Advanced NDIF Method for Eigenvalue Analysis of Arbitrarily Shaped Acoustic Cavities With the Partially Pressure-Release Boundary

In this paper, an advanced non-dimensional dynamic influence function method (NDIF method) for eigenvalue analysis of arbitrarily shaped two-dimensional acoustic cavities with the mixed boundary consisting of the pressure-release and rigid-wall boundaries is proposed. The existing NDIF method has the weakness of having to calculate the singularity of the final system matrix of an analyzed acoustic cavity in the frequency band of interest to obtain the eigenvalues of the cavity because the final system matrix is dependent on the frequency. The newly proposed NDIF method in this paper provides an efficient way to extract accurate eigenvalues and eigenmodes by successfully overcoming the above weaknesses. Finally, the validity and accuracy of the proposed method are shown through verification examples. [DOI: 10.1115/1.4048887]

Keywords: dynamics, modal analysis, propagation and radiation, structural acoustics

1 Introduction

For the first time non-dimensional dynamic influence function method (NDIF method) for the extraction of highly accurate eigenvalues of arbitrarily shaped acoustic cavities with the rigid-wall boundary was introduced [1]. The NDIF method has the advantage of requiring less numerical calculation and offering highly accurate results because basis functions used in the method exactly satisfy the governing differential equation, and at the same time the boundary of an analyzed cavity is discretized by a small number of nodes unlike the finite element method [2] and the boundary element method (BEM) [3].

However, the above-mentioned eigenvalue extraction technique [1] employing the NDIF method has the disadvantage that the final system matrix depends on the frequency parameter unlike the finite element method or the boundary element method. For this reason, it also has the disadvantage that the singularity of the system matrix should be calculated one by one, increasing the frequency in the frequency band of interest in order to extract eigenvalues and eigenmodes from the system matrix. To overcome the above shortcomings, the authors published the results of a study in which the frequency parameter is separated from the final system matrix of an acoustic cavity with the rigid-wall boundary so that the final system matrix equation of the cavity can be formulated as an algebraic eigenvalue problem [4].

In the BEM, which discretizes only the boundary of an analyzed object as with the NDIF method, the disadvantage of the system matrix being dependent on the frequency was first overcome by Nardini and Brebbia [5]. In 1982, Nardini and Brebbia succeeded in formulating a final system matrix equation in BEM as a form of algebraic eigenvalue problem and opened new horizons in BEM research [5]. Since then, BEM researches have focused on improving the accuracy of eigenvalues. Kirkup and Amini introduced a practical way of reducing the nonlinear eigenvalue problem to a standard generalized eigenvalue problem through a polynomial approximation [6]. Ali presented a historical and critical review of BEM in acoustic eigenvalue analysis [7]. Provatidis tested different types of basis functions for more accurate eigenvalues of two-dimensional acoustic cavities using the dual Reciprocity/boundary element technique [8]. Recently, Wang investigated approximation functions such as radial basis functions and thin plate spline functions in the dual reciprocity BEM for accurate acoustic eigenvalue analysis [9]. Gao et al. presented accurate solutions for eigenvalue analysis of three-dimensional acoustic cavities using BEM with the block Sakuri–Sugiura method [10].

A great deal of analytical or semi-analytical research has been performed to increase the accuracy of eigenvalues and eigenmodes for acoustic cavities with simple shapes having no exact solution. Amir studied a method for calculating the eigenmodes of two-dimensional cavities having two axes of symmetry by computing wave propagation in waveguides of arbitrarily changing cross section [11]. Willatzen and Voon solved quasi-analytically a triaxial ellipsoidal acoustic cavity with walls using the Frobenius powerseries expansion method [12]. Koch computed acoustic resonances in rectangular two-dimensional deep shallow open cavities [13]. Lee presented a semi-analytical approach to solve the eigenproblem of an acoustic cavity with multiple elliptical boundaries by using the collocation multi-pole method [14]. Although analytical and semi-analytical methods such as the above-mentioned methods [11–14] give high accurate solutions, there is the limitation that they are not applicable to arbitrarily shaped acoustic cavities unlike the NDIF method. As a recent study dealing with arbitrary shapes, the authors proposed a modified NDIF method for extracting the eigenvalues of arbitrarily shaped membranes [15], which have an analogy with acoustic cavities in the governing differential equation.

