Abstract: In the present paper, stress intensity factor (SIF) analyses and fatigue-crack-growth simulations of corner cracks emanating from loaded pinholes of attachment lugs in structural assemblies are carried out for different load cases. A three-dimensional (3D) symmetric Galerkin boundary-element method (SGBEM) and FEM alternating method is developed to analyze the nonplanar growth of these surface cracks under general fatigue. The 3D SGBEM-FEM alternating method involves two very simple and coarse meshes that are independent of each other: (1) a very coarse FEM mesh to analyze the uncracked lug, and (2) a very coarse SGBEM mesh to model only the growing crack surface. By using the SGBEM-FEM alternating method, the nonplanar growth of cracks in 3D (surfaces of discontinuity) up to the failure of structures are efficiently simulated, and accurate estimations of fatigue lives are made. The accuracy and reliability of the SGBEM-FEM alternating method are verified by comparing them to other FEM solutions, as well as experimental data for fatigue-crack growth available in the open literature. The SIF calculations, crack surface evolutions, and fatigue-life estimations are all in good agreement with other detailed FEM solutions and experimental observations. It is noted that for fracture and fatigue analyses of complex 3D structures such as attachment lugs, a pure FEM requires several hundreds of thousands or even millions of elements, whereas the present 3D SGBEM-FEM alternating method requires only up to 1,000 FEM elements and ~100 SGBEM elements. It thus demonstrates that the present SGBEM-FEM alternating method, among the many Schwartz-Neumann-type alternating methods developed in the past 20–30 years are suitable for analyzing fracture and fatigue-crack propagation in complex 3D structures in a very computationally efficient manner, as well as with very low human labor costs. DOI: 10.1061/(ASCE)EM.1943-7889.0000870. © 2014 American Society of Civil Engineers.

Introduction

In various mechanical systems, attachment lugs are employed to connect different components. They are mostly subjected to cyclic loading and are transferred through a single bolt or pin without clamping. This leads to a combination of high stress concentration and fretting and can potentially result in crack initiation and fatigue-crack growth near the pinhole. To properly ensure the operational safety of these mechanical systems, it is of great importance to perform fatigue-crack growth analyses and damage control of attachment lugs.

Over the past several decades, several numerical and experimental studies have been conducted to understand the fatigue behavior of attachment lugs, with either constant or variable amplitude loads (Smith et al. 1977; Schijve and Hoeymakers 1979; Moon 1980). Meanwhile, several fatigue laws with appropriate criteria for either the initiation phase or the crack growth phase were also developed. As is well known, Paris and Erdogan (1963) introduced an empirical relationship between the crack growth rate and the range of effective stress intensity factor (SIF), which laid a solid foundation for future research. Elber (1971) proposed the crack closure concept by modifying Paris’ law. Erdogan and Roberts (1965), Walker (1970), and Kujawski (2001) examined the effect of mean stress and introduced the two-parameter driving force model, in which both the maximum SIF and the range of SIF are used to determine the crack growth rate. In all these fatigue laws, the evaluation of SIFs, which characterize the singular stress field near the crack front, are crucial for the fatigue-crack growth analyses. Thus, various methods were developed to obtain the approximate solutions of SIFs of different cracked structural components (Carpinteri 1993, 1994; Qian and Fatemi 1996; Tada et al. 2000; Carpinteri et al. 2006; Rozumek 2009; Carpinteri and Vantadori 2009a, b; Boljanović and Maksimović 2011; Maksimović et al. 2011; Boljanović 2012).

The initial crack shape in the attachment lug can often be approximated as either a quarter-elliptical corner crack or a through-the-thickness crack; and the former can grow and become the latter. Raju and Newman (1979) obtained the SIFs for different corner cracks by using a three-dimensional (3D) FEM. Schijve (1985) developed a simple interpolation method to compute the SIFs for corner cracks in lugs. Atluri and Kathiresan (1978) developed the hybrid 3D crack elements to model various embedded and surface cracks. Heliot et al. (1980) obtained the SIFs of cracks by applying the boundary integral equation (BIE) method. Shah and Kobayashi (1972) studied the same problems employing the alternating method. Sih and Li (1990) suggested that the strain energy density function can be used for the computation of SIFs.

