaminar stresses caused by mechanical loading. In recent papers, Kim and Atluri [8, 9] have shown that a thermo-mechanical control near the free edges with through-thickness temperature variations is indeed an efficient means for this purpose, and provided analytical solutions for the optimal temperature distributions. Hanagud et al. [10] have conducted preliminary tests on an orthotropic glass/epoxy laminate with a thermomechanical actuator/sensor device. Their results showed that under uniaxial tension, heating of the orthotropic plate above ambient temperature results in a reduction of the interlaminar stresses as predicted by the theory.

The present paper addresses a continuation of the work presented in Refs. [8–10] with several added features. The objective of this paper is to study the feasibility of using smart structure concepts to control the interlaminar stresses and ultimately the onset of delamination in laminated composites. On the theoretical side, we will include continuous, piecewise linear temperature distribution, and a combined global–local optimization scheme. It is found that for a cross or angle-ply laminate, a temperature rise or drop which is uniform throughout the laminate can completely eliminate all of the interlaminar stresses. On the experimental side, first, some erroneous steps in the previous calibration of the piezoelectric sensors have been eliminated. Second, a correct constitutive equation has been provided to better correlate the voltage measurement to the interlaminar normal stress. Also, by attaching a second thermal heater at the free edge along with the one embedded within the laminate, the efficiency of thermal heating has been enhanced.

2. Problem statement

An active control to reduce the interlaminar stresses needs actuators that can be used to counter the stresses created by applied service loads. Several candidate actuators that were considered are, piezoelectric actuators, electrostrictive actuators, shape memory alloy actuators, and imposed thermal gradients. In this paper, we have restricted our studies to the use of imposed thermal gradients.

To explore the possible effectiveness of a thermal field, a simple problem has been selected. In this problem, a layered composite plate of rectangular shape subjected to a uniform, uniaxial mechanical loading and a thermal field that can vary in the through-thickness direction but otherwise uniform, is considered. To maintain the simplicity of the present feasibility study, no pre-existing delaminations are considered. Instead, the study is focused on the reduction of interlaminar stresses at the free edges of the rectangular plate. Thus, consider a rectangular composite laminate that is subjected at both ends to an extensional load \( N_x \) (force/unit width) and a layer-wise temperature distribution \( \Delta T(z) \) (see Fig. 1). Local coordinates \((x_1, x_2, x_3)\) are established within each layer. The global through-thickness coordinate \(z\) is also defined at the mid-plane of the laminate. The laminate consists of \(N\) plies and can have a general symmetric or unsymmetric stacking sequence. Basic assumptions that hold throughout theory are as follows: (1) all variations can be ignored in the longitudinal direction \(x_1\); (2) through-thickness temperature distribution is assumed to be piecewise linear within each lamina, continuous at all interfaces; (3) the laminate properties do not change with temperature variation; (4) all variables have reached a steady state and transient effects are ignored. In addition, it is assumed that any temperature distribution can be
specified and applied to control the interlaminar stresses. Thus, it is not necessary to consider the heat conduction equation and heat boundary conditions within the laminate.

3. Global and local mismatches in the elastic properties

In a composite laminate with straight free edges, the interlaminar stresses $\sigma_{33}^{(k)}$, $\sigma_{23}^{(k)}$, and $\sigma_{13}^{(k)}$ arise due to steep transverse gradients in the in-plane stress components, $\partial \sigma_{22}^{(k)}/\partial y$ and $\partial \sigma_{12}^{(k)}/\partial y$ near the free edges. More specifically, the in-plane stresses are zero at the free edges and quickly approach nonzero far-field values $\bar{\sigma}_{22}$, $\bar{\sigma}_{12}$ away from the edges. The far-field in-plane stresses develop because of mismatches between individual ply strains and global laminate strains.

