EVALUATION OF K-FACTORS AND WEIGHT FUNCTIONS FOR 2-D MIXED-MODE MULTIPLE CRACKS BY THE BOUNDARY ELEMENT ALTERNATING METHOD

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Abstract—The concept of the Schwartz-Neumann alternating method, in conjunction with the boundary element method to solve for the stresses in an untracked body and an analytical solution for an embedded 2-D crack subjected to arbitrary crack face loading in an infinite domain, is used to determine the mixed-mode K-factors and weight functions for cracks in finite bodies. Situations of edge-cracks, as well as multiple cracks, all under mixed mode loading, are considered. The boundary element method is better suited for these problems since, pointwise evaluation of stresses at the location of the crack in the uncracked body is more accurate and simple once the tractions and displacements on the boundary are determined. It is expected that the above method would yield highly accurate results in the least expensive way, even compared to the finite element alternating method.

1. INTRODUCTION

In a majority of practical cases, cracks in structures may be approximated to be two dimensional straight, embedded or edge cracks. Cost-effective methods are needed to evaluate the fracture parameters such as the stress intensity factors for such cracks. For a repetitive analysis of the same structural and crack shapes, under varied loading situations, the evaluation of weight functions is also important. In general, linear boundary value problems for a body with cracks can be solved by several numerical methods such as the singularity finite element method[1, 2], the boundary integral equation technique[3], and the Schwartz-Neumann alternating technique[4, 5] in conjunction with either the finite element method (FEM) or the boundary element method (BEM). It is generally recognized that the alternating technique may be the simplest and the most cost-effective technique of the methods described above.

For linear elasto-static boundary value problems, the boundary element method has a major advantage over the finite element method, in that it reduces the dimensionality of the problem. Unlike in the finite element method where the stresses at any point in the domain are obtained through the differentiated shape functions associated with nodal displacements at the element level, the boundary element method allows us to evaluate the stresses exactly by differentiating the integral equation for the displacement field in the interior of the domain. Hence the BEM gives rise to a more accurate stress evaluation than the FEM. The BEM requires only a discretization of the boundary as compared to the discretization of the whole domain in the FEM. Hence from a practical point of view where preprocessing plays an important factor in a routine evaluation of fracture mechanics parameters in elastic structures, the BEM stands to have an advantage over the FEM.

This paper presents a methodology for evaluating the fracture mechanics parameters (such as K-factors and weight functions) for two-dimensional elastic structures with embedded or edge straight cracks by the boundary element alternating method. The governing equations and the solution procedures[6, 7] for the problem of arbitrary normal as well as shear loading on the face of the crack embedded in an infinite isotropic domain, is presented. The evaluation of K-factors for the above mentioned problem, and the far field stresses and displacements are considered here. Next, a brief description of the boundary element method is presented. The BEM facilitates a pointwise evaluation of stresses at the location of the uncracked body once the tractions and displacements on the boundary are determined.

A general boundary element alternating solution procedure for multiple embedded cracks is presented. If necessary the above mentioned procedure can be restricted to the case of a single crack.
as well. The case of an edge crack is considered next. As before this procedure could be extended to multiple embedded and edge cracks. An elegant as well as an accurate method of computing weight functions [8, 9] on the boundary by the virtual crack extension technique [10, 11] for arbitrary mixed mode loading is also presented. Numerical results for certain selected illustrative problems which encompass the above mentioned areas are presented. It is shown that the boundary element alternating method gives rise to highly accurate results in a less expensive manner when compared to the earlier mentioned solution techniques. The results of the BEM alternating method is compared with those given in the handbook by Rooke and Cartwright [12].

2. ANALYTICAL SOLUTION FOR AN EMBEDDED CRACK

In an alternating method [4, 5], the tractions at the location of the crack in the uncracked body are erased, so as to create the traction-free crack as in the given problem. The key step in this iterative approach [4, 5] is the analytical solution for an embedded crack, subjected to arbitrary crack-face tractions, in a plane infinite body. This solution procedure is briefly discussed below.

