CRACK-TIP PARAMETERS AND TEMPERATURE RISE IN DYNAMIC CRACK PROPAGATION

S. N. ATLURI, M. NAKAGAKI†, T. NISHIOKA‡ and Z.-B. KUANG§
Center for the Advancement of Computational Mechanics, School of Civil Engineering, Georgia Institute of Technology, Atlanta, GA 30332, U.S.A.

Abstract—This paper presents a discussion of (i) possible crack-tip parameters that may govern elastodynamic crack propagation in the presence of nonuniform temperature fields, material inhomogeneity, etc.; (ii) possible crack-tip parameters in elastoplastic dynamic fracture; and (iii) the temperature field near the crack tip due to the propagating inelastic zone in which heat is generated.

1. INTRODUCTION

The success of modern fracture mechanics is due, in large measure, to the celebrated work of Irwin in showing that for elastic materials, the crack-tip fields are governed by the so-called stress-intensity factor K. Likewise in elastic-plastic materials, the well-known work of Hutchinson–Rice–Rosengren shows that for stationary cracks in quasi-statically and monotonically loaded bodies of power-law-hardening materials, the stress and strain fields in the vicinity of the crack-tip under yielding conditions varying from small scale to full yielding are controlled by the Eshelby–Cherapanov–Rice J integral.

This paper is concerned with a general discussion of crack-tip parameters governing dynamic propagation of cracks in elastic as well as elastic-plastic materials. These crack-tip parameters are, in general, defined, say for two-dimensional problems, as integrals over a circular path Γe, with radius ε tending to zero. The integrand, which involves the crack-tip stress, strain, and displacement fields, is, in general, such that it is of 1/r type near the crack tip, which renders the integral over Γe to be of a finite magnitude. This crack-tip integral parameter is then sought to be represented equivalently as a far-field integral plus a "finite-domain integral," using the divergence theorem. This alternate representation is convenient for computational analysis of fracture problems. Under some special circumstances, the aforementioned finite-domain integral vanishes identically—thus making it possible to express the crack-tip integral parameter solely as a far-field contour integral. These special circumstances are clearly spelled out in this paper. In Section 2, we consider self-similar dynamic crack propagation in an elastic solid subject to nonuniform temperature fields, and when the material is considered to be nonhomogeneous. In Section 3, we discuss crack-tip parameters for dynamic crack propagation in elastic-plastic solids. In Section 4, we present a discussion of the temperature rise near the crack tip due to heat dissipation in the plastic zone near the propagating crack tip.

2. CRACK-TIP PARAMETERS IN SELF-SIMILAR CRACK PROPAGATION IN ELASTIC SOLIDS

We restrict our attention here to solids that are linearly or nonlinearly elastic. We consider the dynamic propagation of a crack in a self-similar fashion such that the crack length increases by da in time (dt), with a nonconstant velocity of propagation, a = da/dt. The energy release to the crack tip per unit of crack extension, denoted by G, is given from global energy considerations as

\[ G a = \frac{D W_{ex}}{D t} + \frac{D H}{D t} - \frac{D U}{D t} - \frac{D T}{D t}, \] (2.1)
wherein, on the right-hand side of (2.1), the first term represents the rate of external work done, the second the rate of heat input to the body, the third the rate of change of internal energy in the body, and the fourth the rate of change of kinetic energy in the body. The heat-flux relation is given by

\[
\frac{DH}{Dt} = - \int_S \mathbf{h} \cdot \mathbf{n} \, ds = - \int_{\nu_0} \nabla \cdot \mathbf{h} \, dv, \tag{2.2}
\]

where \( \mathbf{h} \) is the vector of heat flux, and \( \mathbf{n} \) is the unit outward normal to the boundary \( S \) of the body. We employ a fixed global Cartesian coordinate system such that \( X_i \) are the coordinates of a material particle before deformation, and \( u_i \) are Cartesian components of displacement. We use another “local” Cartesian system such that \( x_1 \) is locally normal to the crack border, \( x_2 \) is normal to the crack plane, and \( x_3 \) is locally tangential to the crack border. In eqn (2.2), \( \nabla = \frac{\partial}{\partial X_i} \). The components of \( \mathbf{n} \) are \( n_i \) in the \( x_i \) system and \( N_k \) in the \( X_k \) system. Also, in any thermomechanical process[1], we have

\[
\frac{DU}{Dt} = \frac{DW}{Dt} + \frac{DH}{Dt}, \tag{2.3}
\]

where \( \frac{DW}{Dt} \) is the stress power, and \( W \) is the density of total stress work. Note that, in general, when finite deformations are considered, \( \frac{DW}{Dt} = t_{ij} \dot{F}_{ji} \), where \( t_{ij} \) is the first Piola-Kirchhoff stress, and \( \dot{F}_{ji} \) is the rate of deformation gradient. On the other hand, if deformations are infinitesimal, \( \frac{DW}{Dt} = \sigma_{ij} \dot{e}_{ij} \), where \( \sigma_{ij} \) is the stress and \( \dot{e}_{ij} \) is the rate of strain. Using (2.3) in (2.1), we obtain

\[
G = \int_S t_i \frac{Du_i}{Da} \, ds + \int_{\nu} \bar{f}_i \frac{Du_i}{Da} \, dv - \frac{D}{Da} \int_{\nu} (W + T) \, dv, \tag{2.5}
\]

where \( V \) is the total volume of the cracked body, \( S_t \) is the surface of \( V \) where tractions are prescribed, \( \bar{f}_i \) are body forces, \( W \) is the density of total stress work (or equivalently, the strain energy density for the case of a nonlinear elastic material), and \( T = \frac{1}{2} \rho U_{ij} U_{ij} \).

