Effective Reasoning Systems for ASP Related Paradigms

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ABSTRACT

This dissertation focuses on improving the effectiveness of two Answer Set Programming ($\mathcal{ASP}$) related paradigms. The first paradigm is $\mathcal{ALM}$, a recent action language that allows the modular specification of ontologies and state transition diagrams to model actions and their effects in finite dynamic domains. Currently there is no publicly available compiler or reasoning system which accepts $\mathcal{ALM}$ System Descriptions as input. In this dissertation we describe and implement $\mathcal{CALM}$, a compiler and reasoning system that translates $\mathcal{ALM}$ System Descriptions to $\mathcal{SPARC}$, a variant of $\mathcal{ASP}$. $\mathcal{CALM}$ enables the development of knowledge libraries and the investigation of best practices in modeling with $\mathcal{ALM}$.

One future extension of $\mathcal{ALM}$ is to incorporate elements of action language $\mathcal{H}$ which enable reasoning about continuous change over time. In order to reason in continuous domains, programs in $\mathcal{H}$ are translated to $\mathcal{AC}(\mathcal{C})$, an extension of $\mathcal{ASP}$ to include constraint logic programming ($\mathcal{CLP}$) reasoning and query answering in continuous domains. Our second area of focus is improving the effectiveness of the $\mathcal{CLP}$ algorithm used in $\mathcal{AC}(\mathcal{C})$ solvers. $\mathcal{AC}(\mathcal{C})$ solvers extend $\mathcal{ASP}$ solvers with an incrementally changing query to the $\mathcal{CLP}$ program derived from the $\mathcal{AC}(\mathcal{C})$ program. Current solvers restart $\mathcal{CLP}$ reasoning from scratch when the query changes, leading to redundant computation in the search for answers to the portions of the query that did not change. In this dissertation we formalize the incremental query problem and provide an incremental algorithm that reuses the solutions of previous queries in the search for answers to the modified query.

CHAPTER 1
INTRODUCTION

This dissertation focuses on improving the effectiveness of two Answer Set Programming ($\mathcal{ASP}$)[17, 16] related paradigms. The first paradigm is $\mathcal{ALM}$[24], a recent action language that allows the modular specification of ontologies and state transition diagrams to model actions and their effects in finite dynamic domains. Currently there is no publicly available compiler or reasoning system which accepts $\mathcal{ALM}$ System Descriptions as input. In this dissertation we describe and implement $\mathcal{CALM}$, a compiler and reasoning system that translates $\mathcal{ALM}$ System Descriptions to $\mathcal{SPARC}$[2, 1], a variant of $\mathcal{ASP}$. $\mathcal{CALM}$ enables the development of knowledge libraries and the investigation of best practices in modeling with $\mathcal{ALM}$.

One future extension of $\mathcal{ALM}$ is to incorporate elements of action language $\mathcal{H}$[7, 6] which enable reasoning about continuous change over time. In order to reason in continuous domains, programs in $\mathcal{H}$ are translated to $\mathcal{AC}(\mathcal{C})$[35, 36, 18], an extension of $\mathcal{ASP}$ to include constraint logic programming ($\mathcal{CLP}$)[25, 26] reasoning and query answering in continuous domains. Our second area of focus is improving the effectiveness of the $\mathcal{CLP}$ algorithm used in $\mathcal{AC}(\mathcal{C})$ solvers. $\mathcal{AC}(\mathcal{C})$ solvers extend $\mathcal{ASP}$ solvers with an incrementally changing query to the $\mathcal{CLP}$ program derived from the $\mathcal{AC}(\mathcal{C})$ program. Current solvers restart $\mathcal{CLP}$ reasoning from scratch when the query changes, leading to redundant computation in the search for answers to the portions of the query that did not change. In this dissertation we formalize the incremental query problem and provide an incremental algorithm that reuses the solutions of previous queries in the search for answers to the modified query.

A dynamic domain can be viewed as a transition diagram whose nodes are possible states of the domain and whose arcs are actions in the domain. Action languages have been used to conveniently specify the state transition diagram, but are restricted to small or medium sized domains [15]. $\mathcal{ALM}$ is a recently developed modular action language that addresses larger domains [24]. It supports the modeling by the concepts of modules, module hierarchy and library.

In this dissertation we present $\mathcal{CALM}$ – a compiler for $\mathcal{ALM}$. It can translate an $\mathcal{ALM}$ system description P into a $\mathcal{SPARC}$ program which specifies the same state transition diagram of P. $\mathcal{CALM}$ also supports language for specifying temporal projection and planning problems. $\mathcal{CALM}$ will translate a given system description and the specification of a temporal projection or planning problem into a $\mathcal{SPARC}$ program whose answer sets contain solutions to these problems.

$\mathcal{CALM}$ is the first compiler to support the complete syntax of $\mathcal{ALM}$ and perform semantic error checking prior to translation into an Answer Set Programming ($\mathcal{ASP}$) program. Prior to $\mathcal{CALM}$, there is a prototype translator for $\mathcal{ALM}$ [24]. It only works with correct system descriptions (syntactically and semantically), It does not implement all of the syntax of $\mathcal{ALM}$, but covers its significant core. Another modular action language is $\mathcal{MAD}$[29, 10, 30]. A $\mathcal{MAD}$ compiler can translate a $\mathcal{MAD}$ program into a program in the language of the Causal Calculator ($\mathcal{CCALC}$)[19, 33] that can be used to carry out reasoning tasks.

One significant difference between $\mathcal{ALM}$ and $\mathcal{MAD}$ in modeling dynamic domains is that the semantics of $\mathcal{ALM}$ are based on the "law of inertia" [34] which says that "things normally stay the same" while $\mathcal{MAD}$ is founded on the causality principle which states that "everything true in the world must be caused"[14]. $\mathcal{ALM}$ and $\mathcal{MAD}$ also differ in their organization of modules, hierarchical ontologies, and instantiation of objects[22, 10].