The basic equations for plane elasticity have the general solution [13] as follows:

\[ \mathbf{z} = (x + iy) \]
\[ 2\mu(u + iw) = K \Omega(z) - z \Omega'(z) - \omega(z) \]
\[ \tau_{xx} + \tau_{yy} = 2\Omega(z) + 2\Omega'(z) \]
\[ \tau_{yy} - i\tau_{xy} = \Omega(z) + \Omega'(z) + z \Omega''(z) + \omega'(z). \]

Here \((x, y)\) refers to the spatial location, \((u, v)\) refers to the displacement components in the \(x\) and \(y\) directions, respectively, and \(\tau_{xx}, \tau_{yy}, \tau_{xy}\) are the stress components at \((x, y)\). The complex potentials \(\Omega(z), \Omega'(z)\) and \(\omega'(z)\) are holomorphic functions in the region occupied by the body. \(\mu\) is the shear modulus and \(K\) takes the following form:

\[ K = 3 - 4\nu \quad \text{for plane strain} \]
\[ = \frac{3 - \nu}{1 + \nu} \quad \text{for generalized plane stress}. \]

Suppose a line crack on \(y = 0, \, |x| \leq a\) in an infinite plane is inflated by equal and opposite tractions over the face of the crack, then this problem could be solved by [6] choosing \(\omega(z)\) such that

\[ \omega(z) = \Omega(z) - z \Omega'(z). \]

The problem reduces to determining the potential \(\Omega(z)\), holomorphic in the plane cut along \(y = 0, \, |x| \leq a\), across which it satisfies

\[ \Omega'^+(t) - \Omega'^-(t) = -[p(t) + is(t)], \quad |t| \leq a. \]

Here \((+)\) denotes a region occupying the upper half plane \(y > 0\) and \((-)\) denotes a region occupying the lower half plane \(y < 0\) and \(p(t)\) and \(s(t)\) are the normal and shear tractions applied on the crack face, respectively. The applied tractions can be approximated in the form

\[ p(t) + is(t) = -\sum_{n=1}^{N} b_n U_{n-1}(t), \quad |t| \leq a \]

where \(U_{n-1}(t)\) is the Chebyshev polynomial of the second kind and is defined as

\[ U_n(t) = \frac{\sin[(n + 1)\theta]}{\sin \theta}, \quad t = a \cos \theta. \]
It could be easily shown that

\[ 2\Omega(z) = \sum_{n=1}^{N} b_n G_{n-1}(z) \]  
\[ 2\Omega(z) = \sum_{n=1}^{N} b_n \frac{R_n(z)}{n}. \]  

Here \( N \) is the number of Chebyshev terms taken into consideration, and \( R_n(z) \) and \( G_{n-1}(z) \) are defined as

\[ R_n(z) = a \{ z - (z^2 - 1)^{1/2} \}^n \]  
\[ G_{n-1}(z) = -(z^2 - a^2)^{-1/2} R_n(z) \]  

where \( z_i = z/a \).

The stresses on \( y = 0, |x| \geq a \) are given by

\[ \tau_{yy} - i\tau_{xy} = -\text{sgn}(x)(x^2 - a^2)^{-1/2} \sum_{n=1}^{N} b_n R_n(x). \]  

The stress-intensity factors \( K_i \) and \( K_{ii} \) at \( x = +a \) are defined as

\[ K_i = \lim_{x \to a} \left[ \sqrt{2\pi(x - a)} \tau_{yy} \right] \]  
\[ K_{ii} = \lim_{x \to a} \left[ \sqrt{2\pi(x - a)} \tau_{xy} \right]. \]  

Making use of eqs (14–16) we obtain

\[ K_i - iK_{ii} = -\sqrt{\pi a} \sum_{n=1}^{N} b_n. \]  

Similarly at \( x = -a \) the stress-intensity factors are

\[ K_i - iK_{ii} = \sqrt{\pi a} \sum_{n=1}^{N} (-1)^n b_n. \]  

Hence the key to an accurate computation of \( K \)-factors depends upon the evaluation of \( b_n \). From eq. (8) \( b_n \) takes the following form

\[ b_n = -\int_{-1}^{+1} [P(at') + is(at')] U_{n-1}(at') \, dt', \quad t' = \frac{t}{a}. \]  

The above mentioned integral may be solved by the related Gauss–Chebyshev integration formula[14]

\[ \frac{1}{\pi} \int_{-1}^{+1} f(t)(1 - t^2)^{1/2} \, dt \approx \sum_{k=1}^{n} \frac{(1 - t_k^2)f(t_k)}{n + 1} \]  

where

\[ t_k = \cos(k\pi/(n + 1)), \quad k = 1, \ldots, n. \]
Here \( n \) is the number of Chebyshev terms used in the evaluation of the integral in eq. (20a). The stresses and displacements at any point in the domain could be evaluated by eqs (1–4) and eqs (6, 8, and 9) once \( b_n \) is determined.