Referring to Fig. 1 for nomenclature, we consider, for instance in a two-dimensional problem, an arbitrarily small loop \( \Gamma_e \) surrounding the crack tip, such that the “volume” (or area in a “plane” problem with unit thickness) inside \( \Gamma_e \) is \( V_e \) (including the crack tip). For instance, in two-dimensional problems, \( \Gamma_e \) may be considered to be a circle of radius \( \epsilon \) while, in three-dimensional problems, \( \Gamma_e \) may be considered to be a toroidal surface whose axis coincides with the crack front and whose cross-section is a circle of radius \( \epsilon \). If we consider the volume \( V - V_e \) which does not include the crack-tip, we see that the following equation of conservation of energy holds:

\[
0 = \int_{\Gamma_e} t_i \frac{Du_i}{Da} \, ds + \int_{V - V_e} \bar{f}_i \frac{Du_i}{Da} \, dv - \frac{D}{Da} \int_{V - V_e} (W + T) \, dv. \tag{2.6}
\]

If \( \mathbf{n} \) is the unit “outward” normal in the conventional sense, it is seen that the “external” boundary of \( V - V_e \) is \( S_e = \Gamma_e \) if the “sense” of \( \Gamma_e \) is as shown in Fig. 1. Using (2.6) in (2.5), it is seen that

\[
G = \lim_{\epsilon \to 0} \left\{ \int_{\Gamma_e} t_i \frac{Du_i}{Da} \, ds + \int_{V - V_e} \bar{f}_i \frac{Du_i}{Da} \, dv - \frac{D}{Da} \int_{V_e} (W + T) \, dv \right\}. \tag{2.7}
\]
Crack-tip parameters and temperature rise in crack propagation

Referring to Figs. 2(a) and 2(b), p₁ and P₂ represent the same material particle at times t and t + Δt, respectively, when the crack propagates by an amount da in time Δt. On the other hand, points p₁ and P₂ are located at the same distance and orientation (i.e. r, θ) as in Figs. 2(a,b)) from the crack tips at times t and t + Δt, respectively. The fundamental idea, in self-similar, elastodynamic crack propagation, is that the crack-tip fields are self-similar at times t and t + Δt, respectively, except that their intensities may differ. From this concept, we see that the changes in displacement, velocity, and stress at the same material particle, due to crack growth by da, are given by

\[
\begin{align*}
\Delta u_i(P_2) - u_i(p_2) &= [u_i(P_2) - u_i(p_2)] - [u_i(p_2) - u_i(p_1)] \\
&= (\partial u_i/\partial a - \partial u_i/\partial x_1) \, da, 
\end{align*}
\]  

(2.8a)  

(2.8b)

with similar relations for changes in \( u_i \) and \( t_{ij} \). It is important to note that \( x_1 \) is along the direction of (self-similar) crack propagation as in Figs. 1 and 2. Note that the terms such as \( \partial u_i/\partial a \) arise due to a change in the strength of the singularities of the crack-tip fields corresponding to an increase in crack length of da, while terms such as \( \partial u_i/\partial x_1 \) occur due to the translation of the crack-tip fields by da. If the material particle is at the external boundary of the specimen, it is easy to see from relations of the type (2.8) that

\[
\frac{\partial u_i}{\partial a} = \frac{\partial u_i}{\partial x_i} \text{ at } S_u; \quad \frac{\partial \sigma_{ij}}{\partial a} = \frac{\partial \sigma_{ij}}{\partial x_1} \text{ at } S_t.
\]

(2.9)
Now consider the third term on the right-hand side of (2.7):

\[ - \frac{D}{Da} \int_{v_{*}} (W + T) \, dv = - \int_{v_{*}} [(W + T)(P_2) - (W + T)(p_2)]/da \, dV. \] (2.10)

It is well known that in the case of the propagating crack, \( W \) and \( T \) may possess singularities of the order \( 1/r \) near the crack tip. Furthermore, \( \partial w/\partial a \) and \( \partial T/\partial a \) merely represent changes in the intensities of the singularities, while the order of their singularities is still \( 1/r \). However, since \( \partial W/\partial x_1 \) and \( \partial T/\partial x_1 \) may lead to “nonintegrable” singularities (which invalidate the application of the divergence theorem to terms of the type \( \int_{v_{*}} (\partial W/\partial x_1) \, dv \)), it is more proper to use the concept of “subtracting-out singularities” as illustrated in Fig. 2(c). Thus, (2.10) may be written as

\[ - \frac{D}{Da} \int_{v_{*}} (W + T) \, dv = - \int_{v_{*}} \frac{\partial}{\partial a} (W + T) \, dv + \int_{1*} (W + T)n_1 \, ds. \] (2.11)

Using eqns (2.8b) and (2.11), we rewrite (2.7) as

\[ G = \int_{1*} \left[ (W + T)n_1 - t_i \frac{\partial u_i}{\partial x_1} \right] \, ds - \left( \int_{v_{*}} \left[ \frac{\partial}{\partial a} (W + T) - f_{i-1} \frac{\partial u_i}{\partial a} - f_{i,1} u_i \right] \, dv - \int_{1*} t_i \frac{\partial u_i}{\partial a} \, ds \right) \] (2.12)

\[ = \lim_{\varepsilon \to 0} \int_{1*} \left[ (W + T)n_1 - t_i \frac{\partial u_i}{\partial x_1} \right] \, ds. \] (2.13)
Equation (2.13) follows from (2.12) since the second term of (2.12) vanishes in the limit \( \epsilon \to 0 \), due to the fact that \( \partial W/\partial a \) and \( \partial T/\partial a \) are still of order \( 1/r \) near the crack tip, \( t_i \) is of \( O(1/\sqrt{r}) \), \( \partial u_i/\partial a \) is \( O(\sqrt{r}) \). The result for \((\partial W)/\partial a\) is obtained by Atkinson and Eshelby[2] and Eshelby[3], even though not conclusively for a crack propagating with an arbitrary history of motion. Using arguments similar to those used above in deriving (2.12) from (2.7), one may likewise derive from (2.5) that

\[
G = \int_S \left[ (W + T)n_1 - t_i \frac{\partial u_i}{\partial x_1} \right] ds + \int_V \left[ \sum_{ij} \frac{\partial}{\partial x_1} (W + T) - t_i \frac{\partial u_i}{\partial x_1} - t_i \frac{\partial u_i}{\partial a} \right] dv - \int_S t_i \frac{\partial u_i}{\partial a} ds,
\]

where \( S \), the external boundary of \( V \), such that \( S = S_t + S_a \), and use has been made of (2.9). Note that the second term in (2.14) does not vanish; its evaluation in the practical problem of crack propagation in an arbitrary finite body involves, however, two solutions for slightly different crack lengths, \( a \) and \( a + da \). However, if one considers the volume \( V_r - V_a \) (where \( \Gamma \) is any path surrounding the crack tip, see Fig. 1) and thus excludes the crack tip, it is a simple matter to apply the divergence theorem and rewrite (2.13) as

\[
G = \int_{\Gamma + S_C} \left[ (W + T)n_1 - t_i \frac{\partial u_i}{\partial x_1} \right] ds - \int_{V_r - V_a} \left[ \sum_{ij} \frac{\partial}{\partial x_1} (W + T) - \sigma_{ij,j} \frac{\partial u_i}{\partial x_1} - \sigma_{ij} \frac{\partial^2 u_i}{\partial x_j \partial x_1} \right] dv.
\]