Other action languages of note include $TAL - C$ [21] and $Modular\ BAT$[20]. While $TAL-C$ provides its semantics in translation to $\mathcal{CCALC}$, it does not present a mechanism for specifying reusable modules of knowledge and formal organization of a hierarchical ontology. $Modular\ BAT$ has similar goals to $\mathcal{ALM}$[22] but is based in the situation calculus. $\mathcal{ALM}$also contains a concept of basic action theory, but our semantics are provided through $\mathcal{ASP}$.

Due to translation of $\mathcal{ALM}$ System Descriptions to $\mathcal{ASP}$, $\mathcal{CALM}$ can only support finite domains for sorts. In order to support to continuous domains and their corresponding constraint relations, $\mathcal{ALM}$ System Descriptions must be translated to a to a different language. The area of Constraint Answer Set Programming ($\mathcal{CASP}$) extends ASP with constraint programming techniques which are capable of handling the real number domain and its constraint relations. The languages $\mathcal{AC}(\mathcal{C})$[36], $\mathcal{CLINGCON}$[40], and $\mathcal{EZCSP}$[3, 4] have all addressed the task in various ways. Both $\mathcal{EZCSP}$ and $\mathcal{CLINGCON}$ are equivalent to $\mathcal{AC}^{-}$[28] a subset of $\mathcal{AC}(\mathcal{C})$.

The *ACSolver* for $\mathcal{AC}(\mathcal{C})$ incorporates both $\mathcal{ASP}$ and $\mathcal{CLP}$ solvers to handle the regular and constraint portions of the program. The original *ACSolver*[35] executes the $\mathcal{CLP}$ query from scratch when the query is extended or backtracking occurs. The *LUNA* solver[37] improved the performance of the *ACSolver* by reusing the answer to the previous query when the query to the $\mathcal{CLP}$ program was extended. If backtracking occurred and the query was reduced, the query would be re-executed against the $\mathcal{CLP}$ program. This re-execution of the query during backtracking performs redundant computation. In order to eliminate this redundant computation, the $\mathcal{CLP}$ solver must be modified to become an incremental solver which preserves solutions to incrementally constructed queries for reuse when the query is decremented during backtracking. For a $\mathcal{CLP}$ solver, both *ACSolver* and *LUNA* used $\mathcal{CLAM}(\mathcal{R})$[27], an extension of the *WAM*[44] to include arithmetic constraints over real numbers. Our eventual goal is to provide a modified instruction set and implementation to the $\mathcal{CLAM}(\mathcal{R})$ solver which saves results of incrementally constructed queries for later reuse. As a significant step towards that goal we have developed a state transition diagram called *IQTD* which models the state of an incremental solver as it searches the sld-derivation tree for the answers to an incremental query of a logic program. As our goal was to extend the capability of the $\mathcal{CLAM}(\mathcal{R})$ solver, we did not investigate the use of tabled logic programs[5] and slg-resolution.

The *ACSolver*'s query to the $\mathcal{CLP}$ solver behaves as a stack. The query is extended by appending a new query increment as a result of unit propagation in the *ACSolver* and the most recently appended query is removed when backtracking removes the context which forced the query to be extended. Our criteria for reviewing the existing approaches to incremental $\mathcal{CLP}$ solvers included the requirements that it can be implemented as a modification to $\mathcal{CLAM}(\mathcal{R})$, support a stack like behavior for query modification, and preserve all solutions discovered for each prefix of the active query stack to eliminate redundant computation during backtracking.

In our literature review we investigated several approaches to incremental constraint logic programming which we could not apply given our criteria. The *Reactive* $\mathcal{CLP}$ *scheme*[11, 12] did account for the query manipulating operations, but the solution performs transformations of the sld-derivation tree which would require significant redesign of the $\mathcal{CLAM}(\mathcal{R})$ solver and its proofs of correctness. Many incremental query efforts provide their solutions as meta-interpreters implemented in Prolog[42, 38] and their translation to a machine instruction set would not be straight forward.

Our solution *IQTD* extends the work of Peter Stuckey's *Expanding query power in constraint logic programming languages*[31] and Pascal Van Hentenryck's *Incremental search in constraint logic programming*[43]. Stuckey's work saves every solution encountered for a query and executes a query extension in the context of each saved solution to derive solutions for the extended query. Stuckey did not address removal of queries except to indicate re-execution of the remaining query would be needed. Van Hentenryck's work re-executes portions of a path of computation when constraints are added or removed from the query, but his work does not address addition and removal of atoms. Our solution combines both approaches. At every level of the incremental query we save all solutions discovered and their paths of computation for future reuse after removal of queries.

CHAPTER 2
CALM

2.1 Basic Action Theory (BAT)

In this chapter we recall the concept of a Basic Action Theory (BAT) [24] by first introducing sorted signature and then the syntax and semantics of BAT.

2.1.1 Sorted Signature

The *sorted signature* of a BAT is $\Sigma = \langle C, O, H, F \rangle$ where C, O, and F are sets of strings over some fixed alphabet, and

1. $C = C_{sp} \sqcup C_{pd} \sqcup C_{ud}$ where $C_{sp} = \{nodes, obj\_constants, universe\}$ elements of C_{sp} are called *special sorts*; $C_{pd} = \{boolean, integer, [m..n]\}$, where m, n are natural numbers and $m < n$, and elements of C_{pd} are called *pre-defined sorts*, and C_{ud} is a set and elements of C_{ud} are called *user defined* sorts. Elements of C are called *sort names* in general.

2. $O = O_{pd} \sqcup O_{ud}$ where $O_{pd} = \{true, false, 0, 1, -1, 2, \ldots\}$, and elements of O_{pd} are called *predefined* object constants, and O_{ud} is a set and its elements are called *user defined* object constants. Elements of O are called *object constants*.