3. THE BOUNDARY ELEMENT METHOD

The BEM is used in the present approach, to solve for the stresses in the uncracked body only. The BEM formulation could be derived by taking the weak form of the equilibrium equation where, the test function is taken as the fundamental (displacement) solution of the Navier equations. In the absence of body forces, the BEM equations become

\[
C_{ij} u_i = \int_{\Gamma} \left[ U^*_j t + T^*_j u_j \right] \mathrm{ds}
\]

where \( C_{ij} = \frac{1}{2} \delta_{ij} \) on a smooth boundary and \( \delta_{ij} \) for a point in the interior. \( U^*_j \) and \( T^*_j \) are the displacement and traction kernels for the Navier's problem. The explicit forms of these kernels in the context of planar problems are given in [15]. The stresses at a point in the interior of the domain are obtained by differentiating eq. (21) at the interior to obtain strains and then Hooke's law. Thus

\[
C_{ij} = \int_{\Omega} \left[ P_{ik} t_k + Q_{ik} u_k \right] \mathrm{ds}.
\]

Here, \( P_{ik} \) and \( Q_{ik} \) are functions of derivatives of \( U^*_j \) and \( T^*_j \). It should be noted here that \( P_{ik} \) and \( Q_{ik} \) do not involve singular expressions. The reader should refer to [15] for a detailed numerical implementation of this method in the context of two-dimensional problems.

4. THE SCHWARTZ–NEUMANN ALTERNATING TECHNIQUE

By employing the Schwartz–Neumann alternating method (herein after called the alternating method), it is possible to obtain the stress-intensity factors for an embedded or edge-crack in a finite solid. Since no general solution exists for a finite body subject to arbitrary externally applied tractions, a boundary element solution is used to evaluate stresses at the location of the crack. The infinite body with an embedded crack has a solution which is valid for an arbitrary distribution of tractions on the crack-face. The procedure for the evaluation of \( K_i \) and \( K_{ii} \) factors in a finite body can be summarized as follows:

(1) Solve the uncracked body under the given external loads by BEM. The uncracked body has the same geometry as the given problem except for the cracks.

(2) Using the BEM solutions the stresses at the location of the crack are computed. The tractions present on the boundary are called \( T_1 \).

(3) Compare the residual stresses calculated in step (2) with the permissible stress magnitude. In the present study one per cent of the maximum external applied stress is used for the permissible stress magnitude.

(4) Reverse the stresses on the first crack from the analytical solution. Find the traction on the boundary and stresses on all other crack locations. Find also the \( K \)-factors for the first crack.

(5) Let's consider the generalised \( n \)th crack. For the \( n \)th crack add the contribution of stresses at the \( n \)th crack location by BEM due to \( T_1 \) and the stresses due to the analytical solution of all the previous \((n - 1)\) cracks. Reverse these cumulative stresses at the \( n \)th crack location. Find the boundary tractions and stresses at all other crack locations by the analytical solution for an infinite domain. Find also the \( K \)-factor for the \( n \)th crack. Carry out this procedure for all cracks except the first crack.

(6) Add all the boundary traction contributions in steps (4–5). This will be reversed and redefined as the present boundary traction.

(7) Add all stresses at the corresponding crack locations from steps (4–5) which have not been reversed.

(8) Repeat steps (2–7) until the criterion described in step (3) is satisfied.

(9) Add the \( K \)-factor solutions from all iterations for the corresponding cracks.
Evaluation of K-factors and weight functions

In the infinite domain solution, the crack face tractions are defined on the entire embedded crack. It is thus necessary, in edge crack problems, for tractions to be defined over the entire crack plane, including the fictitious portion of the crack which lies outside the finite body (Fig. 1). The actual distribution chosen for the fictitious tractions on the crack face defined outside the finite body will only affect the character of convergence[4, 5]. The treatment of edge cracks can be described as follows. The solution procedure for an edge crack (Fig. 1) is the same as the one described for an embedded crack except for minor differences. As before, the stresses at the crack location $AO$ can be evaluated by the BEM. By reversing the stresses to satisfy the boundary conditions on the crack $AO$, the tractions on the crack face can be evaluated. Now a mirror image of the half crack $AO$ and the tractions acting on the crack face are extended to obtain a full crack $AB$. Using the analytical solution for an embedded crack in an infinite domain, the stresses can be evaluated at the boundary of the crack specimen. The rest of the algorithm remains the same as the case of an embedded crack.