In this paper, the advanced way of formulating the NDIF method into algebraic eigenvalue problem for arbitrarily shaped acoustic cavities with the partially pressure-release boundary, which has not been covered so far, is proposed by extending the authors’ previous research [4]. It is confirmed from verification examples that the proposed method successfully resolves the problem of frequency dependency in the final system matrix arising in the existing NDIF method.

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2 Theoretical Formulation

2.1 Governing Equation and Boundary Conditions. The governing differential equation for the eigenmode analysis of the two-dimensional acoustic cavity expressed as the solid line in Fig. 1 has the form of the Helmholtz equation [4] as follows:

\[ \nabla^2 p(r) + k^2 p(r) = 0 \]  

(1)

where \( r \) denotes the position vector for a point \( P \) inside the acoustic cavity (see Fig. 1), \( p(r) \) represents the sound pressure at the point \( P \), and \( k \) is the frequency parameter, which is the frequency (rad/s) divided by the sound speed (m/s).

In general, the boundary of the acoustic cavity is divided by the pressure-release boundary and the rigid-wall boundary, and the boundary conditions for the two boundaries are given by, respectively,

\[ p(r_i) = 0, \quad \frac{\partial p(r_i)}{\partial n} = 0 \]  

(2,3)

where \( r_i \) is the position vector for a point on the boundary of the acoustic cavity and \( n \) means the normal direction at the point, as shown in Fig. 1.

2.2 Applying Boundary Conditions to Assumed Solution. First, the boundary of the acoustic cavity under analysis is discretized with \( N \) nodes \( P_1, P_2, \ldots, P_N \) as shown in Fig. 1. The sound pressure at the point \( P \) inside the acoustic cavity is then assumed as a linear combination of the non-dimensional dynamic influence functions as follows:

\[ p(r) = \sum_{i=1}^{N} A_i J_0(k | \mathbf{r} - \mathbf{r}_i |) \]  

(4)

where \( J_0 \) represents the Bessel function of order zero of the first kind, \( A_i \) is an unknown contribution coefficient, and \( \mathbf{r}_i \) stands for the position vector for node \( P_i \) located on the boundary. Note that the assumed solution, Eq. (4), exactly satisfies the governing differential equation, Eq. (1).

Next, the boundary conditions, Eqs. (2) and (3), given continuously along the boundary are discretized for nodes on the boundary as follows, respectively:

\[ p(r_i^{(o)}) = 0, \quad i = 1, 2, \ldots, N_o \]  

(5)

\[ \frac{\partial p(r_i^{(o)})}{\partial n_i} = 0, \quad j = N_o + 1, N_o + 2, \ldots, N \]  

(6)

where \( r_i^{(o)} \) means the position vector for node \( P_i \) on the pressure-release boundary, \( r_j^{(r)} \) and \( n_j \) means the position vector and the normal direction for node \( P_j \) on the rigid-wall boundary, respectively, and \( N_o \) and \( N_r \) mean the numbers of nodes at the pressure-release boundary and the rigid-wall boundary, respectively (note that \( N_r + N_r = N \)).

Now, if the assumed solution (Eq. (4)) is substituted into the boundary conditions (Eqs. (5) and (6)), the following two equations can be obtained, respectively:

\[ p(r_i^{(o)}) = \sum_{j=1}^{N} A_j J_0(k | \mathbf{r}_i - \mathbf{r}_j |) = 0, \quad i = 1, 2, \ldots, N_o \]  

(7)

\[ \frac{\partial p(r_i^{(o)})}{\partial n_i} = \frac{1}{k} \sum_{j=1}^{N} A_j \frac{\partial J_0(k | \mathbf{r}_i - \mathbf{r}_j |)}{\partial n_i} = 0, \quad j = N_o + 1, N_o + 2, \ldots, N \]  

(8)

Differentiating Eq. (8) for the normal direction \( n_j \) gives

\[ \sum_{j=1}^{N} A_j J_1(k | \mathbf{r}_i - \mathbf{r}_j |) = \frac{1}{k} \sum_{j=1}^{N} A_j J_1(k | \mathbf{r}_i - \mathbf{r}_j |) = 0, \quad j = N_o + 1, N_o + 2, \ldots, N \]  

(9)

where \( J_1 \) represents the Bessel function of order one of the first kind.