3. H is a directed acyclic graph (V, E), where $V \subseteq (\{universe\} \cup C_{pd} \cup C_{ud}) \cup O$. H is called a *sort hierarchy*. Sometimes (C, O, H) is called an *ontology*. For simplicity, we assume the graph has exactly one sink node.

   (a) For any $(c_1, c_2) \in E$, where $c_1, c_2 \notin O$, c_2 is called *parent* of c_1. For any sort names c_1 and c_2, c_1 is a *subsort* of c_2 if c_1 is a descendant of c_2 in H.

   (b) For any (o, c) where $c \notin O$ an $o \in O$, o is called *of sort* c. For any parent c_2 of c_1, if o is of sort c_1, then o is *of sort* c_2.

4. $F = F_{pd} \sqcup F_{sp} \sqcup F_{ud}$ where

   - $F_{pd} = \{+, -, \times, /, \leq, \geq, <, >, \ldots\}$, and elements of F_{pd} are called the *pre-defined* functions.

   - $F_{sp} = F_{spH} \sqcup F_{spD}$ where

     $$\begin{aligned} F_{spH} = \{ & link : nodes \times nodes \rightarrow boolean; \\ & is\_a : universe \times nodes \rightarrow booleans; \\ & instance : universe \times nodes \rightarrow booleans; \\ & subsort : nodes \times nodes \rightarrow boolean; \\ & has\_child, has\_parent, sink, source : nodes \rightarrow boolean\}, \text{ and} \end{aligned}$$

     $$F_{spD} = \{(dom_f : c1, \ldots, c_n \rightarrow boolean) \mid (f : c_1, \ldots, c_n \rightarrow c) \in F_{ud}\}.$$

     Elements of F_{spH} are called *sort hierarchy functions*. Elements of F_{spD} are called *domain functions*. Elements of F_{sp} are called *special functions*.

   - F_{ud} is a set of functions, and its elements are called *user defined* functions.

   - We assume a function $f \in F$ of the form $f : c_1, \ldots, c_n \rightarrow c$ where f is a string, c and c_i $(i \in 1..n)$ are sort names. f is called the *name* of the function, n is called the *arity* of f, $c_1, \ldots, c_n$ are called the *sorts of parameters* or *domain sorts* of f, and c is called the *range* or *range sort* of f. A domain sort or range sort of f is called a *signature sort* of f.

A variable and an object constant is a *term*. If $f : c_1 \times c_1, \ldots, c_n \rightarrow c$ is an element of F, and $t_1, \ldots, t_n$ are terms then $f(t_1, \ldots t_n)$ is a *term*.

Expressions of the form $t_1 = t_2$ and $t_1 \neq t_2$, where t_1 and t_2 are terms, are called *literals*. The former is also called *atoms*. We use standard shorthand and write t and $\neg t$ instead of $t = true$ and $t = false$.

Terms and literals not containing variables are called *ground*.

2.1.2 Basic Action Theory (BAT)

We first introduce action signatures. An *action signature* is a sorted signature

$$\Sigma = \langle C, O, H, F \rangle$$

where C includes the string *actions*, there is a node labeled by *actions* and an arc labeled by $\langle actions, universe \rangle$ in H. In an action signature, its user-defined and special function symbols are divided into three disjoint categories: *attributes, statics, and fluents*. Both statics and fluents are further divided into *basic* and *defined*. The latter are total boolean functions that can be defined in terms of the former.

We next introduce the definitions of axiom statements.

- A *dynamic causal law* is an expression of the form

$$\textbf{occurs}(a) \textbf{ causes } f(\bar{x}) = o \textbf{ if } instance(a, c), cond \tag{2.1}$$

  where **occurs**, **causes** and **if** are keywords, a and o are variables or object constants and $\bar{x}$ is a sequence of terms, f is a basic fluent, c is the sort *actions* or a subsort

  of it, and *cond* is a collection of literals.

  The law says that *an occurrence of an action a of the sort c in a state satisfying property cond causes the value of $f(\bar{x})$ to become o in any resulting state.*

- A *state constraint* is an expression of the form

$$f(\bar{x}) = o \textbf{ if } cond \tag{2.2}$$

  where

o is a variable or an object constant, f is any function except a defined function, and *cond* is a collection of literals.

The law says that *the value of* $f(\bar{x})$ *in any state satisfying condition cond must be* o.

Additionally, $f(\bar{x}) = o$ can also be replaced by the object constant $false$, in which case the law says that *there is no state satisfying condition cond.*

- The *definition* of a defined function p is an expression of the form

$$\begin{aligned} &p(\bar{t}_1) \textbf{ if } cond_1 \\ &\ldots \\ &p(\bar{t}_k) \textbf{ if } cond_k \end{aligned} \tag{2.3}$$

where $\bar{t_1}, \ldots, \bar{t_k}$ are sequences of terms, and $cond_1, \ldots, cond_k$ are collections of literals. Moreover, if p is a static then $cond_1, \ldots, cond_k$ can not contain fluent literals. Statements of the definition will be often referred to as its *clauses.*

The statement says that, for every $\overline{Y}$, $p(\overline{Y})$ is true in a state σ iff there is $1 \leq m \leq k$ such that statements $cond_m$ and $\bar{t}_m = \overline{Y}$ are true in σ.

- An *executability condition for actions* is an expression of the form

$$\textbf{impossible occurs}(a) \textbf{ if } instance(a, c), cond \tag{2.4}$$

where **impossible** is a keyword, a is a variable or an object constant, c is the sort *actions* or a subsort of it, and *cond* is a collection of literals and expressions of the form $occurs(t)$ or $\neg occurs(t)$ where t is a variable or an object constant of the sort *actions*.

This law says that *an occurrence of an action a of the sort c is impossible when condition cond holds.*

Dynamic causal laws and constraints will be sometimes referred to as *causal laws*. We use the term *head* to refer to the literal immediately before **if** in (2.1) and (2.2), and to any of the $p(\overline{t_i})$, $1 \leq i \leq k$, in (2.3). We call *body* the expression to the right of the keyword **if** in statements (2.1), (2.2), (2.4), or in any of the statements of (2.3). Statements not containing variables will be referred to as *ground*.