We now restrict attention, without much loss of generality, to infinitesimal deformations of cracked nonlinear elastic bodies which are subject to nonuniform temperature fields. In such a case, the infinitesimal strain tensor \( \varepsilon_{ij} \) may be decomposed as

\[
\varepsilon_{ij} = \varepsilon^m_{ij} + \varepsilon^t_{ij}
\]

where \( \varepsilon^m_{ij} \) are "mechanical" strains and \( \varepsilon^t_{ij} \) are "thermal" strains \( (\varepsilon^t_{ij} = \alpha \theta \delta_{ij}, \text{where } \alpha \text{ is the coefficient of thermal expansion, } \theta \text{ is the temperature rise } T - T_0, \theta = \theta(x_k), \text{and } T_0 \text{ is the ambient temperature}) \). The total stress-working density is

\[
W = \int_0^{\varepsilon_{ij}} \sigma_{ij} \varepsilon_{ij},
\]

where

\[
\sigma_{ij} = f_{ij}[\varepsilon_{kl} - \varepsilon^t_{kl}, x_k]
\]

and

\[
W = W(\varepsilon_{ij}, \theta, x_k).
\]

In (2.18) \( f_{ij} \) is a tensor-valued function. Note that in thermoelasticity, stress depends on the mechanical strains as well as explicitly on \( x_k \), since the material may be nonhomogeneous (either naturally or due to temperature dependence of the material properties in a nonuniform temperature field). Thus

\[
\frac{\partial W}{\partial x_1} = \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1} + \frac{\partial W}{\partial x_1} \bigg|_{\text{explicit}} = \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1} + \int_0^{\varepsilon_{ij}} \left( \frac{\partial f_{ij}}{\partial x_1} \bigg|_{\text{explicit}} + \frac{\partial f_{ij}}{\partial \varepsilon_{ij}} \frac{\partial \theta}{\partial x} \right) \varepsilon_{ij},
\]

(2.20)
wherein the definition of \((\partial W/\partial x_1)_{\text{explicit}}\) is apparent. Use of (2.20) in (2.15) results in

\[
G = J' = \int_{V + S, \epsilon} \left[ (W + T) n_1 - t_1 \frac{\partial u_t}{\partial x_1} \right] \, ds
- \lim_{\epsilon \to 0} \int_{V - V'} \left( \rho \dddot{u}_t \frac{\partial u_t}{\partial x_1} + (f_i - \rho \dddot{u}_i) \frac{\partial u_t}{\partial x_1} + \frac{\partial W}{\partial x_1} \right) \exp \, dv \tag{2.21a}
- \int_S \left[ (W + T) n_1 - t_1 \frac{\partial u_t}{\partial x_1} \right] \, ds
- \lim_{\epsilon \to 0} \int_{V - V'} \left( \rho \dddot{u}_t \frac{\partial u_t}{\partial x_1} + (f_i - \rho \dddot{u}_i) \frac{\partial u_t}{\partial x_1} + \frac{\partial W}{\partial x_1} \right) \exp \, dv \tag{2.21b}
\]

If the material is homogeneous in the \(x_1\) direction, and \(T = T_0 = 0\), i.e. isothermal conditions prevail in the solid, \((\partial W/\partial x_1)_{\text{explicit}} = 0\). On the other hand, consider for example an isotropic linear elastic material with nonuniform temperature distribution \(\theta(x_k)\) and nonhomogeneous material properties. Here,

\[
\sigma_{ij} = E_{ijkl} \varepsilon_{kl} = (2\mu \delta_{ik} \delta_{jl} + \lambda \delta_{ij} \delta_{kl}) (\varepsilon_{kl} - \alpha \theta \delta_{kl}) \tag{2.22}
\]

such that

\[
\frac{\partial W}{\partial x_1} \bigg|_{\text{explicit}} = \int_{0}^{e_{ij}} \frac{\partial E_{ijkl}}{\partial x_1} \mu_{ij} \, d\mu_{ij} - \int_{0}^{e_{ij}} E_{ijkl} \delta_{kl} \frac{\partial \theta}{\partial x_1} \, d\mu_{ij}
= \frac{1}{2} \epsilon_{ij} E_{ijkl,1} \epsilon_{kl} - \epsilon_{ij} E_{ijkl,1} \epsilon_{kl}^* - (2\mu + 3\lambda \alpha \epsilon_{kk} \theta), \tag{2.23}
\]

where \((\ ),_{1} = \partial(\ )/\partial x_1\). Now from (2.22) one obtains

\[
\sigma_{ij,1} = E_{ijkl,1} \epsilon_{kl} + E_{ijkl,1} \epsilon_{kl,1} - E_{ijkl,1} \epsilon_{kl}^* - E_{ijkl,1} \epsilon_{kl,1}^*. \tag{2.24}
\]

Use of (2.24) in (2.23) results in

\[
\frac{\partial W}{\partial x_1} \bigg|_{\exp} = \frac{1}{2} \sigma_{ij,1} \epsilon_{ij}^m - \frac{1}{2} \sigma_{ij} \epsilon_{ij,1}^m - \frac{1}{2} (2\mu + 3\lambda) \alpha \epsilon_{kk} \theta^2
- (2\mu + 3\lambda) \alpha \epsilon_{kk} \theta, \tag{2.25a}
\]

Note that \(W\) in (2.17) and (2.19) is given for the present linear elastic isotropic case as

\[
W = \mu \epsilon_{ij} \epsilon_{ij} + \frac{\lambda}{2} \epsilon_{kk}^2 - (2\mu + 3\lambda) \alpha \epsilon_{kk} \theta. \tag{2.25b}
\]