It can be shown that the most important mismatches that contribute to the interlaminar stresses under combined thermo-mechanical loading are in the equivalent Poisson's ratios and equivalent coefficients of mutual influences [12]. The first kind of mismatches are the global mismatches defined as follows:

$$\delta \nu_{12E}^{G}(k) = v_{12E}^{(k)} - \bar{v}_{12E},$$  \hspace{1cm} (1)$$

$$\delta \eta_{12E}^{G}(k) = \bar{\eta}_{12,1E} - \eta_{12,1E}^{(k)},$$  \hspace{1cm} (2)$$

where the equivalent Poisson's ratios and coefficients of mutual influence are defined as

$$\bar{v}_{12E} = - \frac{\varepsilon_{22}^{tot}}{\varepsilon_{11}^{tot}},$$

$$\bar{\eta}_{12,1E} = - \frac{\gamma_{12}^{tot}}{\varepsilon_{11}^{tot}},$$

$$v_{12E}^{(k)} = v_{12}^{(k)} \left[ 1 - \frac{1}{\varepsilon_{11}^{tot}} \left( \mu_{3}^{(k)} + \frac{\mu_{3}^{(k)}}{v_{12}^{(k)}} \Delta T^{(k)}(x_{3}) \right) \right]$$

Fig. 1. Ply coordinate system.
\[ \eta_{12}^{(k)} = \eta_{12,1}^{(k)} \left[ 1 - \frac{1}{\xi_{11}^{(k)}} \left( \mu_{x}^{(k)} + \frac{\mu_{x}^{(k)}}{\nu_{12,1}^{(k)}} \right) \Delta T^{(k)}(x_3) \right]. \]

In the above, \( \nu_{12}^{(k)} \) and \( \eta_{12,1}^{(k)} \) are the Poisson's ratio, the coefficient of mutual influence for the \( k \)th ply, respectively. As usual, these are for mechanical loading only. The global mismatches completely determine the in-plane stresses in the far-field. The second kind of mismatches are local mismatches which are defined as, between the \( k \)th and the \((k + 1)\)th plies,
\[ \delta v_{12E}^{L}(k) = v_{12E}^{(k)} - v_{12E}^{(k+1)}, \]
\[ \delta \eta_{12,1E}^{L}(k) = \eta_{12,1E}^{(k)} - \eta_{12,1E}^{(k+1)}. \]

These mismatches affect the interlaminar stresses only near the free edge region, and do not appear in the far-field expressions. Eqs. (1), (2) and (7), (8) now imply that by applying a proper layer-wise temperature distribution, one can minimize the effects of the mismatches and consequently, the interlaminar stresses.

4. Solution of optimal thermal gradients

4.1. Global optimization

An appropriate performance index that is to be minimized is defined in terms of the through-thickness temperature gradients. The first performance index considered is
\[ J_1 = \int_{-h/2}^{h/2} (x \tilde{\sigma}_{22}^2 + \beta \tilde{\sigma}_{12}^2) \, dz, \]
where \( x \) and \( \beta \) are the weighting factors associated with the far-field-in-plane stresses \( \tilde{\sigma}_{22} \), \( \tilde{\sigma}_{12} \), respectively. By minimizing the far-field in-plane stresses through application of a layer-wise temperature distribution, one can insure that the interlaminar stresses will be also minimized.

Minimizing \( J_1 \) with respect to the \((N + 1)\) temperature difference \( \Delta T^{(i)} \) (see Fig. 1), one obtains the following \((N + 1)\) linear simultaneous equations:
\[ B \cdot \Delta T = d, \]
where the elements of \((N + 1) \times (N + 1) B\) and \((N + 1) \times 1 d\) are given in Ref. [9]. The forcing terms \( d_i \) are proportional to the applied load \( N_x \) whereas the \( b_{ij} \) on the left-hand side are load-independent. Therefore, as long as the laminate properties are assumed invariant under temperature changes, the optimal temperatures fields should vary linearly with the amount of the uniaxial load applied.