5. EVALUATION OF WEIGHT FUNCTIONS

In the practical application of fracture mechanics, the determination of weight functions is often more advantageous than the calculation of stress-intensity factors alone. The use of weight functions can obviate the repeated computer calculations of the stress-intensity factors due to load-independent characteristics of weight functions for a given crack geometry. As shown by [9, 10, 11] the weight functions for a mixed mode loading situation could be written as follows:

$$h_{ii}^{(i)} = \frac{H}{2K_{i}} \frac{\partial u_{i}^{(i)}}{\partial a}.$$  \hspace{1cm} (23)

Here $H$ is the effective modulus, $K_{i}$ and $h_{ii}^{(i)}$ are mode I and II stress intensity factors and weight functions (for $i = 1, 2$) respectively. $u_{i}^{(i)}$ are the displacements for mode I and II defined as follows:

$$\begin{align*}
\{u_{i}^{(1)}\} &= \frac{1}{2} \begin{pmatrix} u_{1} + u_{2} \\ u_{2} - u_{1} \end{pmatrix} \\
\{u_{i}^{(2)}\} &= \frac{1}{2} \begin{pmatrix} u_{1} - u_{2} \\ u_{2} + u_{1} \end{pmatrix}.
\end{align*}$$  \hspace{1cm} (24a)

and

$$\begin{align*}
\{u_{i}^{(1)}\} &= \frac{1}{2} \begin{pmatrix} u_{1} - u_{2} \\ u_{2} + u_{1} \end{pmatrix} \\
\{u_{i}^{(2)}\} &= \frac{1}{2} \begin{pmatrix} u_{1} + u_{2} \\ u_{2} - u_{1} \end{pmatrix}.
\end{align*}$$  \hspace{1cm} (24b)
Here $u'$ denotes the value of displacement at a point $p'$ that is a mirror image of the point $p$ with respect to the crack axis. With this weight function concept, the stress intensity factor $K'_i$ and $K''_i$ for any surface traction $t^{(0)}$, can be expressed as an integral of a worklike product as

$$K'_i = \int t^{(0)} h^{(0)}_{i} ds.$$  \hspace{1cm} (25)

When applying the VCE (virtual crack extension) technique\[11, 12, 13\] to the above class of problems, eq. (23) can be written as:

$$h^{(0)}_{i} = \frac{H}{2K''_i \Delta a} [u^{(m)}_i(2a + \Delta a) - u^{(m)}_i(2a)].$$ \hspace{1cm} (26)

The aim here is to separate the difference in displacements into a symmetric part (mode I) and an antisymmetric part (mode II). The displacements to be evaluated by the alternating method are basically the sum of all the displacements in every iteration. The following solution procedure is adopted:

(1) To find the difference in displacements (for mode I and II) between the original crack and the extended crack (Fig. 2), we do not have to consider the displacements when tractions are evaluated initially at the uncracked location. (This is because cases (1) and (2) (Fig. 2) will yield the same displacements and hence the difference in displacements will be zero.)

(2) Next, reverse the tractions and decompose them into normal and shear traction components on the crack face.

(3) Consider the above scenario in step (3) as though we have two problems: (i) one with only normal loading on the crack face, which gives rise to a mode I problem (ii) the other with only shear loading on the crack face, which gives rise to a mode II problem.
The alternating method iteration loop is carried out separately, for the above mentioned problems till the tractions become negligible. It should be noted here that the influence matrices in the BEM formulation are computed only once, for both problems under consideration.

(4) Compute the sum of mode I and II displacements by the above method for the original crack and the extended crack (cases 1 and 2, Fig. 2).

(5) Compute also the mixed mode K-factors at the extended crack tip by the alternating method described in the previous section.

(6) Compute the mixed-mode weight functions at the nodal locations by eq. (26).

A distinct advantage of the present boundary element alternating procedure for determining the weight functions is that a single boundary element model of the uncracked surface is used to find the weight functions for mode I and II by a single virtual crack extension.†

6. NUMERICAL RESULTS

As for the BEM formulation, a linear variation of displacements and tractions are prescribed on straight sided boundary segments. All integrations of kernel functions are done in closed form, which obviates the need for Gaussian quadrature. The jumps in tractions at the corners are modelled by placing double nodes.