If the temperature dependence of \(\mu\) and \(\lambda\) is ignored, the path-independent integral representation for the energy-release rate, (2.21), becomes

\[
J' = \int_{V - S, \epsilon} \left[ (W + T) n_1 - t_1 \frac{\partial u_t}{\partial x_1} \right] \, ds
- \int_{V - V'} \left[ \rho \dddot{u}_t u_{l,1} + (f_i - \rho \dddot{u}_i) u_{l,1} - (2\mu + 3\lambda) \alpha \epsilon_{kk} \theta,_{1} \right] \, dv, \tag{2.26}
\]

where \((\ ),_{1} = \partial(\ )/\partial x_1\). Now, note the identity

\[
\int_{V} (2\mu + 3\lambda) \theta \epsilon_{kk} a n_1 \, ds = \int_{V - S, \epsilon} (2\mu + 3\lambda) \theta \epsilon_{kk} a n_1 \, ds
- \int_{V - V'} [(2\mu + 3\lambda) \alpha \theta \epsilon_{kk,1}] \, dv. \tag{2.27}
\]
Adding (2.27) and (2.26), we may define a parameter \( \tilde{G} \) such that
\[
\tilde{G} = J' + \int_{\Gamma} (2 \mu + 3 \lambda) \theta \epsilon_{kk} \alpha n_1 \, ds = \int_{\Gamma} \left[ (W^* + T)n_1 - t_i u_{i,1} \right] \, ds
\]
\[
= \int_{\Gamma \setminus S_{\alpha}} \left[ (W^* + T)n_1 - t_i u_{i,1} \right] \, ds
\]
\[
- \int_{V_1 - V_2} \left[ \rho \dot{\theta} \dot{u}_{i,1} + (\dot{f}_i - \rho \ddot{u}_i) u_{i,1} + (2 \mu + 3 \lambda) \alpha \theta \epsilon_{kk,1} \right] \, dv.
\]
(2.28)

where \( W^* = \mu \epsilon_{ij} \epsilon_{ij} + (\lambda/2) \epsilon_{kk}^2 \). On the other hand, the Navier equations of isotropic elasticity in the presence of nonuniform temperature fields are given by
\[
\mu \epsilon_{kk} + (\lambda + \mu) \epsilon_{kk,i} - (2 \mu + 3 \lambda) \alpha \theta_{,i} = 0.
\]
(2.29)

Thus
\[
\int_{V_1 - V_2} (\lambda + \mu) \theta \epsilon_{kk,i} \, dv = \int_{V_1 - V_2} [(2 \mu + 3 \lambda) \alpha \theta_{,i} - \mu \theta u_{i,kk}] \, dv
\]
\[
= \mu \int_{V_1 - V_2} \theta_{,k} u_{i,k} \, dv + \int_{V \setminus S_{\alpha} - V_2} \left[ \frac{(2 \mu + 3 \lambda)}{2} \alpha \theta^2 n_i - \mu \theta u_{i,k} n_k \right] \, dv.
\]
(2.30)

If the temperature field in the body is assumed to obey the harmonic equation
\[
\theta_{,ii} = 0,
\]
(2.31)

then
\[
\int_{V_1 - V_2} \theta_{,i} u_{i,k} \, dv = \int_{V \setminus S_{\alpha} - V_2} \theta_{,k} u_{i,k} \, ds.
\]
(2.32)

Thus, when the process is quasi-static (\( T = 0, \ddot{u}_i = 0 \)), it is possible to use (2.30) and (2.32) in (2.28) and define a modified parameter \( \tilde{G} \) such that
\[
\tilde{G} = J'_{QS} + \int_{\Gamma} \left\{ (2 \mu + 3 \lambda) \theta \epsilon_{kk} \alpha n_1 - \frac{(2 \mu + 3 \lambda)}{(\lambda + \mu)} \alpha \left[ \mu \theta_{,k} n_k u_{i} \right.ight.
\]
\[
+ \left. \frac{(2 \mu + 3 \lambda)}{2} \theta^2 n_i - \mu \theta u_{i,k} n_k \right] \} \, ds
\]
\[
= \int_{V \setminus S_{\alpha}} \left\{ W^* n_1 - t_i u_{i,1} - \frac{(2 \mu + 3 \lambda)}{\lambda + \mu} \alpha \left[ \mu \theta_{,k} n_k u_{i} \right.ight.
\]
\[
+ \left. \frac{(2 \mu + 3 \lambda)}{2} \theta^2 n_i - \mu \theta u_{i,k} n_k \right] \} \, ds.
\]
(2.33)

Thus, (2.34) represents a path independent integral expression (without the presence of a domain integral) for \( \tilde{G} \). This result is due to Gurtin[4]. Note that \( J'_{QS} \) is the "quasi-static" value of the energy-release rate \( J' \) (i.e. when the material inertia is ignored). Thus, while \( \tilde{G} \) represents a mathematically convenient integral in the cases (i) of linear isotropic homogeneous elasticity, (ii) when \( \mu \) and \( \lambda \) do not depend on temperature, and (iii) when the temperature satisfies \( \theta_{,ii} = 0 \), its physical significance is somewhat obscure. The point to be made here is that the situation when a meaningful crack-tip parameter can be represented equivalently by a far-field contour integral alone (i.e. without the presence of a domain integral) is rather rare in practice.
Sometimes, in thermoelasticity, it is convenient to define the stress potential

\[ V = \int_0^\infty \sigma_{ij} \, d\epsilon_{ij}^m \]  

such that

\[ V = V(\epsilon_{ij}^m, \chi) \]  

and

\[ \sigma_{ij} = g_{ij}(\epsilon_{ij}^m, \chi). \]

Now that \( V \) is simply a mathematical potential and is not equal to the stress-working density, we have

\[ \frac{\partial V}{\partial x_1} = \sigma_{ij} \frac{\partial \epsilon_{ij}^m}{\partial x_1} + \int_0^\infty \frac{\partial R_{ij}}{\partial x_1} |_{\epsilon^{m}} \, d\epsilon_{ij}^m \equiv \sigma_{ij} \frac{\partial \epsilon_{ij}^m}{\partial x_1} + \frac{\partial V}{\partial x_1} |_{\epsilon^{m}}. \]  

Thus, one may define a crack-tip parameter

\[ G^* = \int_{\Gamma} [(V + T)n_1 - t_i u_{i,1}] \, ds = \int_{\Gamma + S_{i,1}} [(V + T)n_1 - t_i u_{i,1}] \]

\[ - \int_{V_1 - V} \left[ \rho \ddot{u}_i \dot{u}_{i,1} + (f_i - \rho \ddot{u}_i)u_{i,1} - \sigma_{ikl} \alpha \theta_i + \frac{\partial V}{\partial x_1} \right] |_{\epsilon^{m}}. \]  

If the material is linear elastically isotropic, and \( \lambda \) and \( \mu \) depend on temperature, we have

\[ \frac{\partial V}{\partial x_1} |_{\epsilon^{m}} = \int_0^\infty \frac{\partial R_{ijkl}}{\partial x_1} \mu_{ijkl} \, d\mu_{ijkl} \]

\[ \frac{\partial V}{\partial x_1} |_{\epsilon^{m}} = \frac{1}{2} E_{ijkl} \epsilon_{ij}^m \epsilon_{kl}^m. \]

From the relations

\[ \sigma_{ij} = E_{ijkl} \epsilon_{ij}^m, \quad \sigma_{ij,1} = E_{ijkl} \epsilon_{ij}^m + E_{ijkl} \epsilon_{ij,1}, \]

one may write (2.39) as

\[ \frac{\partial V}{\partial x_1} |_{\epsilon^{m}} = \frac{1}{2} \sigma_{ij,1} \epsilon_{ij}^m - \frac{1}{2} \sigma_{ij} \epsilon_{ij,1}. \]

The case when (i) \( T = 0 \), (ii) \( \ddot{u}_i = 0 \), and (iii) the material constants are independent of temperature, i.e. \( (\partial V/\partial x_1)_{\epsilon^{m}} = 0 \) has been reported by Ainsworth et al.[5]. Note, however, that even this case (or in general) \( G^* \neq J' \,(= G) \). It is more natural to deal with the density of stress work, \( W \), in the case of a nonlinear material.