4.2. Combined global–local optimization

With the exceptions of cross-ply and angle-ply laminates, the global minimization alone may not necessarily lead to a local minimization, and vice versa. To include the effects of global and local minimizations in a single combined minimization, a quadratic measure of mismatch terms is added.
as a constraint in the optimization scheme. For this purpose, the following new index is considered:

\[ J_2 = J_1 + \lambda H, \]

where

\[ H = \gamma \sum_{k=1}^{N} \int_0^{\rho^{(k)}} \left[ \delta \nu_{12E}(k, 1) \varepsilon_1^{(k)} f_1^{(k)}(x_3) + \delta \nu_{12E}(k, 2) \varepsilon_2^{(k)} f_2^{(k)}(x_3) \right]^2 \, dx_3 + \rho \sum_{k=1}^{N} \int_0^{\rho^{(k)}} \left[ \delta \eta_{12,1E}(k, 1) \varepsilon_1^{(k)} g_1^{(k)}(x_3) + \delta \eta_{12,1E}(k, 2) \varepsilon_2^{(k)} g_2^{(k)}(x_3) \right]^2 \, dx_3 - R^2 = 0, \]

where \((k, 1), (k, 2)\) represent the top and bottom of the \(k\)th ply, respectively, and \(\varepsilon_1^{(k)}, \varepsilon_2^{(k)}\) the total extensional strain at the top and bottom of the \(k\)th ply, respectively. The through-thickness weightings \(f_1^{(k)}, f_2^{(k)}, g_1^{(k)}\), and \(g_2^{(k)}\) are defined based on the assumed through-thickness distributions of stresses due to the local mismatches in the equivalent Poisson ratios and coefficients of mutual influence \([11, 12]\). \(\gamma\) and \(\rho\) are weighting factors for the local mismatches, and \(R\) is the norm of the local quadratic measure. All of the \(\gamma, \rho, \) and \(R\) are set a priori, and they reflect how much the local mismatches should be minimized compared with the global mismatches. Minimization of \(J_2\) leads to the following new set of \((N + 1)\) linear simultaneous equations.

\[ B^{**}(\lambda) \cdot \Delta T = d^{**}(\lambda), \]

where the elements of the new \(B^{**}d^{**}\) are defined as

\[ b_{ij}^{**} = b_{ij} + \lambda b_{hi}, \]

\[ d_{i}^{**} = d_{i} + \lambda d_{hi}, \]

where \(b_{hi}, d_{hi}\) are given in Ref. [9]. Eq. (13) and the constraint Eq. (12) can be solved simultaneously for \(\Delta T\) and the Lagrange multiplier \(\lambda\).

4.3. Special cases – cross and angle-ply laminates

Because of its uniform stacking structure, a cross-ply or an angle-ply laminate allows the global and local equivalent mismatches to increase or decrease by the same ratio as functions of a uniform temperature difference \(\Delta T\). Therefore, it can be expected that at a critical uniform temperature difference, both the global and local mismatch terms will disappear exactly.

For a cross-ply laminate, minimization of Poisson’s ratio would lead to the minimization of the interlaminar stresses \(\sigma_{33}, \sigma_{23}\). It can be shown that for given mechanical strain \(\varepsilon_{11}^{\text{mech}}\), the unconstrained optimization result can be written in terms of basic uni-ply properties as follows:

\[ \Delta T_{\text{cross, opt}} = \frac{(\nu_{12} - \nu_{LT}) \varepsilon_{11}^{\text{mech}}}{A(\mu_L - \mu_T)} \]
where 
\[ A = \frac{E_L(E_L + E_T - 2E_Tv_{LT} - E_Tv_{LT}^2) + E_T^2v_{LT}^2(2v_{LT} - 1)}{E_L^2 + E_T^2 + 2(E_LE_T - 2E_T^2v_{LT}^2)} \]  
(16)

Here the subscripts L and T represent the longitudinal and transverse direction of a uni-ply, respectively. For this temperature difference (15), the interlaminar stresses \( \sigma_{33}^{(k)} \) and \( \sigma_{23}^{(k)} \) in a cross-ply laminate will be completely eliminated.

