Relational Bayesian Networks and Random Relational Structure Models

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Relevant Papers

Relational Bayesian Network
Manfred Jaeger.
Proceedings of the 13th Conference of Uncertainty in Artificial Intelligence (UAI-13). 1997

The Primula System: user's guide
Manfred Jaeger.
www.cs.auc.dk/~jaeger/Primula

Compiling Relational Bayesian Networks for Exact Inference
Mark Chavira, Adnan Darwiche, Manfred Jaeger.
To appear in special issue of International Journal of Approximate Reasoning, 2006.

Outline

- Introduction and Motivations
- Syntax of the Language
- Semantics of the Language
- Examples

Background of Probabilistic Reasoning

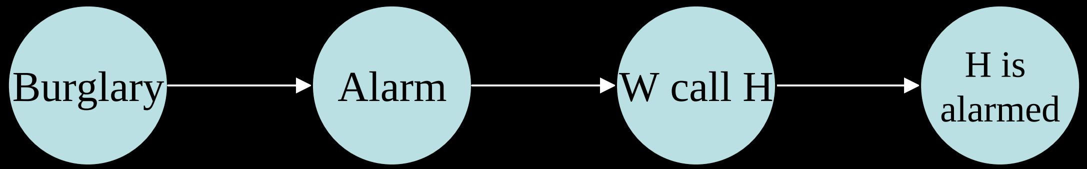
Bayesian Network

A direct acyclic graph that represent a joint probability distribution compactly. Nodes are labeled with random variables X that take values in some finite set.

A Story:

Holmes becomes alarmed if he receives a call from his neighbor Watson. Watson will likely call if an alarm has sounded at Holmes' residence. which is more likely if a burglary occurs. However, Watson is a prankster, so Holmes may receive a call even if the alarm does not sound

Example



Burglary	True	False
	0.005	0.995

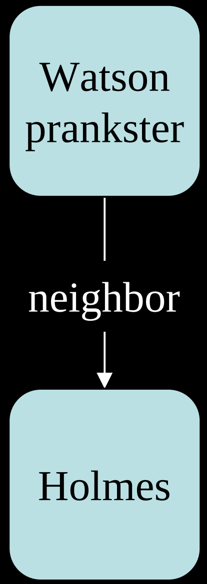
Burglary	True		False	
Alarm	True	False	True	False
	0.95	0.05	0.01	0.99

Alarm	True		False	
W call H	True	False	True	False
	0.7	0.3	0.01	0.99

W call H	True		False	
H is alarmed	True	False	True	False
	1	0	0	1

New Framework: Objects and Relations

$D = \{ \mathit{Holmes}, \mathit{Watson} \}$
$S = \{ \mathit{neighbor}(v,w), \mathit{prankster}(v) \}$
$R = \{ \mathit{calls}(v,w), \mathit{alarmed}(v), \mathit{alarm}(v), \mathit{burglary}(v) \}.$



❖Random Relational Structure Models (RRSMs)
Using general rules and specific instances to represent a joint probability distribution.

❖Relational Bayesian Network Language
A powerful language that represents the RRSMs

Probability Information of the Example

(1)At a given residence, the probability of burglary is 0.005.
(2)A person's alarm sounds with probability 0.95 if a burglary occurs at his house, and with probability 0.01 otherwise.
(3) If an alarm sounds at individual's residence, then each of the individual's neighbors will call with probability 0.9. if the neighbor is a prankster, then the neighbor will call him with probability 0.05 even there is no alarm. One will not call this individual if they are not neighbors.
(4) An individual is alarmed if one or more neighbors call.

Relational Bayesian Network Language Syntax

A Relational Bayesian Network program is a set of declaration of the forms:

RAtom = ProbabilityFormula ;

Probability Formula Syntax

(1) Real Numbers between 0 and 1, like 0.4.

(2) Logic Formula : sformula(a&b)

(3) Probabilistic atom: call(a,b)

(4) Conditional Probabilities: Given a is true, then b is true with probability c_1, else b is true with probability c_2.
$b=(a: c_1,c_2)$;

(5) Functions: We may want to say the probability of an atom p(a) being true is an average of probabilities of other formulas being true. p(a)=mean{f,g,h|u,v:u<v}.