When the material is homogeneous in the \( x_1 \) direction, and when isothermal conditions prevail, eqn (2.21) is reduced to

\[ G = J' = \int_{\Gamma + S_{i,1}} [(W + T)n_1 - t_i \frac{\partial u_i}{\partial x_1}] \, ds \]

\[ - \int_{V_1 - V} \left[ \rho \ddot{u}_i \frac{\partial \ddot{u}_i}{\partial x_1} + (f_i - \rho \ddot{u}_i) \frac{\partial u_i}{\partial x_1} \right] \, dV, \]  

where \( \Gamma \) is any arbitrary contour that encircles the crack tip.

† When, in addition, the so-called steady-state conditions prevail, i.e. \( \ddot{u}_i = du_i/dt = - \partial u_i/\partial x_1 \), etc., and the body forces are zero, the integral over \( V_f - V_1 \) vanishes. Thus, in the steady-state case, one simply has a contour integral (without a domain integral) that is path independent[16, 17, 22]. It can easily be shown that this result for the so-called steady-state conditions holds true even for arbitrary materials when \( W \) is interpreted as the stress-working density[23].
The sense of path independence embodied in eqn (2.42) implies that for any closed volume $V^*$, with boundary $\Gamma^*$ not enclosing the crack tip (as in Fig. 1), we have

$$\int_{V^*} \left[ (W + T)n_i - t_i \frac{\partial u_i}{\partial x_1} \right] ds - \int_{V^*} \left[ \rho u_i \frac{\partial u_i}{\partial x_1} + (f_i - \rho \ddot{u}_i) \frac{\partial u_i}{\partial x_1} \right] dV = 0, \quad (2.43)$$

which may be verified easily under the assumption of material homogeneity along $x_1$, and when $W$ is a single-valued function of $\epsilon_{ij}$.

Because of the use of $J^*$ as defined for any path $\Gamma$ as in (2.42) involves a volume integral, the above notion of path independence has been pronounced by many to be useless. This viewpoint, however, is somewhat orthodox. True, the evaluation of (2.42) involves taking the limit of the volume integral to the crack tip; and thus, on the surface, it appears to involve a "knowledge of the crack-tip fields," which the so-called $J$ integral of elastostatics (when $\ddot{u} = \ddot{u}_i = 0$ in (2.42)) does not involve. First of all, it is clear from (2.42) that its use does not require a knowledge of the crack-tip stress–strain fields, but only of displacement, velocity, and acceleration. Furthermore, a comparison of (2.42) (when evaluated over the external surface $S$) and (2.14) reveals that

$$\lim_{\epsilon \to 0} \int_{V^*} \left[ \rho \ddot{u}_i \frac{\partial u_i}{\partial x_1} + (\dddot{f}_i - \rho \dddot{u}_i) \frac{\partial u_i}{\partial x_1} \right] dV = \int_{V} \left[ \frac{\partial}{\partial a} (W + T) - \dddot{f}_i \frac{\partial u_i}{\partial a} - \dddot{f}_j u_i \right] dV - \int_{S} t_i \dddot{u}_i ds, \quad (2.44)$$

and thus the l.h.s. of (2.44) remains finite in the limit $\epsilon \to 0$. This is interesting if one notes that, in known analytical asymptotic solutions, $u_i \sim O(r^{-1/2})$ and $\dddot{u}_i \sim O(r^{3/2})$ and hence, on first glance, the l.h.s. of (2.44) appears to contain nonintegrable singularities. It has also been verified directly[6] that for known analytical asymptotic solutions for infinite bodies, the volume integral in (2.42) does have a finite limit, due to the fact that the angular variation of the integrand is such that

$$\lim_{\epsilon \to 0} \int_{V^*} \left[ \int_{0}^{\pi} (\rho \ddot{u}_i u_i, r) r d\theta \right] dV \to 0. \quad (2.45)$$

Even though finding the solution of $u_i, \ddot{u}_i, \dddot{u}_i$ near the crack tip in a finite body is a difficult problem analytically, it is a relatively simple task in computational mechanics. This has been demonstrated conclusively by the authors in a variety of crack-propagation problems in finite bodies, even while using the simplest of crack-tip finite elements which do not model any of the singularities in strain, velocity, or acceleration.

If one considers the energy-release rate per unit time in self-similar elastodynamic crack propagation, one sees that this quantity is represented by

$$CG = C \int_{V^*} \left[ (W + T)n_i - t_i \frac{\partial u_i}{\partial x_1} \right] dS \quad (2.46a)$$

$$= \int_{V^*} \left[ (W + T)C_k N_k - C_k t_i \frac{\partial u_i}{\partial X_k} \right] dS, \quad (2.46b)$$

where $C$ is the nonconstant velocity of crack propagation along the $x_1$ direction, and $n_i$ is the component of a unit normal to $\Gamma^*$ along $x_1$, while $C_k$ and $N_k$ are components of the instantaneous velocity vector and the unit normal to $\Gamma^*$, respectively, along the $X_k$ directions (see Fig. 1). (Note that the velocity vector $C$ with $|C| = C$, along the $x_1$ direction in self-similar propagation, may be considered to have components $C_k$ and $X_k$ directions). It is now a simple task to (i) apply the divergence theorem, and (ii) to use the coordinate-invariant forms of the linear-momentum balance laws, under the assumption

$$\frac{\partial W}{\partial X_k} = \frac{\partial W}{\partial u_{ij}} \frac{\partial u_{ij}}{\partial X_k}, \quad (2.47)$$
i.e. \( W \) does not depend explicitly on all the \( X_k \) (or the material is homogeneous in all the \( X_k \) directions), to derive from (2.46b),

\[
CG = C_k J_k' = \left\{ \lim_{\epsilon \to 0} \int_{\Gamma_{a} - \epsilon} \left[ (W + T)N_k - \frac{\partial u_i}{\partial X_k} \right] ds \right\} \text{C}_k.
\]

The sense of path independence embodied in (2.48) is similar to that in (2.42) and (2.43). In the above, \( S_{\Gamma +} \), which is equal to \( S_{\Gamma -} + S_{\Gamma} \) (+ and − referring, arbitrarily, to the crack faces), is the crack surface enclosed within \( \Gamma \), while \( S_\Gamma \) is the total crack surface. Thus, an evaluation of \( J'_k \) not only involves a volume integral, but also an integral along the crack faces.