In all problems considered different numbers of Chebyshev terms were employed for the analytical solution of the infinite domain. Comparisons are made from tables found in the "Compendium of Stress Intensity Factors"[12]. All computations were carried out on a MicroVax II computer. The Young's modulus was taken to be $10^6$ and the Poissons ratio to be 0.3.

6.1. Stress intensity factors

6.1.1. Mixed mode embedded crack. As shown in Fig. 3, a rectangular plate with a centered inclined embedded crack is considered. The plate is subjected to uniform tension as shown. The $K_I$ and $K_{II}$ are tabulated below (with CPU run times) at both crack tip locations (1) and (2) for different number of Chebyshev terms of the analytical solution. Two different boundary mesh discretizations (of equal segments) are considered and comparisons are made from the results obtained from [12]. The agreement is good between the tabulated results and [12]. Moreover, the results appear quite good even for coarse boundary meshes.

†Alternatively however, instead of obtaining the partial derivative of $(\partial u_0/\partial a)$ by a finite difference algorithm (eq. 26), one could analytically differentiate eq. (2) and making use of eqs (10) and (11) obtain a closed form expression for $(\partial u_0/\partial a)$ for the infinite body. The alternating technique in conjunction with the closed form evaluation of $(\partial \sigma_{ij}/\partial a)$ (from eqs 3 and 4) for the infinite body could be made use of, for the evaluation of $(\partial u_0/\partial a)$ for the finite body. By making use of the two load configuration technique described in [16,17] the weight functions could be evaluated. The authors feel however, that the present method would yield more efficient results for weight functions than the method of analytical differentiation.
Results from tables[12]: $K_i = 1.41$, $K_{II} = 1.31$.

<table>
<thead>
<tr>
<th>Nodes = 40</th>
<th>NCHEB = 3, CPU = 29.92 s</th>
<th>NCHEB = 10, CPU = 43.7 s</th>
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<tbody>
<tr>
<td>at (1)</td>
<td>at (2)</td>
<td>at (1)</td>
</tr>
<tr>
<td>$K_i$</td>
<td>1.453</td>
<td>1.457</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>1.374</td>
<td>1.372</td>
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<table>
<thead>
<tr>
<th>NODES = 80</th>
<th>NCHEB = 10, CPU = 153.2 s</th>
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</thead>
<tbody>
<tr>
<td>at (1)</td>
<td>at (2)</td>
</tr>
<tr>
<td>$K_i$</td>
<td>1.453</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>1.377</td>
</tr>
</tbody>
</table>

6.1.2. *Mixed mode edge crack.* As shown in Fig. 4, a rectangular plate with an inclined edge crack is considered. The plate is subjected to uniform tension as shown. The number of Chebyshev terms used are for the combined physical crack and the extended imaginary crack portions. The $K_i$ and $K_{II}$ are tabulated (with CPU run times) for four progressively refined meshes of equal segment lengths. The agreement is good between [12] and the tabulated results.

| From tables[12]: $K_i = 2.49$, $K_{II} = 1.26$. |
|-----------------|----------------|----------------|
| Nodes           | 39             | 58             |
| NCHEB           | 10             | 20             |
| $K_i$           | 2.204          | 2.423          |
| $K_{II}$        | 1.311          | 1.292          |
| CPU (s)         | 34.8           | 78.8           |

6.1.3. *Edge crack near a circular hole.* As shown in Fig. 5, an edge crack is located at a part circular cutout in a square plate which is subjected to uniform tensile loading. A 63 noded mesh is considered here. The number of Chebyshev terms used for the analytical solution is 20. The length of the square plate $"l"$ is progressively increased so that $K_i$ factors obtained, could be compared to the results of the infinite domain solution found in [12]. The $K_i$ factor results converges to that found in[12] as $l$ is progressively increased.

<table>
<thead>
<tr>
<th>$l$</th>
<th>20</th>
<th>40</th>
<th>80</th>
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<tbody>
<tr>
<td>$K_i$</td>
<td>3.966</td>
<td>3.493</td>
<td>2.425</td>
</tr>
<tr>
<td>CPU (s)</td>
<td>106.7</td>
<td>104.5</td>
<td>104.2</td>
</tr>
</tbody>
</table>

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Fig. 4. Evaluation of stress intensity factors for a mixed mode edge crack.