2.3 Formulation Into Algebraic Eigenvalue Problem of the NDF Method. In Eqs. (7) and (9), it may be seen that the Bessel functions \( J_0 \) and \( J_1 \) contain the frequency parameter \( k \). As the first step for the formulation of the NDF method to an algebraic eigenvalue problem, Taylor series expansion [16] is carried out on the two Bessel functions as follows:

\[ J_0(k | \mathbf{r}_i^{(o)} - \mathbf{r}_j |) \approx \sum_{m=0}^{M} (-1)^m (k | \mathbf{r}_i^{(o)} - \mathbf{r}_j | / 2)^{2m} (\Gamma(m + 1))^2 \]  

(10)

\[ J_1(k | \mathbf{r}_i^{(r)} - \mathbf{r}_j |) \approx \sum_{m=0}^{M} (-1)^m (k | \mathbf{r}_i^{(r)} - \mathbf{r}_j | / 2)^{1+2m} (\Gamma(m + 1))^2 (\Gamma(m + 2)) \]  

(11)

where \( M \) represents the number of the series terms. Equations (10) and (11) are briefly rewritten as follows:

\[ J_0(k | \mathbf{r}_i^{(o)} - \mathbf{r}_j |) \approx \sum_{m=0}^{M} k^{2m} \phi_m^{(o)}(i, s) \]  

(12)

\[ J_1(k | \mathbf{r}_i^{(r)} - \mathbf{r}_j |) \approx \sum_{m=0}^{M} k^{1+2m} \psi_m^{(r)}(i, s) \]  

(13)

where \( \phi_m^{(o)}(i, s) \) and \( \psi_m^{(r)}(i, s) \) are given by

\[ \phi_m^{(o)}(i, s) = \frac{(-1)^m (k | \mathbf{r}_i^{(r)} - \mathbf{r}_j | / 2)^{2m}}{(\Gamma(m + 1))^2} \]  

(14)

\[ \psi_m^{(r)}(i, s) = \frac{(-1)^m (k | \mathbf{r}_i^{(r)} - \mathbf{r}_j | / 2)^{1+2m}}{(\Gamma(m + 1))^2 (\Gamma(m + 2))} \]  

(15)

The following equations can be obtained by substituting Eqs. (12) and (13) into Eqs. (7) and (9), respectively:

\[ \sum_{j=1}^{N} A_j \left( \sum_{m=0}^{M} k^{2m} \phi_m^{(o)}(i, s) \right) = 0, \quad i = 1, 2, \ldots, N_o \]  

(16)

\[ \sum_{j=1}^{N} A_j k \left( \sum_{m=0}^{M} k^{1+2m} \psi_m^{(r)}(i, s) \right) \frac{\partial J_1(k | \mathbf{r}_i^{(r)} - \mathbf{r}_j |)}{\partial n_i} = 0, \quad j = N_o + 1, N_o + 2, \ldots, N \]  

(17)
Equation (17) is expressed in a simple form as follows:

\[ \sum_{j=1}^{N} A_j \left( \sum_{m=1}^{M} k^{2(1+m)} \bar{\psi}^{(i)}_m(j, s) \right) = 0, \quad j = N_o + 1, N_o + 2, \ldots, N \]  

where \( \bar{\psi}^{(i)}_m(j, s) \) is given by

\[ \bar{\psi}^{(i)}_m(j, s) = \psi^{(i)}_m(j, s) \frac{\partial}{\partial n_j} | \mathbf{r}^{(i)} - \mathbf{r}_j | \]  

Equations (16) and (18) may be re-arranged as follows, respectively:

\[ \sum_{m=0}^{M} \bar{\psi}^{(m)}(i, s) = 0, \quad i = 1, 2, \ldots, N_o \]  

\[ \sum_{j=0}^{M} k^{2(1+m)} \left( \sum_{n=1}^{N} A_j \bar{\psi}^{(n)}_m(j, s) \right) = 0, \quad j = N_o + 1, N_o + 2, \ldots, N \]  

where \( k = k^2 \).