For an angle-ply laminate, minimizations of coefficient of mutual influence will lead to the minimization of the only interlaminar stresses \( \sigma_{13}^{(k)} \):

\[ \Delta T_{\text{angle, opt}} = \frac{1}{Q_{22}^{(k)}Q_{16}^{(k)}Q_{12}^{(k)}Q_{26}^{(k)}} \left( \frac{Q_{16}^{(k)}Q_{22}^{(k)} - Q_{12}^{(k)}Q_{26}^{(k)}}{Q_{22}^{(k)}Q_{16}^{(k)}Q_{12}^{(k)}Q_{26}^{(k)}} e_{11}^{\text{mech}} \right) + \mu_{i3}^{(k)} + \mu_{i3}^{(k)} \]  
for any \( k \)  
(17)

4.4. Constrained optimizations

Depending on the material properties and a stacking sequence of a laminate, a temperature field may be obtained that is not realizable in practice. Also, one may consider applying “pseudo-optimal” temperatures to reduce the stresses below certain threshold levels. Another constraint is that the laminate should not fail by any mechanisms other than delamination (e.g., by first-ply-failure criterion) for certain combinations of the applied mechanical and thermal loads. In these cases, to obtain temperature fields that are more feasible and less severe in distributions, a proper set of constraints must be introduced in the optimization procedure.

In a strict sense, one must perform a delamination failure analysis to determine the actual amount of thermal control authority. For simplicity, however, one may introduce the following criteria for the second and third constraints to assure that the laminate is free of the delamination and other failure mechanisms.

\[ |\sigma_{ij}|(\Delta T = 0)| < 1, \]  
(18)

\[ N_x < P_y, \]  
(19)

where \( \sigma_{ij} \) are the interlaminar stresses as functions of both the applied load \( N_x \) and thermal gradients \( \Delta T \), and \( P_y \) is the failure load of the laminate as a function of the thermal gradients. See Ref. [9] for other types of constrained optimization problems.

4.5. Numerical results

The optimal algorithms outlined in this paper have been applied to two different Graphite/Epoxy laminates, [90/0], [± 45], lay-ups. The material properties of each ply are:

\[ E_{11} = 138 \times 10^6 \text{ kN/m}^2, \]
\[ E_{22} = E_{33} = 14.5 \times 10^6 \text{ kN/m}^2, \]
\[ G_{12} = G_{13} = G_{23} = 5.86 \times 10^6 \text{ kN/m}^2, \]
\[ v_{12} = v_{13} = v_{23} = 0.21, \]
\[ \mu_L = 0.2 \times 10^{-6} \, 1/^\circ F, \]

\[ \mu_T = 16 \times 10^{-6} \, 1/^\circ F, \]

\[ t = 0.135 \text{ mm (ply thickness)}. \]

For a given uniaxial load, optimal temperature gradients were obtained using the global optimization with \( \alpha = \beta = 1 \). For validation, the optimal temperature results were then used to calculate interlaminar stresses under the combined thermomechanical loading. For this purpose, a global–local-thermo-mechanical stress program called INTGLTM was utilized [12]. The main purpose of this section is to demonstrate the validity of the optimization algorithms. For a fuller presentation of results and discussion, see Ref. [9].

The first case is the \([90/0]_s\) laminate. It is recalled that for a cross-ply or an angle-ply case, both the global and global–local optimization yield the same uniform optimal temperatures. Furthermore, for a Graphite/Epoxy cross-ply laminate, it was found that a tensile load requires a negative \( \Delta T \) for the entire laminate to reduce the interlaminar stresses \( \sigma_{33}, \sigma_{23} \), with a higher magnitude of temperature for a higher load level. Thus, for \( N_x = 100 \, \text{kN/M} \) which represents 65\% of the maximum failure load of the laminate, \( \Delta T = 30^\circ F \) will completely eliminate all the interlaminar stresses. The first two figures, Figs. 2, 3 show the chordwise distributions, predicted by the program INTGLTM, of normalized \( \sigma_{33} \) and \( \sigma_{23} \), after applying uniform temperature gradients of \( 10^\circ F \) and \( 25^\circ F \) along with the mechanical loading, \( N_x = 100 \, \text{kN/M} \). Note that the latter temperature difference is closer to the \( \Delta T_{\text{optimal}} = 30^\circ F \). For this temperature difference the stresses have been reduced approximately by a factor of six, but have not disappeared completely.

