Applications of the three dimensional finite element alternating method

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Abstract

This paper describes some recent applications of the three dimensional finite element alternating method (FEAM). The problems solved involve surface flaws in various types of structure. They illustrate how the FEAM can be used to analyze problems involving mechanical and thermal loads, residual stresses, bonded to composite patch repairs, and fatigue.

Keywords: Finite element alternating method (FEAM); Fracture mechanics; Fatigue; Surface flaw; Bonded composite patch repairs; Residual stress; Thermal gradient

1. Introduction

The presence of cracks in complex three dimensional components is always a cause of concern to engineers. One reason for this concern is the fact that accurate calculation of stress intensity factors using the finite element method or some other numerical technique, is a non-trivial task. Conventional three dimensional finite element fracture mechanics techniques require the construction of an extremely fine mesh in the vicinity of the curved crack front with a large number of elements in this region. This is likely to be very time consuming both from a mesh generation stand-point and also with regard to the computational time. Analysis difficulties are compounded in many instances due to the sub-critical crack growth that can take place under conditions of cyclic loading. The effort required to integrate a fatigue analysis directly with conventional finite element stress intensity factor calculations is prohibitive due to difficulties inherent in ensuring that the mesh must coincide with the crack front during the growth process.

The finite element alternating method (FEAM), which has been developed by Atluri and his coworkers [1, 2] is an innovative computational scheme for stress intensity factor calculation that offers significant savings in time without sacrificing any accuracy. The advantages of utilizing FEAM to analyze a structure containing elliptical or part elliptical cracks are many. However, they are all a result of the fact that only the uncracked structure is modeled with finite elements. The
crack is specified simply by giving the major axis, minor axis and center coordinates of the ellipse which models the crack. As a consequence, the FEAM is extremely efficient from both a computational and manpower point of view when performing parametric studies of crack size, shape and location because the finite element mesh remains the same. This property makes FEAM ideal for performing fatigue crack growth calculations of elliptical or part elliptical cracks.

Over the past 15 years, the FEAM has evolved from the domain of basic research into a tool of use to engineers solving real world problems. The purpose of this paper is to cover some recent applications of the FEAM. They illustrate how the FEAM can be used to analyze problems involving mechanical and thermal loads, residual stresses, bonded composite patch repairs, and fatigue.

2. Finite element alternating method

In the FEAM, two solutions are required.

**Solution 1.** A general analytical solution for an elliptical crack is an infinite body subject to arbitrary crack face tractions [1].

**Solution 2.** A numerical scheme (the finite element method in this instance) to solve for the stresses in an uncracked finite body.

These two solutions are used iteratively in the FEAM as follows (see also Fig. 1).

1. The uncracked finite body under the given external tractions is solved using the finite element method. The uncracked body has the same geometry as in the given problem except for the crack.
2. The finite element solution is used to compute the residual stresses at the crack location.
3. The residual stresses at the crack location, as computed in Step 2, are reversed to create the traction free crack faces as in the given problem.
4. The analytical solution for the infinite body problem with the crack faces subjected to the tractions determined in Step 3 is evaluated.
5. The stress intensity factors for the current iteration are determined from the analytical solution in Step 4.
6. The residual tractions on the external surfaces of the finite body due to the applied tractions on the crack faces of the infinite body are also computed from the analytical solution in Step 4. These residual tractions are then reversed and applied to the uncracked, finite body in Step 1.

All the steps in the iteration process are repeated until the residual stresses on the crack faces become negligible. It has been observed that this iteration process typically takes three to five cycles [2]. The overall stress intensity factor solution is obtained by adding the stress intensity factor solutions for all iterations.

2.1. Fatigue crack growth algorithm

Exploiting the fact that the crack front does not need to be modeled explicitly in the FEAM and thus only one finite element mesh is necessary to model various crack sizes and shapes, it is
relatively straightforward to utilize the FEAM to perform fatigue crack growth calculations. The steps involved in a fatigue crack growth calculation under constant amplitude loading are as follows.

1. Specify initial and final minor axis sizes and divide the interval into a number of subintervals.
2. The number of cycles for minor axis crack growth in a given subinterval is calculated using a crack growth equation and the stress intensity factors (for the crack size and shape at the beginning of the subinterval) generated by a FEAM analysis.
3. The increment in major axis crack growth is determined by utilizing the number of cycles determined in Step 2, the crack growth equation and the stress intensity factors (for the crack size and shape at the beginning of the subinterval) generated by a FEAM analysis.
4. If the minor axis size has reached the specified final size, then stop. Otherwise, go to Step 2.