An *axiom statement* or *axiom* is dynamic causal law, a state constraint, a definition of a defined function, or an executability condition.

A *basic action theory (BAT)* is a pair (Σ, A) where Σ is an action signature, and A is a set of axioms over this signature satisfying the following:

- If f is a basic fluent then
  - A contains a state constraint:

$$dom_f(X_0, \ldots, X_n) \textbf{ if } f(X_0, \ldots, X_n) = Y \tag{2.5}$$

  - No dynamic causal law of A contains an atom formed by dom_f in the head.
- If f is a defined fluent, a static, or an attribute then A contains the definition:

$$dom_f(X_0, \ldots, X_n) \textbf{ if } f(X_0, \ldots, X_n) = Y \tag{2.6}$$

- A contains definitions of special statics of the hierarchy given in terms of functions is_a and $link$:

$$\begin{aligned}
instance(O, C) &\ \ \textbf{if}\ \ is_a(O, C)\\
instance(O, C_2) &\ \ \textbf{if}\ \ instance(O, C_1), link(C_1, C_2)\\
has_child(C_2) &\ \ \textbf{if}\ \ link(C_1, C_2)\\
has_parent(C_1) &\ \ \textbf{if}\ \ link(C_1, C_2)\\
source(C) &\ \ \textbf{if}\ \ \neg has_child(C)\\
sink(C) &\ \ \textbf{if}\ \ \neg has_parent(C)\\
subsort(C_1, C_2) &\ \ \textbf{if}\ \ link(C_1, C_2)\\
subsort(C_1, C_2) &\ \ \textbf{if}\ \ link(C_1, C), subsort(C, C_2)
\end{aligned} \tag{2.7}$$

2.1.3 Semantics of BAT

We introduce the semantics of BAT by first introducing the interpretation of a sorted signature and then the model of a BAT.

2.1.3.1 Interpretation of Sorted Signatures

Definition 1. (Interpretation)

An *Interpretation* I of a sorted signature $\Sigma = \langle C, O, H, F\rangle$ consists of

- A non-empty set $|I|$ of strings called the *universe* of I.
- An assignment that maps
  - every user-defined sort c of H into a subset $I(c)$ of $|I|$ such that
    - if $\langle c_1, c_2\rangle \in H$ then $I(c_1) \subseteq I(c_2)$ and
    - $I(c) = \bigcup\{I(c_i) : c_i\ is\ a\ child\ of\ c\ in\ H\}$
  - every object constant o of Σ into an element of $|I|$ such that if$\langle o, c\rangle \in H$ then $I(o) \in I(c)$;

- every user-defined function symbol $f : c_1 \times \cdots \times c_n \rightarrow c_0$ of Σ into a (possibly partial) function $I(f) : I(c_1) \times \cdots \times I(c_n) \rightarrow I(c_0)$;
- The special function is_a into a function $I(is\_a) : |I| \times nodes \rightarrow booleans$ such that for every $x \in |I|$ and every sort $c \in nodes$, $I(is\_a)(x, c)$ is $true$ iff c is a source node of H and $x \in I(C)$;
- the special function $link$ into function $I(link) : nodes \times nodes \rightarrow booleans$ such that for every two sort nodes c_1, c_2, $I(link)(c_1, c_2)$ is $true$ iff $\langle c_1, c_2 \rangle \in H$;
- the special function dom_f for user-defined function $f : c_1 \times \cdots \times c_n \rightarrow c_0$ into function $I(dom_f)$ such that for every $\bar{x} \in I(c_1) \times \cdots \times I(c_n)$, $I(dom_f)(\bar{x})$ is $true$ iff $\bar{x}$ belongs to the domain of $I(f)$.

• On pre-defined symbols, I is identified with the symbol's standard interpretations.

□

Let signature Σ and an interpretation I be given. I can be partitioned into two parts: the *fluent part* consisting of the universe of I and the restriction of I on the sets of fluents, and the *static part* consisting of the same universe and the restriction of I on the remaining elements of the signature. The static part of I is referred to as the *static interpretation of* Σ.

Let signature Σ and a collection of strings U in some fixed alphabet be given. Σ_U denotes the signature obtained from Σ by expanding its set of object constants by elements of U, which we assume are of sort *universe*.

2.1.3.2 Models of BAT

Given an interpretation I of an action signature Σ, we define *fluent part* of it consisting of the universe of $\mathcal{I}$ and the restriction of I on the sets of fluents. We define *static part* of I consisting of the same universe and the restriction of I to the remaining elements of the signature. Sometimes we will refer to the latter as a *static interpretation* of Σ.

Given an action signature Σ and a collection U of strings in some fixed alphabet, we denote by Σ_U the signature obtained from Σ by expanding its set of object constants by elements of U, which we assume to be of sort $universe$

Definition 2. (Pre-Model)
Let T be a basic action theory with signature Σ and U be a collections of strings over some fixed alphabet. A static interpretation M of Σ_U is called a *pre-model of* T (with the universe U) if $M(universe) = U$ and for every object constant o of Σ_U that is not an object constant of Σ, $M(o) = o$.

□

A pre-model M for basic action theory T defines a *model*, a state transition diagram, T_M. We define T_M through its states and transitions.

Definition 3. (Program S_M)
Let M be a pre-model of basic action theory T. By S_M we denote the logic program that consists of:

- rules obtained from the state consraints and definitions of T by replacing variables with the properly typed object constants of Σ_M, replacing object constants with their corresponding interpretations in M, removing the constant $false$ from the head of state constraints, and replacing the keywork if with $\leftarrow$,
- The Closed World Assumption:

$$\neg d(t_1, \ldots, t_n) \leftarrow not\ d(t_1, \ldots, t_n).$$

  for every defined function $d : c_1 \times \cdots \times c_n \to booleans$ and $t_i \in M(c_i)$, $1 \leq i \leq n$.