The infinitesimal strain counterparts of the \( J'_k \) integrals were first stated in [6], based on a simple modification to the \( J_k \) integrals for dynamic crack propagation given in [9].

It is important to note the meaning of (2.48)—it still governs the energy release per unit time, due to self-similar propagation (along the \( x_1 \) axis). \( J'_k \) would simply characterize the total energy change due to a unit translation of the crack as a whole, rigidly, in the \( X_k \) direction. Thus, \( J'_k \) does not characterize the energy release due to a unit motion of the crack tip in the \( X_k \) direction (and thus kinking the original crack). In fact there are no simple integrals that characterize the energy release due to kinking of a crack, as is often erroneously implied in literature. This is due to the fact that in deriving (2.11), which forms the basis of all the ensuing path integrals thereof, use has been made of the self-similarity of solutions at time \( t \) and \( t + dt \), which is valid only in self-similar crack propagation but not in the case, in general, of a kinked crack.

Assuming for the moment that the global and the crack-tip coordinates coincide, one may define

\[
J_2 = \lim_{\epsilon \to 0} \int_{\Gamma_{\epsilon} - \epsilon} \left[ (W + T)N_2 - \frac{\partial u_i}{\partial X_2} \right] ds - \int_{\Gamma_{\epsilon} - \epsilon} \left[ \rho \tau_i \frac{\partial u_i}{\partial X_2} + (F_i - \rho \bar{u}_i) \frac{\partial u_i}{\partial X_2} \right] dV, \tag{2.49}
\]

which would characterize the total energy change for a unit rigid translation of the crack as a whole (and not a unit growth of the crack tip alone) in the \( X_2 \) direction. Assuming zero body force, traction-free crack faces, and elastostatic deformations, one may reduce (2.49) to

\[
J_2 = \int_{\Gamma} \left( WN_2 - \frac{\partial u_i}{\partial X_2} \right) ds + \lim_{\epsilon \to 0} \int_{S_{\Gamma} \epsilon} (W^+ - W^-) dS, \tag{2.50}
\]

wherein, for a flat crack face, \( N_2^+ = -N_2^- = -1 \). The definition of \( J_2 \) of Budiansky and Rice[10], on the other hand, does not involve the crack-face integral, which accounts for discontinuities of \( W \) along the crack face. Thus, \( J_2 \) as given by [10] is not path independent. Even though (2.50) appears to involve a knowledge of crack tip \( W \) for its successful application as a path-independent integral, the use of (2.50) has been conclusively demonstrated [8, 11] in computational approaches using simple (nonsingular) crack-tip finite elements.

From the above discussions, it should be clear that neither the integrals \( J'_k \) nor any other similarly “path-independent” integrals provide any information as to the kinking of a crack or of the direction of propagation of the crack tip in anything other than a colinear fashion, contrary to speculations often made in literature.

### 3. CRACK-TIP PARAMETERS FOR DYNAMIC CRACK PROPAGATION IN ELASTIC-PLASTIC SOLIDS

In elastodynamic crack propagation, the expression for the rate of energy release per unit of crack extension, written as an integral of crack-tip strain, stress, and displacement variables over a contour \( \Gamma' \) (as in eqns (2.7) or (2.13)) arbitrarily close to the crack tip, served as a natural
"crack-tip parameter." That this crack-tip parameter is alternatively expressed as a far-field contour integral plus a finite-domain integral is simply a mathematical artifact (based on a simple use of the divergence theorem) and is a useful concept in computational applications. However, when material inelasticity is present, the concepts of energy release are, in general, vacuous. In elastic–plastic materials, under quasi-static conditions, it has been shown[12, 13] that the energy-release rate vanishes in the limit of vanishingly small growth increments. Of course, the total energy release for a finite growth step $\Delta a$, denoted as $G^*\Delta$, remains finite and depends on the magnitude of $\Delta a$[13, 14]. It is this dependence on the size of $\Delta a$ that precludes a rational utilization of the original Griffith energy-balance concept in elastic–plastic fracture mechanics. Also, the derivation of crack-tip integrals that may characterize "energy release" even in finite growth steps, along the lines of those given earlier for elasticity, are no longer possible in elastoplasticity since the solutions near the crack tip at times $t$ and $t + \Delta t$ (during which the crack has grown by $\Delta a$) are, in general, no longer self-similar due to the elastic unloading that accompanies crack growth.

Consider first a stationary crack in an elastic–plastic solid that is subject to some arbitrary external loading. If a two-dimensional situation is considered, any integral over an arbitrarily small circular path $\Gamma$, near the crack tip (with radius $\epsilon$ such that $\epsilon \rightarrow 0$), with the integrand being such that (i) it depends on the stress, strain, and displacement state near the crack tip and (ii) it has a $1/r$ variation near the crack tip, would serve as a valid crack-tip parameter. Since the integrand has a $1/r$ variation, it is seen that this integral crack-tip parameter remains finite for the chosen $\Gamma$. Thus, in dynamically loaded elastic–plastic bodies with stationary cracks, of the infinitely many crack-tip parameters that may be defined to satisfy the above requirements we pick, for instance, the parameter

$$T^* = \int_{\Gamma} \left[ (W + T)t_1 - t_1 \frac{\partial u_1}{\partial x_1} \right] d\Gamma. \quad (3.1)$$

By use of the divergence theorem, one may write

$$T^* = \int_{\Gamma_{\text{external}}} \left[ (W + T)t_1 - t_1 \frac{\partial u_1}{\partial x_1} \right] d\Gamma - \int_{v_1 - v} \left[ \frac{\partial (W + T)}{\partial x_1} - \frac{\partial}{\partial x_1} \left( \sigma_{ij} \frac{\partial u_1}{\partial x_1} \right) \right] d\nu \quad (3.2)$$

$$= \int_{\Gamma_{\text{external}}} \left[ (W + T)t_1 - t_1 \frac{\partial u_1}{\partial x_1} \right] d\Gamma - \int_{v_1 - v} \left[ \frac{\partial (W + T)}{\partial x_1} - \frac{\partial}{\partial x_1} \left( \sigma_{ij} \frac{\partial u_1}{\partial x_1} \right) \right] d\nu. \quad (3.3)$$

Note that in an elastic–plastic body under arbitrary load history, $\partial W/\partial x_1 = \sigma_{ij}(\partial \epsilon_{ij}/\partial x_1)$, since $\sigma_{ij}$ is not a single-valued function of $\epsilon_{ij}$. In general, $\partial W/\partial x_1$ is simply evaluated computationally from the values of total stress work $W$ at two neighboring spatial locations.