In order to extract the system matrix for the acoustic cavity, Eqs. (20) and (21) are written in the forms of polynomial equations with respect to \( k \) as follows, respectively:

\[ \gamma^i \sum_{j=1}^{N} A_j \psi^{(i)}_m(j, s) + \gamma^1 \sum_{m=1}^{N_o} A_j \phi^{(i)}_m(j, s) + \ldots + \gamma^M \sum_{j=1}^{N} A_j \psi^{(i)}_M(j, s) = 0, \quad i = 1, 2, \ldots, N_o \]  

\[ \gamma^1 \sum_{j=1}^{N} A_j \psi^{(j)}_m(j, s) + \gamma^2 \sum_{m=1}^{N_o} A_j \phi^{(j)}_m(j, s) + \ldots + \gamma^M \sum_{j=1}^{N} A_j \psi^{(j)}_M(j, s) = 0, \quad j = N_o + 1, N_o + 2, \ldots, N \]  

Erasing common factor \( \gamma \) in Eq. (23) gives

\[ \gamma^i \sum_{j=1}^{N} A_j \psi^{(i)}_m(j, s) + \gamma^1 \sum_{m=1}^{N_o} A_j \phi^{(i)}_m(j, s) + \ldots + \gamma^M \sum_{j=1}^{N} A_j \psi^{(i)}_M(j, s) = 0, \quad j = N_o + 1, N_o + 2, \ldots, N \]  

Equations (22) and (24) are converted into the forms of simple matrix equations as follows, respectively:

\[ (\gamma^i \Phi^{(i)}_0 + \gamma^1 \Phi^{(i)}_1 + \ldots + \gamma^M \Phi^{(i)}_M) A = 0 \]  

\[ (\gamma^i \Psi^{(i)}_0 + \gamma^1 \Psi^{(i)}_1 + \ldots + \gamma^M \Psi^{(i)}_M) A = 0 \]  

where \( \Phi^{(i)}_m \) is a matrix of size \( N \times N \), \( \Psi^{(i)}_m \) is a matrix of size \( N \times N \), and \( A \) is a vector of size \( N \times 1 \). The elements of the two matrices and the vector in Eqs. (25) and (26) are given by

\[ \Phi^{(i)}_m(i, s) = \phi^{(i)}_m(i, s) \]  

\[ \Psi^{(i)}_m(j, s) = \psi^{(i)}_m(j, s) \]  

\[ A(s, 1) = A \]  

Equations (25) and (26) can be expressed in a single matrix equation as follows:

\[ \left[ \gamma^1 \Phi^{(i)}_1 \right] + \gamma^1 \left[ \Phi^{(i)}_1 \right] + \ldots + \gamma^M \left[ \Phi^{(i)}_M \right] A = 0 \]  

Equation (30), a higher-order polynomial matrix equation for \( \gamma \), can be linearized as follows [17]:

\[ \mathbf{S}_M \mathbf{B} = \gamma \mathbf{S}_R \mathbf{B} \]  

where the system matrices \( \mathbf{S}_M \) and \( \mathbf{S}_R \) and the unknown vector \( \mathbf{B} \) are given by

| Table 1 | Eigenvalues \( k_1 \sim k_6 \) of the rectangular acoustic cavity by the proposed method, the exact solution, and FEM (ANSYS) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Computational   | Proposed method (16 nodes) | Exact solution [18] | FEM (ANSYS) (1271 nodes) |
| time            | \( M = 10 \)     | \( M = 15 \)     | \( M = 20 \)     | \( M = 20 \)     |
|                 | 0.23 s           | 0.61 s           | 1.11 s           | 5.48 s           |
| \( k_1 \)       | 1.309 (0.00)     | 1.309 (0.00)     | 1.309 (0.00)     | 1.308 (−0.08)    |
| \( k_2 \)       | 3.728 (0.00)     | 3.728 (0.00)     | 3.728 (0.00)     | 3.729 (0.03)     |
| \( k_3 \)       | 3.927 (0.03)     | 3.927 (0.00)     | 3.927 (0.00)     | 3.926 (−0.03)    |
| \( k_4 \)       | 5.254 (0.15)     | 5.254 (0.00)     | 5.254 (0.00)     | 5.253 (−0.02)    |
| \( k_5 \)       | None 6.547 (0.03) | 6.547 (0.03)     | 6.545 (0.03)     | 6.551 (0.09)     |
| \( k_6 \)       | None 7.108 (0.07) | 7.108 (0.07)     | 7.103 (0.17)     | 7.115 (0.17)     |

