Brain Tissue Fragility—A Finite Strain Analysis by a Hybrid Finite-Element Method

This paper deals with the finite-strain, finite-element analysis of the states of stress and strain in the vicinity of a blunt indenter applied to the exposed surface of the pia-arachnoid of an anesthetized rhesus monkey.

Introduction

Much of the present effort in head injury study has been focused on determining the dynamic states of stress and strain developed during impact load with the ultimate objective of designing protective devices. In this process, tolerable limits of states of stress and strain which cause "damage" to the brain tissue must be established and as a result, a coordinated effort by neuropathologists and bioengineers at the University of Washington was initiated to establish such a minimum threshold of brain injuries. The experimental protocol in this investigation consisted in indenting an exposed surface of the pia-arachnoid of an anesthetized rhesus monkey. The extent of hemorrhaging and cell damage determined by histological studies of sections taken from the vicinity of the loading site was then correlated with the computed states of stress and strain within this section for the purpose of establishing a fragility index of the brain which will then relate the minimum states of strain and possibly stress to brain damage. The objective of this paper is to describe briefly a finite-element technique which was used to determine the state of strain in the vicinity of the indenter and to correlate these strains with brain cell damage in rhesus monkeys.

During the last decade, several investigators have extended the finite-element method to large deformation problems [1-6]. An excellent survey of the developments in this field has been presented in [7, 8]. The finite-element procedures used in the references quoted above are usually based on the compatible displacement method and generally require a large number of elements particularly in the region of high stress gradients, such as near points of singularity. A preliminary discussion on the incremental formulation of assumed displacement hybrid finite-element model for large deflection problems was also presented earlier by Pian and Tong [9].

Recently, two of the authors have used the assumed displacement hybrid finite-element method to compute the stress-intensity factors in cracked two-dimensional structures based on linear elasticity [10, 11]. The quadrilateral finite element used in this procedure is composed of a quadratic boundary and quadratic interior displacement fields and a quadratic set of Lagrange multiplier boundary tractions. In addition, singular elements containing 1/\sqrt{r} stress and \sqrt{r} displacement fields were used in the region surrounding the crack tip. Compatibility between all elements was maintained through the use of the foregoing Lagrangian multipliers. Excellent correlation was obtained between available stress-intensity factor solutions and those computed by using as little as 12 elements for a central notched tension plate [10, 11].

In the following, the foregoing assumed displacement hybrid finite-element method is extended to finite-elastic deformation analysis of a region of high strain gradient. It is believed that the present application of the hybrid finite-element model to incremental analysis of finite-strain problems in regions of high-stress gradients is new.

Finite-Element Formulation

Formulation of the assumed displacement hybrid finite-element method for finite-elastic deformation analysis parallels that of infinitesimal elastic deformation analysis [10, 11]. An incremental procedure with equilibrium checks [5] which is based on the incremental equivalent of the variational principle for the displacement hybrid finite-element model for solid continua [12] is used in this study. With this procedure, the large strains in the vicinity of the...
indenter tip can be determined by assuming appropriate displacement fields in elements under and surrounding the indenter, and yet satisfy interelement deformation compatibility by the use of a Lagrange multiplier technique.

In deriving the finite-element equations, for each increment, we assume:

1. An element interior displacement field which is completely arbitrary and if necessary can include proper singularity terms.
2. An independent, inherently compatible element boundary displacement field, which for the quadratic element used in this analysis, involves 16 degrees of freedom.
3. A set of Lagrange multiplier boundary tractions $\Delta T_{Li}$ which involve 16 parameters.

As a generic case, consider the increments from $(N)$th to $(N + 1)$th load step. The incremental variational principle used is $\delta (\Delta x)$ = 0 for all $m$ elements where

$$\Delta \pi = \sum m \left\{ \int_{A_m} \left[ \frac{1}{2} (E_{ijkl}) \delta \epsilon_{ij} \delta \epsilon_{kl} + \frac{1}{2} \sigma_{ij}^0 \Delta u_{n,i} \Delta u_{n,j} \right] dA + \int_{S_m} \Delta T_{Li} (\Delta \bar{u}_i - \Delta u_i) ds 
- \int_{S_m} \Delta \bar{T}_i \Delta u_i ds - \epsilon_m^* \right\}$$ (1)

and

$$\epsilon_m^* = \int_{A_m} \left( - \sigma_{ij}^0 \Delta \epsilon_{ij} + \overline{F_i}^0 \Delta \epsilon_{i} dA + \int_{S_m} \overline{T_i}^0 \Delta \epsilon_{i} ds \right)$$ (2)

