FATIGUE CRACK CLOSURE AND DELAY EFFECTS UNDER MODE I SPECTRUM LOADING: AN EFFICIENT ELASTIC–PLASTIC ANALYSIS PROCEDURE

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Abstract—Results of an elastic–plastic finite element analysis of crack closure effects under mode I Spectrum loading are presented. Various factors that cause crack growth acceleration or retardation under low-to-high, high-to-low, and single overload cyclic loadings are discussed.

INTRODUCTION

The phenomenon of fatigue-crack-closure, first discovered experimentally by Elber [1, 2], continues to be a subject of several recent experimental and analytical studies; see, for instance, [3, 4]. Analytical models that provide some rationality for the adoption of an effective stress–intensity range, based on closure effects, for the correlation of fatigue crack growth rate, have also been proposed [5]. As for a more general analysis of extending cracks under general block cyclic loading, elastic–plastic finite element analyses were first performed by Newman et al. [6] and by Ohji et al. [7]. Now, since the crack growth was simulated in [6–7] by shifting a finite element node (the current crack-tip) to an immediately adjacent node, and since constant strain triangles were used to model the cracked structure, a very fine finite element mesh (with the smallest element often being of the order of $10^{-3}$ times the crack length) is necessary in the procedures of [6–7]. This can be very expensive especially when arbitrary cyclic loads are considered. One of the objectives of the present work is to present an alternate cost-efficient way to analyse fatigue crack closure and its effects under general spectrum loading. Also presented are the results of an investigation, of various factors that cause crack growth acceleration or retardation under high-to-low, low-to-high, single overload and constant amplitude type cyclic loading in a Mode I situation.

A SYNOPSIS OF THE PRESENT ANALYSIS PROCEDURE

Due to space limitations the mathematical details are omitted here; however, the salient features of the presently reported analysis procedure are given below:

(i) The present elastic–plastic finite element procedure accounts for arbitrary strain-hardening material behavior; the mathematical description of incremental elastic–plastic flow consists of Huber–Mises–Hencky yield criterion, and a Prager–Ziegler type kinematic hardening rule which best models the Bauschinger effects.

(ii) The well-known Hutchinson–Rice–Rosengren type strain and stress singularities, for strain-hardening materials, are embedded in special elements near the crack-tip. This
eliminates the need for a very fine mesh near the tip. Thus the crack-tip elements in the present procedure are of the order $10^{-1}$ of the crack length, as compared to constant strain triangles of the order of $10^{-3}$ to $10^{-4}$ times the crack length generally used in the procedures of [6-7]. A hybrid displacement finite element method [8,9] is used in developing these special elements.

(iii) The above special hybrid elements are of circular-sector shape, centered at the crack-tip; thus enabling one to model crack growth in any arbitrary direction, from the crack-axis, under general mixed mode cyclic loading.

(iv) In the present procedure, crack-growth is simulated by: (a) translation of the core of circular-sector elements, with embedded $H-R-R$ singularities, by an arbitrary amount in the desired direction; (b) reinterpolation of requisite data in the new finite element mesh; and (c) proportional relaxation of tractions in order to create a new crack surface.

(v) In order to determine most accurately the crack-opening stress $\sigma_{op}$ and crack-closure stress $\sigma_{cl}$, the displacements at nodes on the crack-axis before closure (and after opening), as well as the restraining force at the corresponding nodes after crack-closure (and before opening) are extrapolated against the load level. In all the cases studied, these two sets of extrapolated values for $\sigma_{op}$ and $\sigma_{cl}$ were found to correlate excellently.

(vi) A static-condensation procedure is employed wherein the plastic portion of the structure is isolated from the elastic; the stiffness of only the former part keeps changing whereas that of the latter remains fixed. This results in a considerable saving of the computational time. This and the invocation of the appropriate antisymmetric and symmetric conditions for modes II and I problems, along with the use of special elements of a large size as described in (ii) above, reduce the analysis procedure to be feasible as a routine tool in design, if necessary.

(vii) A study is made to arrive at a criterion for the stress level $\sigma_{ex}$, at which fatigue crack growth occurs. In prior literature, this crack extension stress level was chosen arbitrarily. For instance, in [6] the crack is extended at the maximum applied stress in each cycle in a spectrum loading, whereas in [7] the crack was extended at the applied stress level at which the restraining nodal force at the new crack-tip becomes zero. In the present study, for instance in a constant-amplitude (zero to tension) cyclic loading, it was found that $\sigma_{op}$ and $\sigma_{cl}$ were very sensitive to the chosen $\sigma_{ex}$. In the present work, a criterion, $\sigma_{ex} = p(\sigma_{max} - \sigma_{op}) + \sigma_{op}$ where $p$ is a constant of proportionality, is postulated; and $p$ is obtained by calibration such that the calculated $\sigma_{op}$ correlated with that observed in experimental studies as in [1-3]. However, it is not claimed that this is a general criterion; it is recognized that it is dependent on material properties, and to some extent on the analysis procedure itself.