The next case is the \([\pm 45]_s\) laminate. Contrary to the \([90/0]_s\) laminate, for the angle-ply laminate it is necessary to apply a negative \( \Delta T \) to cancel the mechanical extension effect to reduce the stress levels. Thus, for \( N_x = 55 \, \text{kN/M} \) which represents 49\% of the maximum failure load of the laminate, \( \Delta T = -200^\circ F \) will completely eliminate the interlaminar stress \( \sigma_{13} \). However, it must be
emphasized that this temperature value could mean either heating or cooling depending on the zero residual temperature and the ambient temperature. Fig. 4 shows the chordwise distributions of the normalized $\sigma_{13}$ near the free edge, after applying uniform temperature gradients of $-80^\circ F$ and $-150^\circ F$ along with the mechanical loading, $N_x = 55 \text{kN/M}$. In general, for a laminate other than a cross or an angle-ply, optimal temperature distributions are not uniform throughout the laminate thickness, and various optimization schemes result in different stress distributions. Therefore, depending upon the most critical interlaminar stress component in a specific laminate, a proper algorithm with a proper set of weightings need to be utilized.

5. Experimental program

There are three primary objectives to the experimental program: (1) to demonstrate the ability to accurately measure interlaminar normal stress due to a mechanical load, (2) to demonstrate the ability to accurately measure interlaminar normal stress due to a thermal load, and (3) to experimentally prove the feasibility of active control of interlaminar stresses using a thermal load.

During the course of the experiments, several tests have been conducted on three test articles. The first was a preliminary test article, intended primarily to test our measurement method and to identify problems with our test procedure. This specimen was also used for the testing outlined in Ref. [10]. The second is a validation test article and is intended to rigorously validate the assumptions made for the experimental program. This article is a $[0/0/0]$ lay-up, using the same sensor methodology as for the final test. An ideal orthotropic laminate should have no interlaminar stresses. By measuring the stresses experimentally in this specimen, it is possible to quantify the error from ideal of our test results. The final test article provides the results needed to demonstrate
the ability to control interlaminar normal stress. This specimen, as well as the associated sensor methodology, is discussed in detail below.

5.1. Piezoelectric wafers for strain measurement

After considering several types of sensors, piezoelectric wafers were selected as embedded strain sensors. The piezoelectric sensor, when bonded to a structure, produces a voltage potential as a function of the structural strain. Piezoelectric sensors have several advantages over more traditional methods of strain measurement. First, a properly poled piezoelectric responds to forces in all three orthogonal directions. When embedded in a structure, this piezoelectric sensor will produce a charge due to strain in three directions, but not to shear strain. This response is not available in a strain gauge, which can only sense in-plane deformations. Second, the piezoelectric sensor can be completely embedded within the structure. In contrast, eddy-current and Moir methods can only determine a strain distribution on a surface. To minimize the sensor’s effect on the test article stress distribution, sensors which are much thinner than the ply thickness have been chosen. The sensors used in our experiments are 0.006 in thick PZT 5A, embedded in a bonding layer that is approximately 0.01 in thick. Each ply in the test is 0.05 in thick. Third, piezoelectric sensors could eventually be incorporated into an on-line interlaminar stress measurement system. Sensing techniques using acoustic waves or visual measurement are essentially relegated to experimental or off-line use. Finally, piezoelectric sensors produce a high voltage output for a low value of normal strain. Unfortunately, the use of piezoelectric sensors also involves significant difficulties to be overcome: First, piezoelectric sensors cannot be used to measure steady-state quantities. For any sensor-signal amplifier combination, there is a lower-frequency limit below which measurement accuracy decreases with decreased frequency. For steady state, there is no stable signal. Second, piezoelectric sensors suffer from the effects due to heating. A temperature change induces a voltage potential in the sensor independent of strain, adding an error to the signal.