$A_m$ = area of the $m$th element $(m = 1, 2, \ldots m)$ in the $N$th state

$\partial A_m$ = entire boundary of the $m$th element in the $N$th state

$S_m$ = a portion of $A_m$ where surface tractions $\Delta T_i$ are actually prescribed in the $N$th state

$\sigma_{ij}^0$ = the Piola initial stress in the $N$th state

$\Delta u_{n,i} (\Delta \epsilon_{ij})$ = the increments of displacements (strains) from $N$th to $(N + 1)$th state

$\Delta u_i$ = any continuously differentiable displacement field used to check the equilibrium of the $N$th referenced state [5]

$E_{ijkl}$ = the current elasticity tensor

$\Delta T_{Li}$ = the incremental Lagrange multiplier boundary tractions

$\Delta u_{ij}$ = independently prescribed increments of element boundary displacements from $N$th to $(N + 1)$th state

$\Delta \bar{T}_i$ = the increments of surface tractions from $N$th to $(N + 1)$th state

$T_i^0$ = the initial surface tractions

$\Delta F_i$ = the increments of body forces from $N$th to $(N + 1)$th state

$F_i^0$ = the initial body forces

$\epsilon_m^*$ = the correction term to "check" the equilibrium of initial stress state in the $N$th state

At each increment of loading, the geometry is continuously updated. Thus during the $N$th load increment, the geometry at the beginning of the $N$th load increment serves as the reference Lagrangian frame from which the incremental displacements are measured. The initial stress and the incremental strains and stresses are referred to the metric of this continuously updated Lagrangian frame. The linear form of incremental strain-incremental displacement relation is thus used for the functional of equation (1), viz.,

$$\Delta \sigma_{ij,k} + \left[ \sigma_{ij}^0 \Delta \epsilon_{ik,j} \right]_3 + \Delta \bar{F}_i = 0$$ (3)

$$\Delta T_{Li} = \Delta \sigma_{ij,k} \mu_j + \sigma_{ij}^0 \Delta \epsilon_{ik,j} \mu_j$$ (4)

Equation (3) is the incremental equilibrium equation for the stresses developed by the assumed interior displacements, equation (4) states that the tractions developed by the assumed incremental interior displacements match the independently prescribed boundary tractions (Lagrange multipliers), and equation (5) is the statement of interelement displacement compatibility that is enforced in the present technique by means of a Lagrange multiplier method. Equation (6) which follows from the variation, $\delta \epsilon_m^*$, implies that the initial stresses in the $N$th stage are in equilibrium.

$$\sigma_{ij,k}^0 + \bar{F}_i = 0 \quad \sigma_{ij}^0 \mu_j = \bar{T}_i^0$$ (6)

Since the initial stress and tractions, $\sigma_{ij}^0$ and $T_i^0$, are known the integral in equation (2) can be written as

$${\epsilon_m^*} = \left\{ \sigma_0 \right\} \left\{ \Delta q \right\}$$ (6)

where $\{Q_0\}$ can be used to measure the residual error in nodal point equilibrium at the beginning of any load step.

Expressing element nodal displacements in terms of the global coordinates for the assembled structure, equation (1) can be written as

$$\Delta \pi = \frac{1}{2} \left\{ \Delta q^* \right\} \left[ S \Delta q^* \right] - \left\{ \sigma_0^* \right\} \left\{ \Delta q^* \right\} - \left\{ \sigma_0^* \right\} \left\{ \Delta q^* \right\}$$ (7)

where $\{\sigma_0^*\}$, $[S]$, and $\{\Delta q^*\}$ are the assembled matrices for the entire solid.