Despite its widespread popularity, the traditional FEM, with simple polynomial interpolations, is unsuitable for modeling cracks and their fatigue growth. This is partially due to the inefficiency of approximating stress and strain singularities using polynomial FEM shape functions. To overcome this difficulty, embedded singularity elements by Tong et al. (1973) and Atluri et al. (1975), and singular...
quarter-point elements by Henshell and Shaw (1975) and Barsoum (1976), among others, were developed in the 1970s to capture the crack-tip/crack-front singular field. Many related developments were summarized in the monograph by Atluri (1986), and they are now widely available in commercial FEM software. However, the need for constant remeshing makes the automatic fatigue-crack propagation analyses with FEM extremely difficult. In a fundamentally different way, the first paper on a highly accurate finite-element (Schwartz-Neumann) alternating method (FEAM) was published by Nishioka and Atluri (1983). The FEAM uses the Schwartz-Neumann alternation between a crude and simple finite-element solution for an uncracked structure, and the analytical solution for an infinite body containing the crack. The success of this method is largely attributed to the work of Vijayakumar and Atluri (1981), in which analytical solutions were derived for an elliptical crack in an infinite domain subjected to arbitrary crack surface tractions. Subsequent 3D and two-dimensional (2D) variants of the FEAMs were successfully developed and applied to perform structural integrity and damage tolerance analysis of many practical engineering structures (Atluri 1997). Recently, the SGBEM-FEM alternating method, which involves the alternation between the very crude FEM solution of the uncracked structure and a SGBEM solution for a small region enveloping the arbitrary nonplanar 3D crack, was developed for arbitrary 3D nonplanar growth of embedded as well as surface cracks (Nikishkov et al. 2001; Han and Atluri 2002). Dong and Atluri (2012, 2013a, c) also developed a SGBEM superelement for direct coupling of SGBEM and FEM, for fracture and fatigue analysis of complex 2D solid structures and materials. The motivation for this series of works, by Atluri and many of his collaborators since the 1980s, is to exploit the advantages of each computational method: modeling complicated and large-scale uncracked structures with simple FEMs, and modeling crack singularities by mathematical methods such as complex variables, special functions, BIEs, and SGBEMs.

In the present paper, the fatigue growth of corner cracks emanating from the pinholes of attachment lugs is studied in detail by employing the 3D SGBEM-FEM alternating method (Han and Atluri 2002; Dong and Atluri 2013b). The SIFs of the crack front are determined by Paris’ law. The crack paths and number of loading cycles are predicted for damage tolerance of the attachment lugs. The validations of the 3D SGBEM-FEM alternating method are illustrated by the comparison of numerical results with available empirical solutions and experimental observations.

### SGBEM-FEM Alternating Method: Theory and Formulation

The SGBEM has several advantages over collocation/direct and dual BEMs (Rizzo 1967; Hong and Chen 1988). It leads to a symmetric coefficient matrix and avoids the need to have a special treatment of sharp corners. Early derivations of SGBEMs involve regularization of hypersingular integrals (Frangi and Novati 1996; Bonnet et al. 1998; Li et al. 1998; Frangi et al. 2002). A systematic procedure to develop weakly singular symmetric Galerkin BIEs was presented by Han and Atluri (2003, 2007). The derivation of this simple formulation involves only the non-hypersingular integral equations for tractions, based on the original works by Okada et al. (1988, 1989). It was used to analyze 3D solids with embedded and surface flaws by Nikishkov et al. (2001) and Han and Atluri (2002).

For a domain of interest in Fig. 1, with the source point denoted by \( \mathbf{x} \) and the target point denoted by \( \mathbf{\xi} \), 3D weakly singular symmetric Galerkin BIEs for displacements and tractions were developed by Han and Atluri (2003).

![Fig. 1. Solution domain with source point \( \mathbf{x} \) and target point \( \mathbf{\xi} \)](image)

The displacement BIE is

\[
\frac{1}{2} \int_{\partial \Omega} v_p(\mathbf{x}) u_p(\mathbf{x}) \, dS = \int_{\partial \Omega} v_p(\mathbf{x}) \, dS \int_{\partial \Omega} t_j(\mathbf{\xi}) u_j^{\infty}(\mathbf{x}, \mathbf{\xi}) \, dS \xi
\]

\[
+ \int_{\partial \Omega} v_p(\mathbf{x}) \, dS \int_{\partial \Omega} D_{ij}(\mathbf{\xi}) u_j G_{ij}^\infty(\mathbf{x}, \mathbf{\xi}) \, dS \xi
\]

\[
+ \int_{\partial \Omega} v_p(\mathbf{x}) \, dS \int_{\partial \Omega} n_i(\mathbf{\xi}) u_j \varphi_{ij}^{\infty}(\mathbf{x}, \mathbf{\xi}) \, dS \xi
\]