5.2. Interlaminar sensing technique

Our experiments have been designed to counter the disadvantages of the piezoelectric sensors. For normal strain measurements, the piezoelectric sensors are mounted in pairs. One sensor is embedded in the structure at the measurement location, while the second is mounted to the surface directly above, as shown in Fig. 5. Each sensor is connected to a unity gain voltage follower circuit. Since there is no normal stress at the surface sensor, the interlaminar stress can be calculated from the difference in voltages from the two sensors. For a piezoelectric sensor/voltage follower combination, the voltage output from the amplifier is

\[ V = \frac{j \omega R_g C_s}{1 + j \omega R_g C_s} \left( \frac{Q_s}{C_s} \right), \]

where \( R_g \) is the resistance of ground in the voltage follower circuit, \( C_s \) the capacitance of the piezoelectric, which is determined by \( C_s = DA/t \), \( A \) the area of the sensor, \( t \) the piezoelectric wafer
thickness, $D$ the piezoelectric absolute permeability, $d_{31}, d_{33}$ are the piezoelectric electrical constants, $\omega$ the frequency of the loading, and $Q_s$ the charge induced in the sensor by the applied strain.

If one treats strain throughout the sensor as equal to the strain in the structure it is bonded to, then

$$Q_s = E_{\text{piezo}} A [d_{31}(\varepsilon_{11} + \varepsilon_{22}) + d_{33}\varepsilon_{33}],$$  \hspace{1cm} (21)

where $E_{\text{piezo}}$ is the Young's modulus of the sensor. For frequencies above the voltage follower cut-off frequency, the frequency effects become negligible, and the voltage equation can be approximated as

$$V = \frac{Q_s}{C_s} = E_{\text{piezo}} t \frac{E_{\text{piezo}}}{D} [d_{31}(\varepsilon_{11} + \varepsilon_{22}) + d_{33}\varepsilon_{33}].$$  \hspace{1cm} (22)

For this reason, our measurements for mechanical loading are taken during a cyclic load at a frequency higher than the cut-off frequency. For the voltage follower circuit constructed, the cut-off frequency is 0.25 Hz. All mechanical loading measurements were taken at 2 Hz.
If the in-plane strains can be assumed to vary little through the thickness of the test specimen, their effects can be subtracted out. The difference in voltage between the two sensors is

\[(V_{\text{embedded}} - V_{\text{surface}}) = \frac{E_{\text{piezo}}}{D} d_{33} e_{33}.\] \hspace{1cm} (23)

Using the Young’s modulus for the resin, and assuming that the sensor strain represents the resin strain, one can calculate the interlaminar normal stress as

\[\sigma_{33} \approx E_{\text{resin}} e_{33} = \frac{E_{\text{resin}} D (V_{\text{embedded}} - V_{\text{surface}})}{E_{\text{piezo}} d_{33}}.\] \hspace{1cm} (24)

However, it is emphasized that the above approximation can only be justified by more exact analysis such as a finite element method. For the thermal loading, the temperature cannot be controlled cyclically at a frequency above the voltage follower cut-off frequency. The test article can be heated rapidly, but can not be cooled quick enough to consider it sinusoidal. Therefore, the best option is to heat below this cut-off frequency, and use the principle of superposition to add the effects of mechanical and thermal loading. For loading far below the cut-off frequency, the imaginary terms in the voltage equation predominate, and the equation can be taken as

\[V = R_g C_s \left( \frac{Q_{33}}{C_s} \right)\] \hspace{1cm} (25)

and following a logic similar to that above, it is found that

\[\dot{\sigma}_{33} = E_{\text{resin}} \dot{e}_{33} = \frac{E_{\text{resin}} D (V_{\text{embedded}} - V_{\text{surface}})}{E_{\text{piezo}} d_{33} R_g C_s}.\] \hspace{1cm} (26)

The value of stress at any time can be found through Euler integration.

**5.3. Experimental procedure**

There are several assumptions involved in the above derivation of the sensor equations. In addition, it was also assumed that all voltage is caused by specimen strain. It is imperative that the experiment be conducted such that the assumptions are as valid as possible.

As stated before, the most restrictive experimental protocol is caused by the fact that the piezoelectric sensors can only respond to time-varying strains. For mechanical loading, this is not a difficult obstacle: the mechanical load is applied at a frequency above the cut-off frequency, but low enough to neglect dynamic effects. For these tests, a mechanical loading is applied at 2 Hz, using an Instron servo-hydraulic test machine. However, the thermal gradient is more difficult to apply. As the temperature approaches steady state, the signal-to-noise ratio reduces to an unusable level. For this reason, all of our results are for dynamic thermal loads for which the specimen is quickly heated.