The analysis of fatigue crack growth due to a variable amplitude loading spectrum is a problem of great practical interest. While it is possible to perform a cycle by cycle fatigue crack growth analysis using the FEAM, this is not in general necessary. Rather, the present work has combined the software packages SAFEFLAW3D (which implements the FEAM) and NASA FLAGRO [3] for this purpose. NASA FLAGRO contains advanced features that can be used to consider fatigue crack growth under variable amplitude loading. Thus, by combining the efficient stress intensity factor calculation capability of SAFEFLAW3D with the fatigue crack growth capabilities of NASA FLAGRO, a powerful tool for performing fatigue crack growth analysis of elliptical and part elliptical cracks is created. Fig. 2 illustrates this concept.

2.2. Composite patch algorithm

The FEAM, as presented to this point, is a powerful tool for generating stress intensity factors for cracks in homogeneous bodies. However, its application to structure having adhesively bonded composite patches requires the introduction of a two step analysis procedure. This is because the analytical solution for an infinite body containing an elliptical crack, whose faces are subject to arbitrary tractions, is available only for certain homogenous bodies. The two step analysis procedure used to generate stress intensity factors for structures repaired with adhesively bonded composite patches is as follows.

1. Perform a finite element analysis of the entire structure (i.e. original structure, adhesive and composite patch) under the given external loading. In this analysis, release the nodes at the location of the crack, but make no attempt to model the singular stress field which exists at the crack. Determine the equivalent nodal loads which exist at the interface of the adhesive and original structure.
2. Perform the previously described FEAM analysis of the original structure with the initial external loading consisting of the given loading and the equivalent nodal loads calculated in Step 1.

Step 1 can in general be done using any finite element software which allows the user to model the original structure with 3D solid elements, adhesive with shear elements and the composite patch with 3D solid elements or shell elements. Step 2 requires a special purpose code which implements the FEAM. Recently, this two step procedure was automated in the COMPAT_3D software [4].
3. Applications

3.1. C-141 lower wing weep hole cracking

Fatigue cracks which initiate at weep holes located in the risers emanating from the lower wing surface panels have plagued the C-141 for some time (see Figs. 3–5). Such cracking was first observed in full-scale wing/fuselage durability tests. As a result of these observations, a rework procedure which consisted of reaming followed by low-interference cold working was developed for the weep holes. Despite the application of this rework procedure, weep hole cracking continues to adversely affect the structural integrity of the C-141 fleet.

The Air Force has recently begun using composite patches to repair lower wing panels in which weep hole cracks have been found. In support of this repair effort, a computational procedure using the FEAM was developed which allows engineers to perform rapid Damage Tolerance Analyses (DTA) of repaired and unrepaired lower wing panels. This procedure allows for the efficient performance of residual strength and fatigue calculations of repaired and unrepaired lower wing panels containing part elliptical cracks.
The modeling of fatigue crack growth of a corner crack located at the weep hole in a C-141 lower wing panel/riser and the calculation of the reduction in stress intensity factor due to the application of adhesively bonded composite patches is demonstrated in this section. Numerical results presented consist of a comparison of fatigue calculations performed on a model of an unrepaired weep hole with test data as well as repaired and unrepaired stress intensity factors of part elliptical weep hole cracks. The geometry and material parameters of the C-141 lower wing panel/riser are given in Fig. 6. A critical stress intensity for a part elliptical crack in the riser was calculated to be 57.6 ksi√/in using NASA FLAGRO. This number was based on a yield stress of 65 ksi, a plane
strain fracture toughness of 27 ksi\(\sqrt{\text{in}}\) and a riser thickness of 0.18 in. The limit stress for the lower wing panel/riser is 34 ksi. All applied loads in the analyses are uniformly distributed over the cross section of the panel/riser as shown in Fig. 7. The mesh in Fig. 7 consists of 59,720 node brick elements and is used in all analyses except as noted.
Fig. 7. Weep hole mesh.
3.1.1. Fatigue results

The fatigue crack growth calculations were done assuming a Forman model. The Forman relation is given by:

$$\frac{da}{dn} = \frac{C(\Delta K)^n}{(1 - R)K_c - \Delta K}$$

where $a$ is the crack length, $n$ is the number of cycle, $R$ is the ratio of $K_{max}$ to $K_{min}$, and $K_c$ is the critical stress intensity which was given previously. Experimental fatigue crack growth data for Al 7075-T651 was input into NASA FLAGRO, which determined the constants $C$ and $n$ to be 0.2112E-05 and 2.531, respectively. The crack was assumed to grow from a corner flaw below the weep hole as shown in Fig. 8. The initial crack dimensions were 0.07 in along the bore of the weep hole and 0.05 in down the riser.