And the corresponding traction BIE is

\[
-\frac{1}{2} \int_{\partial \Omega} w_b(\mathbf{x}) v_b(\mathbf{x}) \, dS = \int_{\partial \Omega} v_b(\mathbf{x}) \, dS \int_{\partial \Omega} t_j(\mathbf{\xi}) G_{ij}^{\infty}(\mathbf{x}, \mathbf{\xi}) \, dS \xi
\]

\[
- \int_{\partial \Omega} w_b(\mathbf{x}) \, dS \int_{\partial \Omega} n_i(\mathbf{\xi}) \varphi_{ij}^{\infty}(\mathbf{x}, \mathbf{\xi}) \, dS \xi
\]

\[
+ \int_{\partial \Omega} v_b(\mathbf{x}) \, dS \int_{\partial \Omega} D_{ij}(\mathbf{\xi}) u_j(\mathbf{\xi}) H_{ij}^{\infty}(\mathbf{x}, \mathbf{\xi}) \, dS \xi
\]

In Eqs. (1) and (2), \( D_{ij} \) is a surface tangential operator

\[
D_{ij}(\mathbf{\xi}) = n_r(\mathbf{\xi}) \varphi_{ij}^{\infty} \left( \frac{\partial}{\partial x_r} \right)
\]

Kernels functions \( u_j^{\infty}, G_{ij}^{\infty}, \varphi_{ij}^{\infty} \), and \( H_{ij}^{\infty} \) can be found in the paper by Han and Atluri (2003), which are all weakly singular, making the implementation of the current BIEs very simple.

As shown in Fig. 2, by applying Eq. (1) to \( S_a \), where displacements are prescribed, and applying Eq. (2) to \( S_t \) where tractions are prescribed, a symmetric system of equations can be obtained

\[
\begin{bmatrix}
A_{pp} & A_{pq} & A_{pr} \\
A_{qp} & A_{qq} & A_{qr} \\
A_{rp} & A_{rqr} & A_{rr}
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} =
\begin{bmatrix}
f_p \\
f_q \\
f_r
\end{bmatrix}
\]

where \( p, q, \) and \( r = \) unknown displacements at \( S_a \), unknown displacements at \( S_t \), and unknown displacement discontinuities at \( S_c \), respectively.
With the displacements and tractions being first determined at the boundary, the displacements, strains, and stress at any point in the domain can be computed using the non-hypersingular BIEs in the papers by Okada et al. (1988, 1989). Therefore, the path-independent or domain-independent integrals can be used to compute the SIFs. However, with the singular quarter-point boundary elements at the crack face, SIFs can also be directly computed using the displacement discontinuity at the crack-front elements, as shown by Nikishkov et al. (2001). For fatigue-crack growth, there is no need to use any other special technique to describe the crack surface, such as the level sets. The crack surface is already efficiently described by boundary elements. In each fatigue step, a minimal effort is needed: one can simply extend the crack by adding a layer of additional elements at the crack tip/crack front. This greatly saves computational time for fatigue-crack growth analyses.

However, a SGBEM is unsuitable for carrying out large-scale simulations of complex structures. This is because the coefficient matrix for a SGBEM is fully populated. To further explore the advantages of both the FEM and SGBEM, Han and Atluri (2002) coupled the FEM and SGBEM indirectly using the Schwartz-Neumann alternating method. As shown in Fig. 3, a simple FEM is used to model the global uncracked structure and a SGBEM is used to model the local cracked subdomain. By imposing residual stresses at the global and local boundaries in an alternating procedure, the solution of the original problem is obtained by superposing the solution of each individual subproblem.

The great flexibility of this SGBEM-FEM alternating method is obvious. The SGBEM mesh of the cracked subdomain is totally independent of the crude FEM mesh of the uncracked global structure. Because a SGBEM is used to capture the stress singularity at the crack-tip, the SGBEM mesh can be efficiently refined as the crack propagates.