**ANALYSIS OF FATIGUE CRACK GROWTH UNDER MODE I CYCLIC LOADING**

*Description of the problem*

The present analyses consider fatigue crack growth under Mode I loading of a thin centrally-cracked rectangular plate under far-field uniaxial tensile stresses normal to the crack-axis (see Fig. 1). Plane-stress conditions are assumed to exist in the present series of Mode I analyses. The dimensions of the plate are: half width $w = 230$ mm with a half-crack length $a_0 = 27.3$ mm. The material is a 2024 T3 aluminum alloy, with the mechanical properties yield stress, $\sigma_{ys} = 350$ MN/m$^2$; and Young's modulus,
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\[ W = 230 \text{ mm} \]
\[ a = 27.3 \text{ mm} \]
\[ \rho = 2.8 \text{ mm} \]

**Fig. 1. Finite element model of a center-cracked specimen. Singular sector elements shown within detail "A".**

\[ E = 70,000 \text{ MN/m}^2 \]. The material is assumed to be elastic perfectly-plastic. Figure 1 shows the finite element breakdown that is used. A total of six sector-shaped singularity elements near the crack-tip, and 43 conventional quadratic isoparametric elements are employed, with a total number of degrees of freedom of 311. The radius of the sector-shaped singularity elements is chosen as \( \rho = 2.8 \text{ mm} \); i.e. \( \rho/a_0 = 0.103 \). While the crack-extension per cycle of loading, \( \Delta a \), can be arbitrary (i.e. not related to the finite element mesh size) it is chosen to be \( \Delta a = 0.14 \text{ mm} \) in the present series of computations.

**Monitoring of crack-closure and crack-opening in the finite element model**

During the unloading part of any cycle, at the instant the displacement at one or more nodes on the crack surface becomes negative in the direction of applied tension, further unloading is stopped. The computational procedure is then switched to a displacement control type, and the above negative displacements are precisely enforced to be zero; thus finding the precise stress level at which the closure constraint on the respective node must be enforced. After the crack-closure is detected, the respective node(s) are constrained thereafter, until the restraining force(s) at the node(s) just becomes zero and begins to be tensile in nature. The corresponding applied stress level defines the crack-opening stress. The above processes are illustrated for the case of constant amplitude cyclic loading in Fig. 2. Here the vertical displacement and the restraining force at a node, at which the crack closes and opens respectively, are plotted against the applied stress level. It can be seen from Fig. 2 that the crack-opening stress \( \sigma_{op} \) (or the closure stress \( \sigma_{cl} \)) can be most
Fig. 2. Graphical illustration of procedures to find \( u_{op} \) and \( u_{cl} \) by extrapolation of COD and crack-tip nodal restraint force.

accurately determined by extrapolating to zero the respective nodal displacement after opening (or just before closure, to find \( \sigma_{cl} \)), as well as by extrapolating to zero the restraining force at the respective node just before opening (or just after closure to find \( \sigma_{cl} \)). It can be seen that the values \( u_{op} \) and \( u_{cl} \) obtained by the above two sets of extrapolation procedures agree excellently. This indicates that the present numerical procedure of determining \( u_{op} \) and \( u_{cl} \), by monitoring crack-surface displacements and the nodal restraining forces, is very accurate.

RESULTS AND CONCLUSIONS

Analysis of a center-cracked specimen under mode I cyclic loading

Using the above mentioned procedures, fatigue crack growth was considered under the following four cases of Mode I cyclic loading: (i) constant amplitude zero to tension; (ii) low-to-high block loading; (iii) high-to-low block loading; and (iv) single overload in an otherwise constant amplitude block loading. After calculating the crack opening stress \( \sigma_{op} \) in each, the effective stress intensity factor \( \Delta K_{eff} \) was calculated as a function of the current crack length \( "a" \), in each of the above four cases. Here

\[
\Delta K_{eff} = C_1 \sqrt{\pi(a_0 + N\Delta a)(\sigma_{max} - \sigma_{op})}
\]

where \( C_1 \) is the finite-size correction factor for the present crack geometry, \( N \) is the number of cycles, \( \Delta a \) is the crack growth per cycle. The variations of \( \Delta K_{eff} \) with \( "a" \) in each of the
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Fig. 3. Variation of $\Delta K_{\text{eff}}$ with cyclic crack growth for constant amplitude loading.

Fig. 4. Variation of $\Delta K_{\text{eff}}$ with cyclic crack growth for low-to-high block loading.
Fig. 5. Variation of $\Delta K_{\text{eff}}$ with cyclic crack growth for high-to-low block loading.