The stationary conditions of $\Delta \pi$ with regard to $\Delta q^*$ then yield

$$\left\{ \sigma_0^* \right\} \left\{ \Delta q^* \right\} - \left\{ \sigma_0^* \right\} \left\{ \Delta q^* \right\} = \left\{ \sigma_0^* \right\} \left\{ \Delta q^* \right\} \cdot$$ (8)

The final incremental algebraic equation can be written as

$$[K(q_i, \rho_i)] \left\{ \Delta Q_i \right\}_{i+1} = \left\{ \Delta Q_i \right\}_{i+1}' \cdot$$ (9)

A fixed Cartesian coordinate system is used in the aforementioned incremental procedure, wherein the equilibrium geometry for each successive increment is continuously updated. For example, consider a finite element of rectangular shape in the initial undeformed, unstressed configuration. After the first incremental load step, the boundaries of this rectangle, according to the present formulation, deform to quadratic curves which are uniquely defined by the displacements at 3 nodes along each boundary segment. Thus, after deformation, the position coordinates of the nodes of the element, which is now of the shape of a quadrilateral with quadratically curved boundaries, can be referred to the original fixed Cartesian coordinate system. In the next increment, one has to assume displacement functions in the curved quadrilateral such that they will be compatible along the interelement boundary. Geometric transformations to map this quadratically curved
quadrilateral to a unit square for ease of element formulation can be found easily. A deformation assumption for this unit square, of the isoparametric type \([13]\), would automatically insure interelement compatibility. This deformation as implied by isoparametricity should be a quadratic function. In regions of high-stress gradient, such as near the indenter in the present problem where the stress state can be singular depending on the root radius of the indenter, additional deformation assumptions of higher order (including built-in "singular" type) may be necessary. The present hybrid displacement formulation enables one to select a completely arbitrary displacement field (including a proper singular type) within the element and enforce the interelement compatibility \(a posteriori\) through the variational principle.

Thus, in the present formulation, at any stage of the incremental process, the nodal position coordinates of a quadrilateral element, with quadratic boundary curves, are first computed with respect to a fixed Cartesian coordinate system. Using fixed Cartesian coordinate systems of \(x_i, y_i\) (\(i = 1-8\) for an eight-node quadrilateral), first a geometrical transformation of this quadrilateral element (with quadratically curved boundaries) to a nondimensional square is made. In applying the hybrid displacement formulation to this unit-square, a displacement variation of arbitrarily high order (including any singular type) is assumed within the element. In addition, a boundary displacement that can be uniquely interpolated in terms of the displacements of the boundary nodes (and may include proper singular type functions) is independently assumed, thus enforcing the interelement compatibility condition. The foregoing discussion also serves to explain the motivation for adopting the present hybrid formulation for problems with regions of high-stress gradient.

In the repetitive application of the incremental procedure, equation \((9)\) is solved for \([\Delta x_i]^{11}\) which is used to update the geometry, and find the stiffness matrix for \((i + 1)th\) step with appropriate modifications in geometry, and a given constitutive relation. A tangent modulus method with iteration could be employed to account for nonlinear constitutive law. We also note that stresses, \(\sigma_{ij}^0 + \Delta \sigma_{ij}\), resulting from load step \(N\), become initial stresses for the step \(N + 1\). For step \(N\), the stresses, \(\sigma_{ij}^0 + \Delta \sigma_{ij}\), are treated as Piola-Kirchhoff stresses referred to the state before the addition of the \(N\)th load step. Thus, for treating the incremental problem corresponding to the \((N + 1)th\) load increment, these total stresses at the end of step \(N\) must be converted to Piola-Kirchhoff initial stresses, \(S_{ij}^0\), referred to the state before the addition of \((N + 1)th\) load increment. The ratio between the stresses, \((\sigma_{ij}^0 + \Delta \sigma_{ij})\) and \(S_{ij}^0\), is

\[
S_{ij}^0 = \left[\frac{\partial x}{\partial X}\right]^{-1} \left[\frac{\partial x}{\partial X}\right] \left[\frac{\partial x}{\partial X}\right] \left[\sigma_{ij}^0 + \Delta \sigma_{ij}\right]. \quad (10)
\]

\([\partial x/\partial X]\) is the determinant of \([\partial x/\partial X]\), \(x_i = X_i + \Delta u_i\), and \(x_i = \delta_{ik} + \Delta u_{ik}\). Furthermore, it can be seen that \([\partial x/\partial X]^{-1}\) can be approximated as \([\partial x/\partial X]^{-1} = (1 - \Delta \sigma_{ij})^{-1} - \Delta u_{ij}\). A similar transformation is used for transforming the surface tractions at the end of \(N\)th load step to initial stresses in the new configuration at the beginning of the \((N + 1)th\) load step.