The variable amplitude spectrum used was comprised of peak–valley pairs representing six 504.5 flight hour passes or 3027 total flight hours. The number of peak–valley pairs is somewhat different for each pass in order to account for cycles that occur as little as once in 3027 flight hours and for an accumulation of fractional occurrences in the correct sequence. The data was filtered to remove
stress differences of less than 2 ksi. The highest peak in the data was 17.89 ksi while negative loads were treated as zero in the present analysis.

The fatigue crack growth results are presented in terms of major and minor axis growth in Fig. 8. These results were produced using the previously outlined combination of NASA FLAGRO and SAFEFLAW3D. The results indicate that the crack would grow through the riser in approximately 1513 flight hours. The results were obtained in 1 hour on an HP 9000/735. Limited comparison was possible with weep hole fatigue test data. A corner crack in a test specimen, which had an initial crack size and shape very close to that of the numerical analysis, propagated completely along the bore somewhere between 1475 and 3027 flight hours. While the 1513 flight hours predicted by analysis falls in this range, it appears that the test data indicates a somewhat higher time for the
Fig. 10. Repair of broken riser above the weep hole and corner crack below.

crack to propagate along the bore of the weep hole. There are several reasons which likely contribute to discrepancies that exist between the computational model and the test data. Firstly, the initial crack size along the bore of the weep hole was not known exactly. More precise measurements are needed to describe the progression of the crack in this direction. Also, the crack growth constants used in the fatigue calculation were not based on crack growth data of the Al 7075-T651 used in the tests. Finally, crack closure effects, which would reduce fatigue crack growth rates were not accounted for in the present computational model.

3.1.2. Composite patch results

Reductions in stress intensity factor due to adhesively bonded composite patch repairs of cracked panels/risers were studied for four separate cracking cases. The patch and adhesive geometry and material parameters are given in Fig. 9 (These are generic in nature). The patch
material properties are representative of an 8 ply, uni-directional boron-epoxy. For each cracking case, two patching schemes are compared. Patch 1 models the riser patches only while Patch 2 models the riser patches as well as a patch on the outer surface of the lower wing panel. This is done to study the effect that parasitic stiffening of the structure due to the patches has on stress intensity factors. The parasitic stiffening tends to induce local bending of the structure in the case of Patch 1 while this effect is lessened in the case of Patch 2.

Results for the four cracking cases are given in Figs. 10–14. The stress intensity factors presented are for a 1 ksi unit load. The CPU time required for each run on a HP 9000/735 was about 11 min for the 597 element mesh. Results for case 1, which required an 868 element mesh due to the severe stress concentration (CPU time = 25 min), indicate that while Patch 1 and Patch 2 significantly reduce the stress intensity factors, they do not restore sufficient residual strength to allow the structure to sustain the limit load of 34 ksi. Results for cases 2, 3 and 4 show significant reductions in stress intensity factors due to the composite patches. In all these cases, the Patch 2 scheme is
Table 1
Repaired and unrepaired stress intensity factors

<table>
<thead>
<tr>
<th></th>
<th>COMPAT 3D</th>
<th>Jones and Callinan</th>
<th>Newman and Raju</th>
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<td>$K_{I}$</td>
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<td>$K_{II}$</td>
<td>12.3</td>
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<tr>
<td><strong>SIF with patch repair</strong></td>
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<tr>
<td>$K_{I}$</td>
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<td>7.2</td>
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<tr>
<td>$K_{II}$</td>
<td>3.1</td>
<td>3.5</td>
<td>*</td>
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Fig. 12. Repair of corner crack above the weep hole.
superior to that of Patch 1. This is particularly true for cracks below the weep hole which tend to be affected to a greater extent by the local bending induced by the parasitic stiffening of Patch 1.

3.2. Macchi and Mirage III main landing gear repair

Another application of the FEAM to the analysis of composite patch repairs dealt with a generic composite patch repair typical of ones made to the Macchi and Mirage III main landing wheels [5]. The COMPAT_3D software was used in this analysis. COMPAT_3D has the ability to analyze the effect of size, shape, thickness, tapering and material properties of the composite patch on the crack-tip stress intensity factors and adhesive shear stresses.