Fig. 5. Evaluation of stress intensity factors for an edge crack near a circular hold.
6.1.4. **Multiple cracks.** A square plate loaded in tension with two embedded cracks as shown in Fig. 6 is considered here. Due to the inherent symmetry present the crack tips denoted by (A) will have the same $K$-factors. The same applies to the crack tips (B). Two mesh discretizations of 40 and 80 nodes are considered here. The results presented are for 10 Chebyshev terms of the analytical solution. The results are compared with the infinite domain solution found in [12]. The agreement is good between [12] and the tabulated results.

<table>
<thead>
<tr>
<th>NODES</th>
<th>40</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_I$</td>
<td>$K_{II}$</td>
</tr>
<tr>
<td>at (A)</td>
<td>1.73</td>
<td>-0.197</td>
</tr>
<tr>
<td>at (B)</td>
<td>1.87</td>
<td>-0.068</td>
</tr>
<tr>
<td>CPU (s)</td>
<td>38.5</td>
<td>173.8</td>
</tr>
</tbody>
</table>

**Infinite Solution from [12]**

<table>
<thead>
<tr>
<th></th>
<th>$K_I$</th>
<th>$K_{II}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>at (A)</td>
<td>1.77</td>
<td>0.216</td>
</tr>
<tr>
<td>at (B)</td>
<td>1.86</td>
<td>-0.0813</td>
</tr>
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</table>

6.2. **Weight functions**

The mixed mode weight functions are computed here by a single virtual crack extension. In order to test the load independent character of the weight functions, the $K$-factors obtained through weight functions for some applied load is compared to those obtained by the alternating method.

6.2.1. **Embedded crack.** An inclined embedded crack in a rectangular plate subjected to uniform tension (as shown in Fig. 7a) is used to generate weight functions at nodal locations. The perturbed crack tip (i.e. the crack tip where the $K$-factors need to be evaluated) is denoted by (A). 63 nodes were used for the boundary discretization. 10 Chebyshyev terms were used for the analytical solution. The $K_I$ and $K_{II}$ generated at (A) for uniform loading through weight functions and the alternating method are tabulated below.
Alternating method  |  Weight function method  |  % Error
---|---|---
$K_I$ at (A)  |  1.453  |  1.455  |  0.13
$K_{II}$ at (A)  |  1.385  |  1.364  |  1.51

The $K$-factors are in excellent agreement between the two methods. Now, the above generated weight functions are used for a different load configuration as shown in Fig. 7(b). The $K$-factors so computed at (A) are compared with the $K$-factors generated by the alternating method for the same load configuration. The results are tabulated below.

Alternating method  |  Weight function method  |  % Error
---|---|---
$K_I$ at (A)  |  3.73  |  3.77  |  1.07
$K_{II}$ at (B)  |  3.57  |  3.52  |  1.12

The $K$-factors generated by the two methods are in excellent agreement with each other. Hence, the above mentioned results corroborate the load independent character of the weight functions generated by the present method.

6.2.2. Edge crack. An edge crack in a square plate subjected to uniform tensile loading (as shown in Fig. 8a) is used to generate weight functions at nodal locations. Unlike in the former case, virtual crack extensions are given to both the actual crack tip and the extended imaginary crack tip. This was found to increase the stability and accuracy of the weight functions. The $K_I$ obtained through the weight functions and the alternating method are tabulated below.

Alternating method  |  Weight function method  |  % Error
---|---|---
$K_I$  |  2.371  |  2.386  |  0.63

The above generated weight functions are used for a different load configuration as shown in Fig. 8(b). The $K_I$ so computed (through weight functions) is compared with the $K_I$ obtained through the alternating methods. The results are tabulated below.

Alternating method  |  Weight function method  |  % Error
---|---|---
$K_I$  |  2.82  |  2.79  |  1.06
Evaluation of \( K \)-factors and weight functions

Fig. (8a). Evaluation of weight functions for an edge crack in a rectangular plate subjected to uniform tension.

Fig. (8b). Evaluation of stress intensity factors through weight functions for a linearly varying load.

All the above mentioned results are in excellent agreement and hence confirms the load independent nature of weight functions for edge cracks by the present method.

7. CONCLUSION

The present method of evaluation of \( K \)-factors and weight functions for 2-D mixed-mode cracks by the boundary element alternating technique leads to accurate and efficient results. All influence matrices are computed only once and almost all problems were observed to converge within 3 to 5 iterations. The ability of the weight functions to evaluate \( K \)-factors for arbitrary loading renders the present method powerful.

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