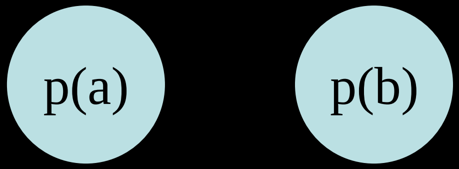
Semantics of Probability Formula

Example:

p (v) = 0.4;

Random relation p(v) is true for any object v with probability 0.4

p(a)	True	False
	0.4	0.6



p(b)	True	False
	0.4	0.6

R=c says prob(R)=c

Semantics of Probability Formula

Example:

p (v) = q(v);

Random relation p has same probability distribution as q

R=Q says prob(R)=prob(Q)

know(a,b)=sformula(neighbor(b,a)|friend(b,a))

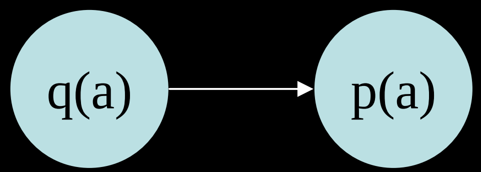
The probability of "a knows b" being true equals to 1, if either "b" is a neighbor of "a" or "b" is a friend of "a".

Note: relation "neighbor" and "friend" are predefined relations.

R=sformula(F) define prob(R) as:

prob(R)=1 if F is true, else prob(R)=0.

Semantics of Probability Formula



q(a)	True		False	
p(a)	True	False	True	False
	0.4	0.6	0.9	0.1

The probability of attribute p(a) being true is 0.4 when attribute q(a) is true, and 0.9 otherwise.

p (a) = (q(a):0.4,0.9);

$F=(F_1:F_2,F_3)$ *defines prob(F) as:*
$prob(F)=prob(F_1)*prob(F_2)+(1-prob(F_1))*prob(F_3)$.

Example:Convex Combinations

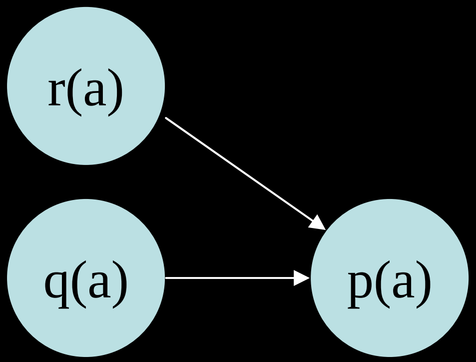
Example:

p (a) = (q(a):

(r(a):0.2,0.4),

(r(a):0.1,0.3)

);



q(a)	True				False			
r(a)	True		False		True		False	
p(a)	True	False	True	False	True	False	True	False
	0.2	0.8	0.4	0.6	0.1	0.9	0.3	0.7

Examples

Example 1: aveg=mean{lose(u),success(u)|u:invest(u)}

D={ p, q, r}

S={ invest(p), invest(q) }.

R={lose(p),lose(q),lose(r),success(p),success(q),success(r),aveg}

aveg=mean{lose(p), success(p), lose(q), success(q)}.

prob(aveg)=(prob(lose(p))+prob(success(p))+prob(lose(q))
+prob(success(a)))/4

Example 2: edge(a,b)=mean{sformula(u=a)|u:u=u};

D={a, b, c}

{1(u/a),0(u/b),0(u/c)}

Three functions

Name	Syntax	Definition	Multilinear
noisy-or	n-or	$1-\prod_{i=1}^{k}(1-p_i)$	yes
mean	mean	$(1/k)\sum_{i=1}^{k}p_i$	yes
inverse sum	invsum	$\min\{1,1/\sum_{i=1}^{k}p_i\}$	no

Burglary-Alarmed

```
burglary(v)=0.005;
alarm(v)=(burglary(v):0.95,0.01);
calls(v,w)
=(neighbor(v,w):
        (prankster(v):
                (alarm(w):0.9,0.05),
                (alarm(w):0.9,0))),0);
alarmed(v)=n-or{calls(w,v)|w:neighbor(w,v)}

DOMAIN: Holmes, Watson
prankster(Watson).
neighbor(Holmes,Watson). neighbor(Watson,Holmes).
```

Conclusion

1. *Relational Bayesian Networks Language is a new method to represent probabilistic relations. It is easier to modify than Bayesian Network.*
2. *The system Primula take Relational Bayesian Networks Language as input, it's output is a Bayesian Network where each random variables are Boolean valued (true/false).*
3. *It's output Bayesian Network could be very large, need special algorithm to handle it.*