□

Definition 4. (Program S_I)

For every interpretation I of Σ with static part M, by S_I we denote the logic program obtained by adding to S_M the set of atoms obtained from I by removing the defined atoms.

□

Definition 5. (State)

Let M be a pre-model of a basic action theory T. An interpretation σ with static part M is a *state* of the transition diagram T_M defined by M if σ is the the only answer set of the logic program S_σ.

□

Definition 6. (Program P_M)

Program P_M is obtained from a basic action theory T and pre-model M by

1. replacing variables by properly typed object constants of Σ_M;
2. replacing object constants by their corresponding interpretation in M;
3. removing the object constant $false$ from the head of state constraints;
4. replacing every occurrence of a fluent term $f(\bar{t})$ in the head of a dynamic causal law by $f(\bar{t}, I+1)$;
5. replacing every other occurrence of a fluent term $f(\bar{t})$ by $f(\bar{t}, I)$;
6. removing "$occurs(a)$ **causes**" from every dynamic causal law and adding $occurs(a)$ to the body.
7. replacing "**impossible** $occurs(a)$" in every executability condition by $\neg occurs(a)$;

8. replacing $occurs(a)$ by $occurs(a, I)$ and $\neg occurs(a)$ by $\neg occurs(a, I)$;

9. replacing the keyword **if** by $\leftarrow$;

10. adding the Closed World Assumption:

$$\neg d(t_1, \dots, t_n, I) \leftarrow not\ d(t_1, \dots, t_n, I)$$

for every defined fluent $d : c_1, \times \dots \times c_n \rightarrow booleans$ and $t_i \in M(c_i)$, $1 \leq i \leq n$;

11. adding the rule:

$$\neg f(t_1, \dots, t_n) \leftarrow not\ f(t_1, \dots, t_n)$$

for every defined static of the form $f : c_1 \times \dots \times c_n \rightarrow booleans$ and $t_i \in M(c_i)$, $1 \leq i \leq n$;

12. adding the Inertia Axiom:

$$dom_f(t_1, \dots, t_n, I+1) \leftarrow dom_f(t_1, \dots, t_n, I), not\ \neg dom_f(t_1, \dots, t_n, I+1)$$
$$\neg dom_f(t_1, \dots, t_n, I+1) \leftarrow \neg dom_f(t_1, \dots, t_n, I), not\ dom_f(t_1, \dots, t_n, I+1)$$

for every basic fluent $dom_f : c_1 \times \dots \times c_n \rightarrow booleans$, and $t_i \in M(C_i)$, $1 \leq i \leq n$;

13. adding the Inertia Axiom:

$$f(t_1, \dots, t_n, I+1) = t \leftarrow dom_f(t_1, \dots, t_n, I+1),\ f(t_1, \dots, t_n, I) = t,$$
$$not\ f(t_1, \dots, t_n, I+1) \neq t$$

for every basic fluent $f : c_1 \times \dots \times \rightarrow c_0$ not formed by dom, and $t_i \in M(c_i)$, $1 \leq i \leq n$, and $t \in M(c_0)$.

$\square$

Definition 7. (Program $P(M, \sigma_0, a)$)

Let σ_0 be a state of the transition diagram defined by a pre-model M, and let $a \subseteq M(actions)$. By $P(M, \sigma_0, a)$ we denote the logic program formed by adding to P_M the set of atoms obtained from σ_0 by replacing every fluent atom $f(t_1, \ldots, t_n) = t$ by $f(t_1, \ldots, t_n, 0) = t$ and adding the set of atoms $\{occurs(x, 0) : x \in a\}$.

□

Definition 8. (Transition)

Let σ_0 and σ_1 be states of the transition diagram defined by a pre-model M and let $a \subseteq M(actions)$. The triple $\langle \sigma_0, a, \sigma_1 \rangle$ is a *transition* of the transition diagram defined by a pre-model M of a basic action theory T if program $P(M, \sigma_0, a)$ has an answer set A such that $f(t_1, \ldots, t_n) = t \in \sigma_1$, iff

- f is an attribute or static and $f(t_1, \ldots, t_n) = t \in A$, or
- f is a fluent and $f(t_1, \ldots, t_n, 1) = t \in A$.

□

Definition 9. (Model)

A transition diagram T_M defined by pre-model M of a basic action theory T is called a *model* of T if it has a non-empty collection of states.

□

2.2 Syntax and Semantics of $\mathcal{ALM}$ System Description

The $\mathcal{ALM}$ paper presents $\mathcal{ALM}$ through examples. In this chapter we will make a more explicit treatment of the syntax and semantics of $\mathcal{ALM}$. The complete ANTLR4 grammar is contained in Appendix A.

2.2.1 Syntax of $\mathcal{ALM}$

2.2.1.1 Syntax of $\mathcal{ALM}$ Theories

A *sort declaration* is of the form:

$$id_1, \dots, id_n :: sort_1, \dots, sort_m$$
$$attr_1 : c_{1,1} \times \cdots \times c_{1,k_1} \to c_{1,0}$$
$$\dots$$
$$attr_l : c_{l,1} \times \cdots \times c_{l,k_l} \to c_{l,0}$$

where for $i \in [1..n]$, $j \in [1..m]$, id_i and $sort_j$ are identifiers and called *sort names* and id_i is called a *declared sort* and a *declared subsort of* $sort_j$. All $attr_1, \dots, attr_l$ are optional attribute function declarations. The syntax for attribute function declaration is a shorthand: for $i \in [1..m], j \in [1..l]$, the signature of the attribute function has the form $attr_j : sort_i \times c_{j,1} \times \cdots \times c_{j,k_j} \to c_{j,0}$. If $sort_j$ is the only domain argument to the signature, the attribute declarations have the form $attr_j : c_{j,0}$.