**Indentation of Monkey Brain**

Four holes, 13 mm in dia, were drilled, two on the frontal suture and two close to the occipital ridge of the anesthetized rhesus monkey skull. The dural matter at the incision was removed and the arachnoid layer of the meninges was exposed. An indenter device with a blunt steel blade of 10 mm width and 1 mm thickness was then inserted into these holes and the indenter blade was depressed by a drop weight. Attempts were made to avoid sites of major folds, convolutions, and arteries in the actual indentation tests. Obviously the complex and irregular geometries could not be avoided entirely in actual experiments, but these effects were minimized by averaging many experimental data. After the last indentation, the monkey was perfused through the heart with formalin. The fixed brain was then removed and sectioned serially normal to blade indentation, stained with Verhoeff Van Giesen and observed under a 200 power microscope for neuron darkening.

Fig. 1 shows a typical density pattern of such darkened neurons in approximately a 2.5 mm square region under the indenter. This density pattern was obtained by averaging 30 density diagrams of darkened neurons obtained from the four indentation sites of 15 rhesus monkeys. Many experimental data were discarded due to a variety of causes, such as obvious geometric interferences caused by folds, poor perfusion, and excessive hemorrhaging. Since the darkening of the neurons indicated a trauma in the nerve cells, efforts were then made to correlate the minimum threshold of neuron darkening with the mechanical state at the time of indentation.

**Finite-Element Idealization**

In the modeling of the foregoing indentation experiment, the large strain gradient in the vicinity of the indenter in this problem introduces additional complexity in the finite-element analysis of nonlinear material. Additional complexity peculiar to biological structural analysis such as possible nonhomogeneous and anisotropic mechanical properties of the pia mater and its complex folds in the subarachnoid space made it necessary to introduce some gross simplification. First, the brain was assumed to be a homogeneous and isotropic material with a smooth exposed surface. This assumption was necessary despite our capability of incorporating nonhomogeneous and anisotropic material properties in the developed computer code, due to the lack of such experimental data in the literature.

Since only the local states of stress and strain in the vicinity of the indenter were of interest in this analysis, only a local section, the brain cross section, consisting of a rectangular block of 6 mm \(\times\) 16 mm and constrained in parts by the interior surface of the skull as shown in Fig. 1, was considered. Displacement boundary conditions were prescribed at the left and lower edges of the rectangular cutout. These prescribed displacements were obtained from another extremely simplified analysis of the idealized entire rhesus monkey brain, 40 mm \(\times\) 60 mm in size, subjected to the same indentation \([14]\).

The assumed state of plane strain in Fig. 1 was justified since an indenter of 10 mm width and 1 mm thickness was used in the indentation tests. Thus one can reasonably assume that for a 1 to 2 mm indentation depth a two-dimensional plane-strain state existed in the central planar cross section of 2.5 mm \(\times\) 2.5 mm along 1/3 of the indenter width. Also note that the contact area of the indenter modeled in this finite-element analysis is about 0.2 mm and not the full indenter thickness of 1 mm.

For the moderately fast drop weight velocity, 1 m/sec, involved in the indentation test, elastic response of the brain was postula-
The load duration of 1 to 2 sec after indentation justified the use of quasi-static analysis to study the structural response of the brain subjected to the indentation test.

As mentioned previously, the anisotropic elastic properties of a rhesus monkey brain under multiaxial loading conditions are not available nor could they be determined experimentally within the scope of this investigation. The elastic properties of the brain were thus taken from literature which showed that the isotropic, nonlinear compressive moduli of free-standing specimens taken from human and from rhesus monkey brains were almost identical [15]. The compressive true stress versus true strain curves for human brains at the available slowest loading rate of 0.1 sec\(^{-1}\) were used in analyzing the rhesus monkey brain. In terms of effective stress, \(\sigma_e\), versus effective strains, \(\varepsilon_e\), the uniaxial experimental results in reference [15] can be represented as

\[
\sigma_e = 10.00 \left( \frac{\mu_{d0} \varepsilon_e}{1.0} \right) \text{ gwt/mm}^2
\]

\[
\sigma_e = \sqrt{\frac{1}{3} S_{ij} S_{ij}}
\]

\[
\varepsilon_e = \sqrt{\frac{1}{3} \varepsilon_{ij} \varepsilon_{ij}}
\]

\(S_{ij}, \varepsilon_{ij}\) are stress and strain deviators, respectively.