Consider the first term on the right-hand side of (3.3), which is labeled here as $J^*$. We now explore the meaning of $J^*$. Consider two identical and identically loaded cracked bodies with crack lengths $a$ and $a + da$, respectively. We assume that both the cracks remain stationary under this identical dynamic loading. The equilibrium process in each body with a stationary

† Here we include nonsteady crack propagation.
‡ For a more comprehensive discussion on the size of $\epsilon$ used in defining the path $\Gamma$, see Ref.[23].
crack implies that

\[
\int_v [W + T]^{(\alpha)} \, dV = - \int_{S_r} \int_0^{\bar{t}_i} u^{(\alpha)} \, d\bar{t}_i \, dS + \int_{S_r} \int_0^{\bar{\vec{\tau}}_i} \bar{u}_i \, dS + \int_v \int_0^{\bar{\vec{\tau}}_i} \bar{u}_i \, d\vec{f}_i \, dV,
\]

(3.4)

where \( \bar{t}_i \) are prescribed tractions at the surface \( S_r \), \( \bar{u}_i \) are prescribed displacements at the surface \( S_r \), \( \vec{f}_i \) are body forces, and the superscript \( (\alpha) = 1, 2 \) denotes bodies 1 and 2, respectively. In the elastic–plastic body, \( W \) is now meant to be the total stress-working density, i.e.

\[
W = \int_0^{\epsilon} \sigma_{ij} \, d\epsilon_{ij}.
\]

(3.5a)

From (3.4) it follows that

\[
\frac{d}{da} \int_v (W + T) \, dV = - \int_{S_r} \int_0^{\bar{t}_i} \frac{du_i}{da} \bar{t}_i \, dS + \int_{S_r} \int_0^{\bar{\vec{\tau}}_i} \frac{du_i}{da} \, dS + \int_v \int_0^{\bar{\vec{\tau}}_i} \frac{du_i}{da} \, d\vec{f}_i \, dV.
\]

(3.5b)

Thus, if \( dA \) is the total difference in areas under the load-deformation curves (note that the “load” is imposed by \( \bar{t}_i \) at \( S_r \) as well as \( \bar{u}_i \) at \( S_r \)), for the two identical and identically loaded cracked bodies, then it follows from (3.5b) that

\[
dA = \int_{S_r} \int_0^{\bar{t}_i} \frac{du_i}{da} \bar{t}_i \, dS - \int_{S_r} \int_0^{\bar{\vec{\tau}}_i} \frac{du_i}{da} \, dS - \frac{d}{da} \int_v (W + T) \, dV + \int_v \int_0^{\bar{\vec{\tau}}_i} \frac{du_i}{da} \, d\vec{f}_i \, dV.
\]

(3.6)

As long as the cracks remain stationary in both bodies which are considered to be subject to identical loading, the crack-tip solutions in both bodies may be considered to be self-similar at all times. Thus, using techniques outlined in Section 2, we may rewrite the extreme right-hand side of (3.6) in the form

\[
dA = \int_{1, \text{external}} \left( (W + T) n_1 - t_i \frac{\partial u_i}{\partial x_1} \right) - \int_v \int_0^{\bar{\vec{\tau}}_i} \frac{\partial u_i}{\partial x_1} \, d\vec{f}_i \, dV
\]

\[
+ \left\{ \int_{S_r} \frac{\partial u_i}{\partial a} - \int_v \frac{\partial (W + T)}{\partial a} \, dV \right\} + \int_v \int_0^{\bar{\vec{\tau}}_i} \frac{\partial u_i}{\partial a} \, d\vec{f}_i \, dV.
\]

(3.7)

Now, consider the case when (i) each cracked body is only loaded quasi-statically, i.e. the kinetic energy \( T = 0 \); (ii) body forces \( \vec{f}_i \) (due to, say, thermal gradients, gravity, electromagnetic or other sources) are zero; (iii) the elastic–plastic body is loaded only monotonically and proportionally so that the deformation theory of plasticity (for in essence the nonlinear elasticity theory) is valid, thus implying, for instance, that \( \partial W/\partial a = \sigma_{ij} (\partial \epsilon_{ij}/\partial a) \), etc. Thus, when all these conditions prevail, the following simplifications result in the left-hand side of (3.7): (i) \( T = 0 \) in the first term, (ii) the second term disappears, and (iii) when \( T = 0 \), the third term vanishes by virtue of the principle of virtual work. Thus, under these restrictions,

\[
dA = \int_{1, \text{external}} \left[ WN_1 - t_i \frac{\partial u_i}{\partial x_1} \right] = J.
\]

(3.8)
Thus, the well-known $J$, as a far-field integral, evaluated, say, at the boundary of the specimen, is equal to the area difference under the load-deformation curve to two identical cracked bodies with slightly differing crack lengths only under the above restrictions and only so long as the crack remains stationary (see discussion leading to eqn (3.7)). Using the divergence theorem, we see that

$$
J = \frac{dA}{d\Gamma} = \int_{\Gamma} (W_{n1} - t_1 \frac{\partial u_1}{\partial x_1}) d\Gamma = \int_{\Gamma} \left( W_{n1} - t_1 \frac{\partial u_1}{\partial x_1} \right) d\Gamma \\
+ \int_{\Gamma_{c,1}} \left[ \frac{\partial W}{\partial x_1} - \frac{\partial}{\partial x_1} \left( \sigma_{ij} \frac{\partial u_1}{\partial x_1} \right) \right] d\Gamma.
$$

The second term (i.e. the integral over the volume) on the extreme right-hand side of (3.9) is seen to vanish only when (i) the material is nonlinear elastic and homogeneous along $x_1$, such that $\frac{\partial W}{\partial x_1} = \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1}$; (ii) body forces (due to thermal strains, electromagnetic forces, etc.) are zero; (iii) for elastoplastic materials, only conditions of monotonic and proportional loading must prevail; and (iv) the solid is in equilibrium.