A *sort declarations section* is of the form

`sort declarations` $sd_1 \ \dots \ sd_n$

where `sort declarations` are reserved identifiers, sd_i is a sort declaration.

We next define constant declaration which introduces a collection of named object constants and indicates to which sorts they belong.

A *constant declaration* is of the form

$$id_1, \dots, id_n :: sort_1, \dots, sort_m$$

where for $i \in [1..n]$, $j \in [1..m]$, id_i is called a *declared constant* and $sort_j$ is called a *declared sort of constant* id_i. A *constant declarations section* is of the form

`constant declarations` $cd_1 \ \dots \ cd_n$

where `constant declarations` are reserved identifiers, cd_i is a constant declarations.

A *function declaration* is of the form

`[total]` $f : sort_1, \ldots, sort_n \rightarrow sort$

where f is an identifier and called *function name* and for $i \in [1..n]$, $sort_i$ is a sort name. Each function declaration is also called a *user-defined function signature*. A *functions declaration section* is of the form

> `function declarations`
>
>   `statics`
>
>   `basic` $bf_1 \ \ldots \ bf_{n_1}$
>
>   `defined` $bd_1 \ \ldots \ bd_{n_2}$
>
>   `fluents`
>
>   `basic` $ff_1 \ \ldots \ ff_{n_3}$
>
>   `defined` $fd_1 \ \ldots \ fd_{n_4}$

where `function declarations, statics, basic` and `defined` are reserved words, $n_1, \cdots, n_4$ are numbers, and every bf_i, bd_i, ff_i and fd_i is a function declaration.

An *axioms section* is of the form

> `axioms` $a_1 \ \ldots \ a_n$

where `axioms` is a reserved identifier, and a_i is an axiom as defined in BAT.[1]

A *module dependencies section* is of the form

> `depends on` $M_1, \ldots, M_n$

where M_i is a module name (which will be defined later).

A *module* is of the form

> `module` *mname mdep sdec cdec fdec axioms*

[1] The notion of literals is used in the definition of axioms in BAT. This notion can be redefined using the terminologies here, but we do not repeat the definitions.

where `module` is a reserved identifier, *mname* is an identifier and called a *module name*, *mdep* is a module dependencies section, *sdec* is a sort declarations section, *cdec* is a constant declarations section, *fdec* is a function declarations section, and *axioms* is an axioms section.

A *theory* is of the form

`theory` *tname* $m_1 \ \ldots \ m_n$

where `theory` is a reserved identifier, *tname* is an identifier and called a *theory name*, and m_i is a module.

2.2.1.2 Syntax of ALM Structures

A *structure* consists of *constant definition*, *instance definition*, *instance schema definition*, and *statics definition*. Those concepts will be defined below in order.

A *constant definition* is of the form

$$id = groundterm$$

An *instance definition* is of the form

$$groundterm \ \textbf{in} \ sort.$$

An *instance schema definition* is of the form

$id(V_1, \ldots, V_n)$ **in** c **where** $l_1, \ldots, l_m$

$f_1(\bar{t_1}) = V_1$

$\ldots$

$f_n(\bar{t_n}) = V_n$

where $f_1, \ldots, f_n$ are attribute functions declared for c and its super sorts, $V_1, \ldots, V_n$ are variables, the sort of V_i is the range sort of f_i, $l_1, \ldots, l_m$ are literals, and $\bar{t_1}, \ldots \bar{t_n}$ are vectors of terms. If a vector $\bar{t_i}$ is empty, the containing parenthesis () are removed.

A *statics definition* is of the form

$$l_0 \textbf{ if } l_1, \ldots, l_n.$$

where l_0 is a static function literal and $l_1, \ldots, l_n$ are static literals.

A *structure* is of the form

`structure` *sname consdef insdef statdef*

where `structure` is a reserved identifier, *sname* is an identifier and called a *structure name*, *consdef* is a sequence of constant definitions, *insdef* is a sequence of instance definitions and instance schema definitions, and *statdef* is a sequence of statics definitions.

2.2.1.3 ALM System Descriptions

An *ALM system description* is of the form

`system description` *name theory structure* where `system description` are reserved identifiers, *name* is an identifier and called a *system description name*, *theory* is a theory and *structure* is a structure.

2.2.1.4 Well Defined System Descriptions

In this dissertation, we consider only a special class of system descriptions. First, we define the notion of sort dependency.

Definition 10. The Sort Dependency Graph of a System Description

Given a system description P with core BAT $T = (\Sigma = \langle C, O, H = (V, E), F\rangle, A)$, the *sort dependency graph* is the minimal directed graph (V', E') such that $V' \subset V$ and E' is defined as follows:

- $\langle c_2, c_1\rangle \in E'$ where $\langle c_1, c_2\rangle \in E$ and $c_1 \in C$.
- For any instance schema definition of P with the following form:

$$
\begin{array}{l}
\texttt{id(}V_1\texttt{, ..., }V_n\texttt{) in } c_1 \texttt{ (where } l_1, \ldots, l_m\texttt{)?} \\
\qquad f_1\texttt{(}\bar{t_1}\texttt{) = } V_1 \\
\qquad \ldots \\
\qquad f_n\texttt{(}\bar{t_n}\texttt{) = } V_n
\end{array}
$$

  $\langle c_1, c_2\rangle \in E'$ where c_2, $c_2 \neq c_1$, is user defined signature sort of some attribute function f_i ($i \in [1..n]$), or a user defined signature sort of a function occurring in $l_1, \ldots, l_m$, or occurs in any sort hierarchy functions that occurs in $l_1, \ldots, l_m$.

- For any static function definition r in the structure of P such that c_1 is a signature sort of a function occurring in the head of r and c_2 is a signature sort of a function occurring in the body of r

A sort c_2 *depends on* c_1 if there is a path from c_2 to c_1 in (V', E').

□

Definition 11. Well Defined System Descriptions

A system description P is *well defined* if its sort dependency graph is acyclic.