Since the maximum calculable Lagrangian strain is of the order of 0.2, the effective stress-strain curve represented by equation (11) is essentially linear in this strain range and thus a constant shear modulus of \(G = 4.6 \text{ gwt/mm}^2 = 4.5 \times 10^4 \text{ dynes/cm}^2\) was used. This value is about two-thirds the \(G_1\)-value of a recently determined average complex shear modulus, \(G = G_1 + iG_2\), for human brain tissue [16].

The Poisson’s ratio necessary for this isotropic elastic analysis is normally assumed to be 0.5 for biological structures which are considered incompressible. Structural analysis involving incompressible materials introduces an added undetermined quantity, the hydrostatic state of stress, to the yet-to-be-determined six stress components and complicates the solution procedure. Although various solution procedures have been proposed for handling structural analysis involving incompressible materials, these procedures were not incorporated in this investigation due to the added complexity of the already complex solution procedure. A physical justification for not considering incompressible material is that liquid in this liquid-filled porous medium tends to pour out under compression and thus the apparent Poisson’s ratio would be less than 0.5. In this analysis, a Poisson’s ratio of 0.49 was thus used.

Computation Procedure

Fig. 2 shows the finite-element breakdown of one half of the 6 mm \(\times\) 16 mm plane-strain cross section of a segment of the rhesus monkey brain. A total of only 48 elements were necessary in this finite-element breakdown despite the large strain gradient in the vicinity of the plunger. Since much of the pathological evidence associated with the indentation was concentrated in a 2.5 mm \(\times\) 2.5 mm brain region surrounding the indenter, these local regions were sufficiently large to encompass the high strains in this small region.

For the first increment of loading, which is about 0.49 mm indentation, three iterations of the equilibrium check of equation (9) were necessary which increased the indentation to 0.55 mm. Convergence of the numerical procedure was determined by the decrease in the ratio of the norms of the residual vector and total load vector of \(\varepsilon^{**} = (\varepsilon^{**})^T/\varepsilon^{**}\). The iteration was then repeated and this ratio of \(\varepsilon^{**}\) decreased to a value lower than 0.05.

The second increment of loading for indentation of 1.15 mm also required three iterations before \(\varepsilon^{**}\) became smaller than 0.05. The equilibrium check and associated iteration procedures increased the indentation to 1.29 mm.

Results

Figs. 3 and 4 show distributions of the maximum principal strain.
for the two indentations of 0.55 mm and 1.29 mm, respectively. The deformed free surfaces of the brain under these two indentations are shown in Fig. 2. The obvious region of maximum strain is immediately under the indenter. These strain contours follow a characteristic inverted bell-shape at all levels of indentation.

A comparison of the maximum strain distributions in Figs. 3 and 4 and the neuron density pattern shown in Fig. 1 indicates a threshold maximum normal strain of 0.2 ~ 0.4 where the brain cells retain the effects of the indenter. This definite correlation suggests that the mathematical model of the rhesus monkey brain subjected to a constant indentation, upon refinement, could be used to delineate structural parameters and loading conditions which affect the brain cells to retain memory of the indenter loading for a considerable period after impact.

Fig. 5 shows the distributions of shear strain, $\varepsilon_{xy}$, for the indentation of 1.29 mm. This shear strain could mechanically shear the horizontal arterioles or venules emanating from arteries vertical to the cortex surface and may thus cause hemorrhage close to the indenter. Although the brains were perfused prior to fixing in actual experiments, regions of residual hemorrhage suggest that some arterioles in the high $\varepsilon_{xy}$ regions did shear, thus providing some verification of this hypothesis.

Conclusions

An assumed displacement hybrid finite-element method suitable for finite-elastic deformation of anisotropic material was developed. This numerical procedure was then used to determine the strain distribution in the vicinity of a blunt indenter applied to the exposed surface of the pia-arachnoid of an anesthetized rhesus monkey. A comparison of the numerical results and neuro-pathological results indicates that a maximum strain of $0.2 ~ 0.4$ is necessary for the brain cells to retain the effect of indentation.

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