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**Fig. 2.** Defective solid with arbitrary cavities and cracks

**Fig. 3.** Superposition principle by Han and Atluri (2002) for SGBEM-FEM alternating method: (a) the uncracked body modeled by simple FEM; (b) the local domain containing cracks modeled by SGBEM; (c) FEM model subjected to residual loads; (d) alternating solution for the original problem
at crack tips/fronts, a very coarse mesh can be used for a FEM model of the uncracked global structure. Because a FEM is used to model the uncracked global structure, large-scale structures can be efficiently modeled.

In this study, the SGBEM-FEM alternating method is used to model the fatigue growth of corner cracks emanating from the pinholes of lugs. The detailed configurations of numerical experiments are given in the next section.

Geometry of Specimens and Configuration of Experiments

The FEM-SGBEM models in the current study are created based on the geometries given in the comprehensive report by Kathiresan and Brussat (1984), in which a large number of experiments were carried out. Four models are considered with different maximum gross stresses (represented by $\sigma_{\text{0max}}$) and different stress ratios (represented by $R$). The corresponding specimens’ model numbers and load cases are given in Table 1. All four models have the same geometrical parameters, as shown in Fig. 4. Note that the initial crack surface is considerably small compared with the size of the attachment lug. In fact, the thickness of the attachment lug is 20 times the size of the radius of the initial crack surface ($T/R_c = 0.5/0.025 = 20$), so the initial crack surface is magnified in Fig. 4.

The applied external load is an axial cyclic load with a constant amplitude of $\sigma_{\text{0max}}$, as shown in Fig. 5. The contact pressure between the pin and lug is assumed to have the following form, based on Raju and Newman (1979):

$$\sigma(\theta) = A \sin^2 \theta$$  \hspace{1cm} (5)

From the condition of equilibrium

$$\int_0^\pi \sigma(\theta) R_i \sin \theta d\theta = \sigma_{\text{0max}} H$$  \hspace{1cm} (6)

And thus the contact pressure is determined by

$$\sigma(\theta) = \frac{3\sigma_{\text{0max}} H}{4R_i} \sin^2 \theta$$  \hspace{1cm} (7)

An attachment lug with a considerably small initial quarter-circular surface crack emanating from one corner of the pinhole is modeled by the SGBEM-FEM alternating method. As shown in Fig. 6, the meshes for the SGBEM-FEM alternating method consist of only 268 finite elements and 15 boundary elements. However, for pure FEMs, including the popular extended FEM (XFEM), it is typical to use more than 1 million degrees of freedom to model a cracked 3D complex structure [see Levén and Rickert (2012) for examples]. Thus, the current SGBEM-FEM alternating method greatly reduces the burden of computation as well as the human labor of pre-processing, compared with FEM-based numerical methods, including XFEM, by several orders of magnitude.

The attachment lugs are manufactured with 7075-T651 aluminum alloy. The Young’s modulus of this material is 71,705,475.8491 kPa (10,400 ksi), and Poisson’s ratio is 0.33. Paris’ law (Paris and Erdogan 1963) is used to analyze the fatigue-crack growth. Based on Paris’ law, the crack growth rate is determined by

$$\frac{da}{dN} = C(\Delta K)^n$$  \hspace{1cm} (8)

Based on the report by Kathiresan and Brussat (1984), the parameters of Paris’ law for this material are $C = 1.1897 \times 10^{-9}$ and $n = 3.8585$ for stress ratio $R = 0.1$, and $C = 6.296 \times 10^{-9}$ and $n = 3.6999$ for stress ratio $R = 0.5$. The units for $\Delta K$ and $da/dN$ are 34,748.4823 kPa$\sqrt{\text{mm}}$ (1 ksi$\sqrt{\text{in.}}$) and 25.4 mm/cycle (1 in. /cycle), respectively. Refer to Tables 1-4 and 1-5 in the report by Kathiresan and Brussat (1984) for more details.