Parenthesized values denote errors (%) with respect to the exact solution.
\[ \text{SM}_L = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
-\Phi_0^{(o)} & -\Phi_1^{(o)} & -\Phi_2^{(o)} & \cdots & -1
\end{bmatrix} \]

\[ \text{SM}_R = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix} \]

\[ \mathbf{B} = \begin{bmatrix}
A^T & \gamma A^T & \gamma^2 A^T & \cdots & \gamma^{M-1} A^T
\end{bmatrix} \]

where \( \mathbf{0} \) and \( \mathbf{I} \) represent the zero matrix and the identity matrix, respectively. Finally, Eq. (31) can be formulated to the algebraic eigenvalue problem as follows:

\[ \text{SM} \mathbf{B} = \gamma \mathbf{B} \]

where the final system matrix \( \text{SM} \) is given by

\[ \text{SM} = \text{SM}_R^{-1} \text{SM}_L \]

The final system matrix \( \text{SM} \) in Eq. (35) may be seen as not dependent on the frequency parameter, \( k (= \sqrt{\gamma}) \). Although the unknown vector \( \mathbf{B} \) involves the frequency parameter, it should be noted that \( \mathbf{B} \) is not used to solve the algebraic eigenvalue problem given by Eq. (35).

Lastly, the \( n \)th eigenvalue \( k_n (= \sqrt{\gamma_n}) \) and the \( n \)th eigenvector \( \mathbf{B}_n \) of the acoustic cavity can be obtained from Eq. (35). If referring to Eq. (34), the \( n \)th acoustic mode shape of the cavity can be drawn by substituting \( \mathbf{A}^T \) that is the first element of the eigenvector \( \mathbf{B}_n \) into Eq. (4). Note that the NDIF method does not need to divide the inside space of the cavity like BEM because mode shapes are plotted using Eq. (4), which is a function of the Cartesian coordinates \((x, y)\) in Fig. 1.

3 Verification Examples

To verify the validity and accuracy of the method established in this paper, example studies for three shapes of acoustic cavities are conducted.

3.1 Rectangular Acoustic Cavity With Exact Solution

Figure 2 shows a rectangular acoustic cavity with dimensions 1.2 m × 0.9 m, which is considered as the first example. Note that the rectangular acoustic cavity whose one side (dotted line) is the pressure-release boundary as shown in Fig. 2 has an exact solution for the eigenmode analysis [18]. The rectangular acoustic cavity is discretized with 16 nodes of which three nodes are used for the pressure-release boundary and 13 nodes are used for the rigid-wall boundary. The connecting points between the pressure-release boundary and the rigid-wall boundary are defined as rigid-wall nodes to properly represent the shape of the cavity consisting the three rigid walls.

In addition, the normal directions at the four corners of the cavity were marked with four arrows that denote normal vectors in Fig. 2. The normal direction at the corner where the rigid-wall boundaries meet each other is determined by the sum of the two normal vectors for the adjacent boundaries. The normal direction at the corner where the rigid-wall boundary and the pressure-release boundary meet each other is set as the normal direction for the rigid-wall boundary. It has been confirmed in the study that establishing the normal directions at the corner in these ways provides the most accurate eigenvalues.