The problem is idealized as a semi-elliptical surface crack centrally located in a rectangular block. Fig. 15 shows one quarter of the problem modeled. COMPAT-3D was used to calculate the stress intensity factors along the crack front in both the repaired and unrepaired structure. Table 1
Fig. 14. Repair of corner crack above and below the weep hole; lower crack solution.

Fig. 15. Repair of surface flaw (1/4 of structure modeled).
Fig. 16. Adhesive stresses acting on aluminium block.

Fig. 17. Repaired and unrepaired crack opening displacements.

compares the stress intensity factors calculated with COMPAT_3D with those given in [4] (point $d$ is the point of deepest penetration while point $s$ is the point where the crack intersects the free surface). In addition, the Newman and Raju solution for the unrepaired case is given in Table 1. The agreement between solutions is good. It is noted that the solution obtained in [4] and the
Fig. 18. Geometry of plate and surface flaw.

Fig. 19. Finite element mesh of plate with residual stresses.
Residual Stress: $a/t = a/c = 0.2$

![Graph showing normalized stress intensity factors for $a/t = a/c = 0.2$.](image)

Fig. 20. Normalized stress intensity factors for $a/t = a/c = 0.2$.

Residual Stress: $a/t = a/c = 0.6$

![Graph showing normalized stress intensity factors for $a/t = a/c = 0.6$.](image)

Fig. 21. Normalized stress intensity factors for $a/t = a/c = 0.6$. 
Newman and Raju solution used explicit crack tip meshing, which is not necessary with COMPAT_3D. Thus, solutions of at least the same accuracy can be obtained with COMPAT_3D with only a fraction of the effort (i.e. no remeshing for different crack sizes and locations). Additional output which can be obtained from COMPAT_3D are shown in Figs. 16 and 17. Fig. 16 shows the adhesive stresses acting on the aluminium block while Fig. 17 compares the crack opening displacements of the repaired and unrepaired case. In Fig. 17, note how the crack is not modeled explicitly with finite elements.

3.3. Surface crack in a welded plate

A surface crack in a plate was analyzed by the FEAM. The loading of the plate was due to a welding induced residual $\sigma_{zz}$ stress field. Fig. 18 shows the plate geometry. The residual stress field
is defined as:

\[ \sigma_{zz} = \begin{cases} 
(x/c)^3 - 4.5((x/c)^2 + 1), & -c \leq x \leq c, \\
0, & |x| > c.
\end{cases} \]

Fig. 19 contains two views of the mesh used to solve the problem with the FEAM. Due to symmetry, only one quarter of the problem is modeled. The center of the elliptical surface crack is located at (0, 0, 0). The normalized stress intensity factors for \( a/t = 0.2 \) and 0.6 are shown in Figs. 20 and 21, respectively. Here \( a \) is the length of the ellipse semi-minor axis, \( c \) the length of the ellipse semi-major axis and \( t \) the thickness of the plate. The agreement with the finite element solution due to Shiratori, Miyoshi and Tanikawa is seen to be very good. In order to demonstrate that the FEAM does not require explicit crack tip modeling in order to capture the deformation near the crack front, Fig. 22 shows crack opening displacements for the \( a/t = a/c - 0.2 \) and 0.6 cases.
3.4. **Surface flaw in a pipe with thermal gradient**

A surface crack in the inner wall of pipe was analyzed by the FEAM. The loading was due to a through the wall temperature gradient applied to the pipe. Fig. 23 shows the pipe geometry and mesh used to solve the problem. Due to symmetry, only one half of the problem was modeled. The center of the semi-elliptical crack is at \((0, 10, 0)\). The equation defining the temperature gradient is:

\[
T(r) = T_o + (T_i - T_o) \frac{\ln (R_o/r)}{\ln (R_o/R_i)}
\]

The temperature on the outer surface \((T_o)\) was taken to be 200 while that of the inner surface \((T_i)\) was 100. The normalized stress intensity factor for \(a/t = 0.5, a/c = 0.4\) and \(R_i/t = 10\) is shown in Fig. 24. The agreement with the finite element solution of Raju and Newman is very good.

4. **Conclusions**

Over the past 15 years, the FEAM has evolved from the domain of basic research into a tool of use to engineers solving real world problems. The technique provides accurate solutions and is very efficient in terms of manpower and computational times. This paper has reviewed some recent applications of the FEAM. They illustrate how the FEAM can be used to analyze problems
involving mechanical and thermal loads, residual stresses, bonded composite patch repairs, and fatigue.

References