□

2.2.2 Semantics of ALM System Descriptions

In the following treatment of the semantics of ALM system descriptions, we assume a single module theory is present. A multi-module theory can be flattened into a single module [24].

An ALM system description P defines a BAT and its premodel(s).

2.2.2.1 Core BAT Defined by a System Description

Definition 12. The Core BAT Defined by the Theory of a System Description

Given a System Description P, *the core basic action theory* $T = (\Sigma = \langle C, O, H, F \rangle, A)$ defined by P's theory is as follows:

- $C = C_{sp} \cup C_{pd} \cup C_{ud}$ where C_{sp} and C_{pd} are the set of special sort names and predefined sort names as previously defined for BAT, and $C_{ud} = \{s \mid$ where s is a declared sort in $P\}$,

- $O = O_{pd} \cup O_{ud}$ where O_{pd} is the set of predefined constants as previously defined for BAT and $O_{ud} = \{o \mid$ where o is a declared constant in $P\}$,

- $H = (V, E)$ is the directed acyclic graph such that the set of nodes $V = \{universe, actions\} \cup C_{pd} \cup C_{ud} \cup O$ and the set of edges $E = E_{sp} \cup E_{pd} \cup E_{ud}$ where E_{sp} and E_{pd} are as defined previously for BAT and $E_{ud} = \{\langle v_1, v_2 \rangle \mid$ either $v_1 \in C_{ud}$, $v_2 \in C_{ud} \cup \{actions, universe\}$ and v_1 is a declared subsort of v_2 in P, or, $v_1 \in O_{ud}$, $v_2 \in C_{ud}$ and v_2 is a declared sort for constant v_1 in $P\}$,

- $F = F_{sp} \cup F_{pd} \cup F_{ud}$ where F_{pd} is as previously defined for BAT, $F_{ud} = \{f : c_1 \times \cdots \times c_n \to c \mid f : c_1 \times \cdots \times c_n \to c$ is a user-defined function signature in

$P\}$, and the previous definition of F_{sp} for BAT is extended by the set $\{dom_f : c_1 \times \cdots \times c_n \rightarrow booleans \mid f : c_1 \times \cdots \times c_n \rightarrow c \in F_{ud}\}$, and

- $A = A_{sp} \cup A_{ud}$ where A_{sp} are axioms from equations (2.5) to (2.7), and $A_{ud} = \{a \mid a$ is a user-defined axiom in $P\}$.

□

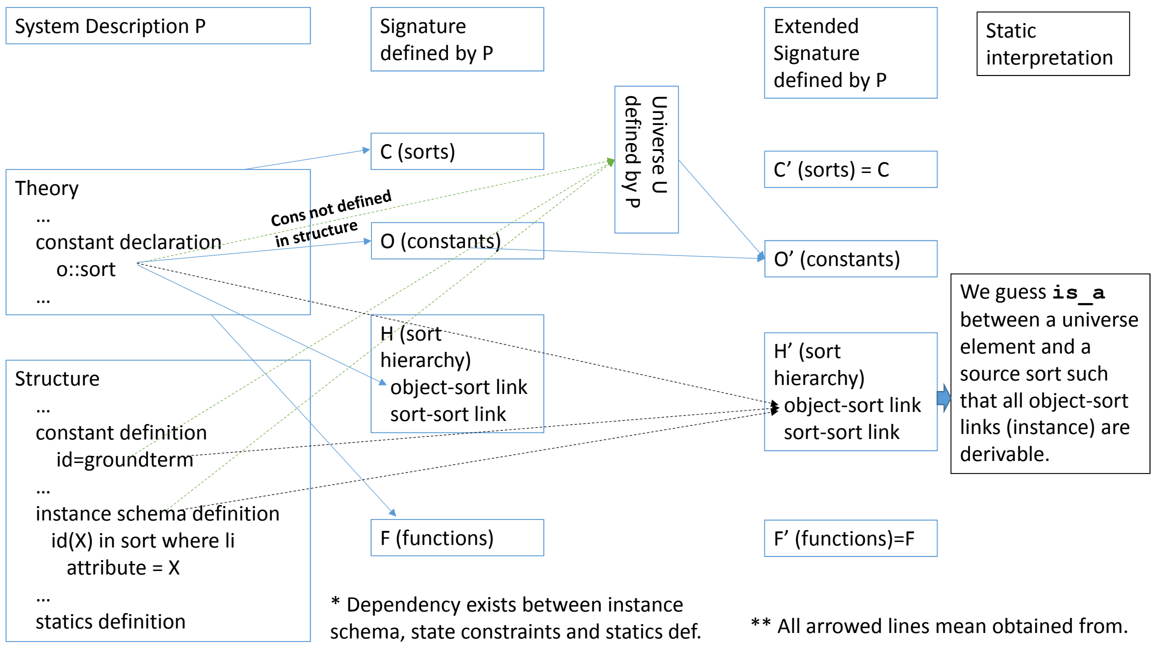
Proposition 1. The core BAT defined by the theory of a system description is a BAT.

Proof. By comparing the definition of BAT and Definition 12, we can verify that this proposition holds.

□

2.2.2.2 Static Interpretations Defined by ALM System Descriptions

An ALM system description is designed to specify an interpretation. We focus here on the static portion of the interpretation. The following diagram illustrates the key components leading to a static interpretation: the signature defined by a system description, the extended signature defined by such a description and the establishment of the `is_a` relation.



Due to the possible interactions among the instance schema definition, the state constraints on static functions and the statics definitions, we will use an ASP program to define the static interpretation specified by a system description.