A more difficult assumption is to insure output is the due only to stress. As mentioned before, there is a piezoelectric effect which causes a voltage due to the temperature increase of the sensor. Although the voltage follower has been chosen as a signal amplifier for its low response to temperature effects, it is not possible to eliminate this error completely. To minimize the error, each
sensor in the comparison pair should be at the same temperature at any given time. This requirement limits the thermal gradients which can be applied with reliable results. For the gradient not to cause a significant pyroelectric error, the article must either be heated at a ply between the two sensors, or from the edge. For this series of experiments, both types gradients were tested.

5.4. Test specimen

The test article is shown in Fig. 6. It is a Epoxy-glass [0/90]_S lay-up with an embedded ni-chrome wire. The ni-chrome wire is connected to a power source to heat the test article from the center. For heating from the edge, another ni-chrome wire is held at the edge. Each ply is 0.05 in thick, with a 0.01 in epoxy bonding layer. The bonding layer is required in order to embed the 0.006 in PZT-5A piezoelectric sensors plus wiring terminals and leads. Each ply is cured in the autoclave, but the plies are bonded with a room temperature cure epoxy to avoid damaging the sensors. Test article width is 0.8 in. The composite specimen is fitted with four primary sensor pairs, so that the interlaminar stress can be calculated at four locations across the width. In addition, two back-up pairs are installed in case of primary sensor failure. The voltage comparisons are done within the circuit, and only the voltage difference is read to the oscilloscope. The oscilloscope is
a digital oscilloscope and, therefore, all of the data for an entire test heating can be recorded on one screen. One sensor circuit is required for each sensor pair.

5.5. Experimental results

The results for mechanical loading are shown in Fig. 7. Also presented are analytic results predicted by INTGLTM. Measurements were taken at the mid-surface (90/90 interface) at varying levels of load, 10–60 lbs, then normalized to nominal stress and averaged. There is a very good agreement between the analysis and the experiment at the three transverse locations shown in the figure. This correlation is close enough to support our sensor assumptions. For the thermal loading, readings were taken for a constant axial load of 60 lbs. Linear superposition is assumed, and the stress due to the thermal gradient are added to that of the mechanical load. The heat source is at the edge of the test article. The thermal gradient is a step decent from the edge, with almost no temperature change at the center. The temperature gradients were measured during subsequent tests, and is shown in Fig. 8 at several time intervals. The measured interlaminar stress is shown in Fig. 9. The stress near the edge is significantly reduced as predicted by our theory, but near the center it is increased. This may be attributed to the nonzero temperature gradient and transient heat transfer effect. However, since the edge is a singularity in the analytical modeling and an important failure location for free edge composite plates, the benefits of the edge stress decrease outweigh the increase at the center.

6. Conclusion

In this research, feasibilities of applied thermal gradients as the actuation mechanism for control of interlaminar stresses near the straight free edges of composite laminates have been examined both analytically and experimentally. The optimal static temperature gradients are in the through-thickness direction, and are obtained by minimizing objective functions which are strictly dependent upon the far-field properties. It was analytically shown that for cross-ply or angle-ply
laminates uniform temperature rises or drops can eliminate all of the interlaminar stresses, provided that the applied load is within the range of thermal control capability. The nonuniformity of the results, however, is unavoidable for other types of laminates. In the experiment, it has been shown that a pair of piezoelectric sensors can be efficiently used to measure the interlaminar normal stresses due to mechanical or thermal loading. The experimental results, which include effects of dynamic loading, confirms the general trend anticipated by the analysis. In view of the fact that a realistic active thermal control could be performed only in a transient fashion, the present experimental procedure suggests a new direction for the analysis. That is, future analytical research must include the transient effects including heat transfer and time-varying loading. On the experimental side, future work should focus on improving the viability of the interlaminar stress determination using piezoelectrics and its applicability to more complicated structures and load patterns.

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References