Thus, in summary, we see that the well-known $J$ integral, which is experimentally determined from the area under the load-deformation curves of cracked specimens, is a valid crack-tip parameter in elastic-plastic bodies only under the following restrictions: (i) isothermal conditions, (ii) material homogeneity, (iii) monotonic and proportional loading, (iv) no unloading, (v) no body forces and inertia, (vi) only up to the initiation of quasi-static crack growth under monotonic loading, and (vii) perhaps for very small amounts of crack growth.

To arrive at fracture criteria that may be theoretically valid in the context of rate theories of inelasticity, studies[9, 15-17] were recently aimed at incremental (or rate) path-independent integral parameters. An incremental crack-tip parameter $\Delta T^*$ was defined such that (i) it involves the incremental stress work in the integral over $\Gamma_e$, such that it can be defined appropriately for any material model such as rate-independent elastoplasticity, viscoplasticity, etc.; (ii) the far-field definition of $\Delta T^*$ is inherently path independent but involves a domain integral; (iii) it is easily defined for nonisothermal, nonhomogeneous material conditions; (iv) it is a path-independent integral type crack-tip parameter even for large amounts of crack growth and general nonsteady conditions; (v) it is a valid crack-tip parameter for arbitrary histories of loading and unloading; and (vi) when specialized to the case of monotonic proportional loading of homogeneous isothermal elastoplastic bodies containing stationary cracks, the integral of $\Delta T^*$ over the load path is equal to the well-known $J$.

Detailed studies of $\Delta T^*$ in the context of (i) large amounts of stable crack growth in elastoplastic bodies under rising load, (ii) stable crack growth in elastoplastic bodies subject to arbitrary load histories, and especially crack growth following a large unloading and reloading cycle, and (iii) creep crack growth at elevated temperatures have recently been presented[18-20]. The reader is referred to these publications for a comprehensive account of further theoretical and computational details concerning incremental crack-tip parameters.

4. TEMPERATURE RISE NEAR THE CRACK TIP IN DYNAMIC CRACK PROPAGATION

We consider elastoplastic dynamic crack propagation in which, between times $t$ and $t + dt$, the crack propagates by $da$, with velocity $a = da/dt$. In a differential volume in the plastic zone near the propagating crack tip, the density of plastically dissipated stress power is given by

$$
\dot{w}_p = \sigma_{ij} \dot{\varepsilon}_{ij}^p,
$$

which is mostly converted to heat. Thus, the rate of heat production in the plastic zone may
be written as

$$\dot{q} = \dot{\hat{w}}^p = \sigma_{ij} \dot{\varepsilon}_{ij}^p / \text{unit time/unit volume.}$$  \tag{4.2}$$

Even in brittle crack propagation, the so-called energy-release rate, $\dot{g}$, is mostly converted to heat, so that one may write \( \dot{q} \approx \dot{g} \alpha \). Note also from eqns (4.1) and (4.2) that the rate of heat production has an arbitrary spatial distribution in the plasticity zone, as determined by the spatial dependence of $\sigma_{ij}$ and $\varepsilon_{ij}$ in the plastic zone. Furthermore, the heat-production zone (plastic zone) is propagating with the crack tip. With these in mind, the temperature distribution near the propagating crack tip is governed by the transient heat-conduction equation:

$$\begin{align*}
(K\theta_{ij})_{ij} + \dot{q} &= C_0 \frac{\partial \theta}{\partial t},
\end{align*}$$

where $K$ is thermal conductivity (may depend on temperature), \( \theta \) the temperature, \( \dot{q} \) the rate
of heat production in the propagating plastic zone, \( C \) the specific heat (may depend on temperature), and \( t \) the time, and ( \( \frac{\partial}{\partial x_i} \) denotes \( \partial / \partial x_i \)).

If \( Q \) (having spatial dependence in the plastic zone) is the energy dissipated per unit crack growth per unit volume in the plastic zone, and \( a \) is crack velocity, it is seen that \( \dot{a} = Q a. \) In general, \( Q \) may also be an arbitrary function of \( a. \) Also, near the propagating crack tip, since the stress work has, in general, a singularity, \( Q \) may have a singularity.

In [21] a comprehensive study of the temperature field near the propagating crack tip, due to the above-described moving heat source, is presented, wherein (i) \( Q \) is considered to be a constant that is independent of \( a; \) (ii) the spatial distribution of \( Q, \) in the propagating plasticity zone, is assumed, alternatively, to be either uniform or to possess a \( 1/r \) type singularity; (iii) the solution is presented for the transient temperature field as seen by an observer moving with the crack tip at the same velocity; (iv) the temperature dependence of material properties \( K \) and \( C \) is accounted for; and (v) the effects of convective and radiative heat transfer to the surrounding medium are accounted for. The problem, mathematically, is a strongly nonlinear moving-boundary problem, which has been solved[21] by a moving-mesh finite-element procedure.

For small velocities of crack propagation, the process near the crack tip is nearly isothermal, i.e. heat diffuses quickly away into the remainder of the cracked solid; for fast, realistic speed of crack propagation, the process is nearly adiabatic, i.e. all the heat is dissipated in the process zone near the crack tip (thus leading to substantial increase of temperature near the crack tip). Moreover, the temperature gradients near the crack tip are also severe. It has also been found[21] that for low velocities, the maximum temperature \( T_{\text{max}} \) occurs at the center of the propagating heat source, while for higher velocities \( T_{\text{max}} \) occurs in the wake of the heat source.

If one considers a rectangular (height \( 2\delta, \) width \( d \)) heat source wherein \( Q \) has a uniform spatial distribution, it has been shown[21] that one may write

\[
(T - T_0)_{\text{max}} = \frac{Q}{(2\delta)(pc)} \beta(\psi, d),
\]

where \( \psi = (a\delta/2\alpha). \) Here \( \alpha \) is the thermal conductivity coefficient. The dependence of \( \beta \) on \( \psi \) as discerned from the calculations in [21] is shown in Fig. 3.

The calculations in [21] show that at realistic crack-propagation speeds in structural steels, while the front of the propagating heat source remains cold, higher temperatures (and severe temperature gradients) may persist in and behind the propagating heat source, even at distances of the order of the size of the heat source (plastic zone). An analysis of the coupled-thermoplastic problem is under way to study the mechanical effects of the aforementioned temperature rise.

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