<table>
<thead>
<tr>
<th>Table 1. Different Models of Attachment Lugs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>7075-T651 aluminum alloy</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Fig. 4. Geometry of the attachment lug (unit: inches) (1 in. = 25.4 mm)
SIF Analyses of the Initial Crack

To verify the accuracy of the SGBEM-FEM alternating method, the computed SIFs are compared with the empirical solution of Newman (1973)

$$K_I = S \sqrt{\frac{\pi a}{Q}} M_f f_1 G_1 \sqrt{\cos \frac{\pi D}{2H}} g_\phi$$

(9)

where $K_I$ = empirical mode I SIF; $S$ = applied gross stress; $f_1$ = Bowie correction factor (Bowie 1956) for a single crack emanating from a circular hole; $G_1$ = stress correction factor for a loaded lug with one crack emanating from the pinhole; $D$ = diameter of the pinhole of the lug; $H$ = width of the lug; $g_\phi$ = curve fitting angular function; $Q$ = elastic shape factor; and $M_e$ = boundary correction factor. See Božanović and Maksimović (2014) for further details of the calculation for each correction factor.

For the geometry and load cases shown in Table 1 and Figs. 4 and 5, the SIFs of the initial quarter-circular corner crack is computed using a SGBEM-FEM alternating method with 268 finite elements and 15 boundary elements, as shown in Fig. 6. Note that the initial crack surface is quarter-circular, so the initial semimajor and semiminor axes are the same, i.e., $a_0 = a_0 = 0.635$ mm (0.025 in.). Comparisons between the computed SIFs by the alternating method and by the empirical solution are given in Tables 2 and 3, which demonstrate good agreement. It is also worth mentioning that the computation time for the SGBEM-FEM alternating method presented here is less than 2 s in a regular PC with an Intel i7 CPU (Core i7-3820 CPU at 3.60 GHz, Intel Corporation, Santa Clara, California). In addition, the cost of human labor to generate the independent coarse meshes of the FEM and SGBEM is also very minimal.

Tables 2 and 3 show that the empirical results of Newman (1973) are approximately 5–7% in error compared with the present solutions. However, it should be emphasized that the Newman solutions are empirical formulas based on brutal-force finite-element analyses. They are only valid for small quarter-elliptical cracks as compared with the size of the hole. The crack shape during the fatigue growth does not always stay strictly quarter-elliptical, and there is no analytical or empirical solution as the crack starts to break through the thickness. Because the computation burden for the SGBEM-FEM alternating method is very small and the independent meshing of the structure and the crack takes only minimal human labor, it is very convenient to

Table 2. Computed SIFs for Quarter-Circular Crack of Lug Using SGBEM-FEM Alternating Method [$\sigma_{\text{omax}} = 41,368.5438$ kPa (6.0 ksi)]

<table>
<thead>
<tr>
<th>$\phi$ (degrees)</th>
<th>$K_{\text{empirical}}$ [$34,748.4823$ kPa/ mm (1 ksi/ in.)]</th>
<th>$K_{\text{SGBEM-FEM}}$ [$34,748.4823$ kPa/ mm (1 ksi/ in.)]</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>232.064.264 (6.6784)</td>
<td>242.249.044 (6.9715)</td>
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</tr>
<tr>
<td>9.02</td>
<td>226.073.625 (6.506)</td>
<td>226.869.361 (6.5289)</td>
<td>0.35</td>
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<tr>
<td>18.04</td>
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<td>211.458.4142 (6.0854)</td>
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<td>217.462.9519 (6.2582)</td>
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</tr>
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</table>

Numerical Results
Simulate the entire fatigue-crack growth process for the damage tolerance analyses of attachment lugs.

Simulations of Crack Growth Path

The process of fatigue-crack growth for each specimen is divided into 40 analysis steps. In each fatigue analysis step, the SIFs along the crack front are computed by the SGBEM-FEM alternating method. Then, the crack growth is modeled by adding a layer of additional elements at the crack front, in the direction determined by the Eshelby force vector (Eshelby 1951), with the size determined by Paris’ law.

The development of fatigue cracks of Model No. ABPLC6.56 is shown in Fig. 7. Crack shapes of other models are similar to those of Model No. ABPLC6.56, which are given in Figs. 8 and 9.

As shown in Fig. 7, the process of fatigue-crack growth in the attachment lugs can be broadly divided into four phases: the initial quarter-circular crack; a quasi-quarter-elliptical crack in growth; transition from a quasi-quarter-elliptical to a through-the-thickness crack; and, finally, a through-the-thickness edge crack. Initially, the crack grows faster in the depth direction than in the surface direction; once the crack cuts through the thickness of the lug, it grows very fast in the surface direction and quickly becomes a through-the-thickness edge crack.