The eigenvalues obtained by the proposed method, the exact solution, and finite element method (FEM) (ANSYS) are presented.
in Table 1. The unit of the eigenvalue is the same as that of the frequency parameter \( k \). The natural frequency \( f \) (Hz) of the cavity is calculated from \( f = ck/(2\pi) \) where \( c \) is the sound speed (m/s). By comparing the eigenvalues by the proposed method with those by the exact solution, it can be seen that the use of 15 Taylor series terms \( (M = 15) \) shows very accurate results almost converging on those by the exact solution, even though only 16 fewer nodes were used. It is assumed in the case of \( M = 10 \) that the proposed method does not provide the fifth and sixth eigenvalues because more terms of Taylor series are required due to the complexity of the corresponding mode shapes. On the other hand, the eigenvalues by FEM (ANSYS) using 1271 nodes can be seen to have greater errors than those by the proposed method using only 16 nodes.

Figure 3 shows the first to sixth mode shapes obtained by the proposed method using 16 nodes and 15 Taylor series terms. Although the six mode shapes (Fig. 3) use only 16 fewer nodes, it can be confirmed that they exactly match those (Fig. 4) obtained by FEM (ANSYS) using 1271 nodes. In Figs. 3 and 4, the purple and red colors indicate the regions with the minimum and maximum sound pressures in the cavity, respectively, and the purple regions are called the nodal lines. Furthermore, it may be said that the nodal lines of the \( n \)th mode in Fig. 3 are exactly the same as those in Fig. 4 in position and shape.

### 3.2 Arbitrarily Shaped Quadrilateral Acoustic Cavity

As a second example of verification, the arbitrarily shaped quadrilateral acoustic cavity shown in Fig. 5 is considered. Among the four sides of the cavity, the left side (dotted line) is the pressure-release boundary and the other three sides (solid lines) are the rigid-wall boundary. The pressure-release and rigid-wall boundaries are, respectively, discretized by three nodes and 13 nodes, with a total of 16 nodes discretizing the cavity.

Normal directions at the four corners of the cavity are marked with arrows in Fig. 5. In the same way as in the first example, the

![Fig. 5 Arbitrarily shaped quadrilateral acoustic cavity discretized with 16 boundary nodes (dotted line: pressure-release boundary, solid line: rigid-wall boundary)](image)

<table>
<thead>
<tr>
<th>Table 2 Eigenvalues ( k_1 \sim k_6 ) of the arbitrarily shaped quadrilateral acoustic cavity by the proposed method and FEM (ANSYS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method (16 nodes)</td>
</tr>
<tr>
<td>( M = 10 )</td>
</tr>
<tr>
<td>( k_1 )</td>
</tr>
<tr>
<td>( (-0.38) )</td>
</tr>
<tr>
<td>( k_2 )</td>
</tr>
<tr>
<td>( (-0.66) )</td>
</tr>
<tr>
<td>( k_3 )</td>
</tr>
<tr>
<td>( (0.27) )</td>
</tr>
<tr>
<td>( k_4 )</td>
</tr>
<tr>
<td>( (-1.73) )</td>
</tr>
<tr>
<td>( k_5 )</td>
</tr>
<tr>
<td>( (-0.03) )</td>
</tr>
<tr>
<td>( k_6 )</td>
</tr>
<tr>
<td>( (-0.06) )</td>
</tr>
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</table>

Parenthesized values denote errors (%) with respect to FEM (ANSYS) using 1390 nodes.
normal direction at the corner where the rigid-wall boundaries meet each other is determined by the sum of the two normal vectors for the adjacent boundaries. Also the normal direction at the corner where the rigid-wall boundary and the pressure-release boundary meet each other is set as the normal direction for the rigid-wall boundary. It has been confirmed that the alternative way of determining the normal directions for the four corners by the sum of the two normal vectors for the adjacent boundaries are less accurate than the way proposed in the paper.

Table 2 shows the eigenvalues obtained by the proposed method and FEM (ANSYS). It is said in Table 2 that the eigenvalues by FEM using 1390 nodes are sufficiently convergent values compared to those by FEM using 856 nodes and 623 nodes. Since there is no exact solution for the current acoustic cavity, the errors of the eigenvalues by the proposed method are calculated by comparing them with the eigenvalues by FEM using 1390 nodes.