Fig. 6. Variation of $\Delta K_{\text{eff}}$ with cyclic crack growth for a single overload.
above four cases of spectrum loading are indicated in Figs. 3–6, respectively. In the case of high-to-low loading, $\Delta K_{eff}$ and hence the growth rate, will eventually return to the lower base line value after further application of the lower level load cycles, unless cyclic stress-strain properties change [10]. In the case of a single overload, Fig. 6, depending on the overload stress ratio, $\Delta K_{eff}$ may actually become zero as further discussed in [11].

Further elaborate results on crack-closure stresses, crack surface deformation profiles, and crack-closure patterns in various cases of spectrum cyclic loading are given in [11]. Based on these results the following conclusions appear to be possible:

(i) If one assumes that in mode I the crack-growth rate is related to the effective stress-intensity factor in the form:

$$\frac{da}{dN} = C(\Delta K_{eff})^n$$

where $\Delta K_{eff} = c_1 \sqrt{\pi a (\sigma_{max} - \sigma_{op})}$ and $n$ is an exponent, the present results for four illustrative types of block loading indicate that: (a) growth retardation occurs in high-to-low and single overload cases, and acceleration in low-to-high block loading, (b) significant delay effects prior to retardation are observed in the case of a single-overload in an otherwise constant amplitude cyclic loading.

(ii) The observed crack-surface deformation profiles, in the different mode I loading cases, have distinctly different characteristic shapes as schematically represented below.

Each of the above crack surface deformation profiles leads to a different pattern of crack-closure when the specimen is unloaded. In the cases of low-to-high loading and single overload, a considerable amount of crack-surface blunting occurs at the instant when the applied load is stepped up; this blunting precludes the possibility of crack-closure behind the location of this blunting in the subsequent load cycling. Thus in these two cases of loading, immediately after the step-up in load level takes place, crack closure occurs only in the small area adjacent to the current crack-tip. Consequently, the total compressive force acting on the closed surfaces in these two cases is also small immediately after the load step-up as shown in Figs 7 and 8 respectively. In Fig. 7, the maximum opening displacement, in each cycle (i.e. at maximum applied load indicated by point A in Fig. 7) at the node closest to the current crack-tip, as well as the total compressive force over the closed area of the crack upon total unloading in each cycle (such as at point B in Fig. 7) are shown for the case of low-to-high loading; while similar results are shown in Fig. 8 for the case of single-overload. In both the cases, it is seen that the total compressive force over the closed crack-surface becomes a minimum right after the step up in the applied load; thus resulting in a lower value of $\sigma_{op}$ at which the crack-opens in the tensile portion of the subsequent loading cycle. These phenomena (brought about essentially by a smaller area of crack-closure due to crack-blunting that occurs at the instant of load step-up) may be responsible for the "delay" effects, e.g. the delayed retardation in the single overload case, and the delayed transition of opening stress levels from the base-line value for lower amplitude block
Fig. 7. Maximum opening displacement at a node closest to crack-tip and total compressive force over closed area near crack-tip in each cycle, in low-to-high block loading.

Fig. 8. Maximum opening displacement at a node closest to crack-tip and total compressive force over closed area near crack-tip in each cycle, in a single overload case.

loading to the higher base-line value for the higher amplitude block loading in the low-to-high loading case.

(iii) As mentioned, growth retardation occurs in the cases of single overload and high-to-low block loading. Observing the crack-surface profiles (not shown here due to space limitation) at the instant of complete unloading in a cycle, for several cycles after the initial overload it is seen that the area of crack closure for both the cases is much larger in comparison with the other two cases. Consequently, the total compressive force in the closed crack-surface is relatively larger in the single-overload and high-to-low cases than in
the other two cases. Thus, in the single overload and high-to-low cases, after the change in applied level, the crack-opening stresses level off at a much higher value than the baseline value corresponding to the level of applied loading after the forementioned change in loading level. From these observations, it appears that the pattern of crack-closure has a significant effect on growth retardation in general spectrum loading.

(iv) The crack surface profiles for the high-to-low loading case possess certain novel features for several cycles after the load-step-down. Crack closure was found to be possible not only at the node closest to the crack-tip, but also at nodes far removed from it, with no closure being observed at nodes in between. In this case also, growth retardation was found to be possible.

(v) Thus, a correlation between the process of a significant crack-blunting at the instant of transition in the amplitude of applied load, and the possibility of growth retardation is found to exist, as summarized below.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Blunting of crack at load-transition</th>
<th>Dip in $\sigma_{op}$ in the subsequent cycle</th>
<th>Delay</th>
<th>Area of crack closure</th>
<th>Retardation/acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant amplitude</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Small</td>
<td>None</td>
</tr>
<tr>
<td>Low-to-high</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Small</td>
<td>Acceleration</td>
</tr>
<tr>
<td>High-to-low</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Large</td>
<td>Retardation</td>
</tr>
<tr>
<td>Single overload</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Large</td>
<td>Retardation</td>
</tr>
</tbody>
</table>

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REFERENCES