Table 3. Computed SIFs for Quarter-Circular Crack of Lug Using SGBEM-FEM Alternating Method [$\sigma_{0\text{max}} = 103,421.3594$ kPa (15.0 ksi)]

<table>
<thead>
<tr>
<th>$\phi$ (degrees)</th>
<th>$K_{\text{empirical}}$ [34,748.4823 kPa/\text{m}] (1 kși/\text{in.})</th>
<th>$K_{\text{SGBEM-FEM}}$ [34,748.4823 kPa/\text{m}] (1 kși/\text{in.})</th>
<th>Difference (%)</th>
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</tbody>
</table>

Fig. 7. Growth of a quarter-circular corner crack of the attachment lug (Model No. ABPLC6.56): 2D view
Fig. 8. Crack shapes for Model No. ABPLC6,56 [initial crack $a_0 = c_0 = 0.635$ mm (0.025 in.)]: (a) $a = 2.2352$ mm (0.088 in.); (b) $a = 3.5052$ mm (0.138 in.); (c) $a = 4.6736$ mm (0.184 in.); (d) $a = 6.9342$ mm (0.273 in.); (e) $a = 9.1186$ mm (0.359 in.); (f) $a = 11.3284$ mm (0.446 in.)
Fig. 9. Crack shapes for model number ABPLC5.54 [initial crack $a_0 = c_0 = 0.635$ mm (0.025 in.)]: (a) $a = 3.6068$ mm (0.067 in.); (b) $a = 3.6068$ mm (0.142 in.); (c) $a = 5.6388$ mm (0.222 in.); (d) $a = 7.9756$ mm (0.314 in.); (e) $a = 10.2616$ mm (0.404 in.); (f) $a = 12.2682$ mm (0.483 in.)
Estimation of Fatigue Lives

In this section, the fatigue lives of the attachment lug are estimated by employing the SGBEM-FEM alternating method together with Paris’ crack growth law. Paris’ law is used to characterize the fatigue-crack growth in the attachment lug. According to Paris’ law, it is easy to estimate the fatigue life of the cracked attachment lug by

\[
N = \frac{a_f}{a_0} \frac{1}{C(\Delta K)^n} da
\]  

At each fatigue growth step, the maximum value of \( \Delta K \) along the crack front is computed by the SGBEM-FEM alternating method, and Eq. (10) is numerically evaluated using the trapezoidal rule.

The crack extensions in both the depth direction and surface direction of the lug are investigated, and the results are shown in Figs. 10–13, where it is clear that the stress ratio plays an important role for the fatigue behavior of the attachment lugs. Generally speaking, for the same geometry and the same amplitude of cyclic loading, the smaller the stress ratio, the faster the fatigue crack propagates and the shorter the life of the attachment lug.

![Fig. 10. Crack length in the depth direction versus number of load cycles up to failure for the attachment lug (stress ratio \( R = 0.1 \)) (1 in. = 25.4 mm)](image1)

![Fig. 11. Crack length in the surface direction versus number of load cycles for the attachment lug (stress ratio \( R = 0.1 \)) (1 in. = 25.4 mm)](image2)
Conclusions

In this paper, several numerical models of attachment lugs with an initial quarter-circular surface crack emanating from the pinhole are considered. The SIF analyses and fatigue-crack growth simulations with different load cases are carried out by employing the 3D SGBEM-FEM alternating method. By carefully comparing the numerical results with empirical solutions and experimental observations available in the literature, the following conclusions can be drawn:

1. The SGBEM-FEM alternating method requires independent and very coarse meshes for both the uncracked structure and the crack; it requires only minimal computational burden and human labor efforts for modeling the fatigue growth of 3D cracks.
2. The computed SIFs for the initial crack using the SGBEM-FEM alternating method are in good agreement with empirical solutions.
3. By employing the SGBEM-FEM alternating method, the whole crack growth path up to failure of the attachment lug can be easily simulated.
4. The predicted fatigue life of lug specimens by the SGBEM-FEM alternating method agrees well with experimental observations. It is thus concluded that the 3D SGBEM-FEM alternating method, among the many alternating methods developed in the past.
20–30 years by Atluri and his many collaborators, are considerably efficient, accurate, and reliable for analyzing 3D fracture and fatigue-crack propagations, which is crucial for the damage tolerance of attachment lugs as well as other 3D complex civil and mechanical structures. The implementation of the 3D SGBEM-FEM alternating method in general-purpose off-the-shelf commercial software is being pursued by the authors.

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