It is confirmed that the proposed method using 16 nodes and 15 Taylor series terms ($M=15$) gives accurate eigenvalues within 0.88% error. It is assumed that the fifth and sixth eigenvalues were not obtained for $M=10$ because the corresponding mode shapes (Figs. 6(e) and 6(f)) are too complex to be represented by ten Taylor series terms. Figures 6 and 7 show mode shapes obtained

Fig. 6  Mode shapes of the arbitrarily shaped quadrilateral acoustic cavity obtained by the proposed method using 16 nodes and $M=15$: (a) first mode, (b) second mode, (c) third mode, (d) fourth mode, (e) fifth mode, and (f) sixth mode

Fig. 7  Mode shapes of the arbitrarily shaped quadrilateral acoustic cavity obtained by FEM (ANSYS) using 1390 nodes: (a) first mode, (b) second mode, (c) third mode, (d) fourth mode, (e) fifth mode, and (f) sixth mode
by the proposed method and FEM (ANSYS), respectively. It can be confirmed that the mode shapes by the proposed method using only 16 nodes exactly matches those by FEM (ANSYS) using 1390 nodes, especially when the position and shape of the nodal lines in Fig. 6 are compared with those in Fig. 7.

### 3.3 Arbitrarily Shaped Acoustic Cavity

In Fig. 8, an arbitrarily shaped acoustic cavity whose boundaries are composed of a semicircle of unit radius and two equilateral edges $\sqrt{2}m$ in length is shown to illustrate the location of 16 nodes used and the normal directions at the corners. The normal directions are determined in the same way as in the previous verification examples.

In Table 3, the eigenvalues of the cavity obtained by the present method are compared with the ANSYS result. It may be said from the comparison of the present method using 16 nodes and the ANSYS result using 1378 nodes that the present result has a small error within 3.83%. In particular, the first eigenvalue has a relatively large error, which requires further research. On the other hand,

<table>
<thead>
<tr>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
<th>$k_5$</th>
<th>$k_6$</th>
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<tbody>
<tr>
<td>Proposed method (16 nodes, $M = 15$)</td>
<td>1.021</td>
<td>2.204</td>
<td>2.846</td>
<td>3.534</td>
<td>4.076</td>
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<td></td>
<td>(3.83)</td>
<td>(−2.19)</td>
<td>(−0.99)</td>
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<td>FEM (ANSYS) (1378 nodes)</td>
<td>0.974</td>
<td>2.253</td>
<td>2.874</td>
<td>3.621</td>
<td>4.115</td>
</tr>
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</table>

Parenthesized values denote errors (%) with respect to FEM (ANSYS).

![Image of mode shapes](image)
with the mixed boundary conditions. In addition, the basic theory used in the present study can be extended to solve complexly shaped membranes, plates, and acoustic cavities. In that the domain of interest is divided into several convex domains to handle complex shapes. Note that NDIF method uses the subdomain method to analyze concave or multiply connected acoustic cavities with more developed method were confirmed by two verification examples.

It is expected that the method presented in this work can be applied to analyze concave or multiply connected acoustic cavities with more complex shapes. Note that NDIF method uses the subdomain method that the domain of interest is divided into several convex domains to solve complexly shaped membranes, plates, and acoustic cavities. In addition, the basic theory used in the present study can be extended to the free vibration analysis of arbitrarily shaped membranes and plates with the mixed boundary conditions.

4 Conclusion

In this paper, we have developed the advanced NDIF method that can efficiently extract the accurate eigenvalues and mode shapes of arbitrarily shaped two-dimensional acoustic cavities with the partially pressure-release boundary. The validity and accuracy of the developed method were confirmed by two verification examples.

Acknowledgment

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Conflict of Interest

There are no conflicts of interest.

References


Fig. 9 shows mode shapes obtained by the proposed method using 16 nodes. These results agree well with those of the ANSYS mode shapes using 1378 nodes in Fig. 10.

Fig. 10 Mode shapes of the arbitrarily shaped acoustic cavity obtained by FEM (ANSYS) using 1378 nodes: (a) first mode, (b) second mode, (c) third mode, (d) fourth mode, (e) fifth mode, and (f) sixth mode