Given an ALM system description P, let $T = (\Sigma = \langle C, O, H, F\rangle, A)$ be the core BAT defined by the theory of P. We need the following predicates:

- $universe(X)$ meaning X is an element of the universe U to be specified,
- $constantInTheory(X)$ meaning X is a constant declared in the theory of P,
- $extendedConstants(X)$ meaning X is *an extended constant*, i.e., a constant from constant declaration, constant definition or instance schema definition,
- $I\_extendedConstants(X, Value)$ meaning extended constant X is mapped to $Value$ of universe U,
- $I\_sorts(o, c)$ meaning the interpretation of the sorts, i.e. o is an element of sort c,

- $link\_constantInTheory(o, c)$ meaning that $(o, c) \in H$,
- $link\_constantInStructure(o, c)$ meaning that (o, c) occurs in an instance schema definition or a constant definition,
- $link\_extendedConstant(o, c)$ meaning that an object sort edge (o, c) occurs either in H or in structure,
- $consInTheoryDef(id, groundterm)$ meaning that the $id = groundterm$ occurs in the structure,
- $link(c1, c2)$ meaning that $(c_1, c_2) \in H$.

2.2.2.3 ASP Program Π_1 for Universe and Extended Signature

We define Π_1 as follows:

- Represent H from the core BAT of P.

  %% 2-1 for every syntactic constant o, there is a sort for it. This is from H, i.e., for every $(o, c) \in H$,

  $link\_constantInTheory(o, c)$.

  %% 2-2 for every $(c_1, c_2) \in H$ where $c_1, c_2 \in C$

  $link(c_1, c_2)$.

  % syntacticConstants from H

  $constantInTheory(X)$:- $link\_constantInTheory(X, C)$.

- Define syntactic constants from constant definition of the structure of P.

  % the constant definition id = groundterm in structure is represented as

  $consInTheoryDef(id, groundterm)$.

%% We introduce $consInTheoryDefined(X)$ meaning syntactic constant X is defined in structure

$consInTheoryDefined(X)$:- $consInTheoryDef(X, Term)$.

- Define/Represent other user-defined elements of universe. They may be from constant definition or instance (schema) definition in the structure, or from constants not defined in the structure.

  %% 3-1 for every constant definition $id = groundterm$ such that the declared sort of id is c,

  $link\_constantInStructure(groundterm, c)$.

  %% 3-2 for every instance definition t **in** c

  $link\_constantInStructure(t, c)$.

  %% 3-3 for every instance schema $id(V1, \ldots, Vn)$ in c where $l_1, \ldots, l_m$ (with) $f_i(\bar{t_i}) = V_i$, $1 \leq i \leq n$.

  $link\_constantInStructure(id(V1, \ldots, Vn), c)$:-

  $l_1, \ldots, l_m,$

  $instance(\bar{t_1}, \bar{c_{11}}), \ldots, instance(\bar{t_n}, \bar{c_{1n}}),$

  $instance(V1, c_{21}), \ldots, instance(Vn, c_{2n})$.

  where $\bar{c_{1i}}$ are the domain sorts of f_i and c_{2i} is the range sort of f_i.

  %% This schema also defines the attribute functions. For $i \in 1..n$,

  $f_i(id(V1, ..., Vn), Vi)$:- $link\_constantInStructure(id(V1, \ldots, Vn), c)$.

  % universe is all the instances in instance (schema) definitions in structure, i.e., $link\_explicitUniverseConstants$, and the syntactic constants not defined in the structure.

  $universe(X)$:- $link\_constantInStructure(X, C)$.

$universe(X)$:- $constantInTheory(X)$, *not* $consInTheoryDef(X)$.

- Extended constants of the extended signature with universe

  % extendedConstants are the union of syntactic constants and explicit element in universe

  $extendedConstants(X)$:- $constantInTheory(X)$.

  $extendedConstants(X)$:- $link\_constantInStructure(X, C)$.

2.2.2.4 ASP Program Π_2 for Interpretation of Extended Constants, is_a, and Sorts

- Map from extended constants to universe

  % the map as defined in the constant definitions in structure

  $I\_extendedConstants(Id, Groundterm)$:-
  $constantInTheory(Id)$,
  $consInTheoryDef(Id, Groundterm)$.

  %% for element in the universe, it is mapped to itself

  $I\_extendedConstants(Id, Id)$:- $universe(Id)$.

- Object-sort links in the hierarchy on extended constants. Note that there is NO object-sort link between a defined syntactic constants and any sort.

  % build link between universe constants and sorts

  %% 2-1 link between explicit universe constants and their sorts

  $link\_extendedConstant(O, C)$:- $link\_constantInStructure(O, C)$.

  %% 2-2 link for H where syntactic constants are not defined in structure, i.e., mapped to themselves

  $link\_extendedConstant(O, C)$:- $link\_constantInTheory(O, C)$,
  $I\_extendedConstants(O, O)$.

- Common axioms for BAT are useful for later definition of is_a relation, e.g., the definition of source sorts, subsorts etc.

  %% 2-1 For every state constraint or definition that involve static functions only, include ASP rules for it. (These rules correspond to axioms 2.2 and 2.3.)

  %% 2-2 for every static definition in structure of P, include ASP rules for them.

- Define is_a. Intuitively, is_a is a relation relates any universe element to a source sort such that all object sort links can be "derived".

  %% 2-1: if (o,c) is an "extended" link and c is source sort.

  $is\_a(O, C)$:- $link\_extendedConstant(O, C)$, $source(C)$.

  %% 2-2: if (o,c) is in "extended" link and c is NOT a source sort . We introduce o into any leaf (source) subsort of c.

  $1\{is\_a(O, SubC) : subsort(SubC, C), source(SubC)\}1$:-
  $link\_extendedConstant(O, C)$, $not\ source(C)$.

2.2.2.5 Pre-model Defined by a System be Description

Given a system description P, we call the program $\Pi_A = \Pi_1 \cup \Pi_2$, where Π_1 and Π_2 are defined as above, the *ASP program defined by* P. Given P, let $T = (\Sigma = \langle C, O, H, F\rangle, A)$ be the core BAT defined by the theory of P. Σ is called the *signature* defined by P.

Definition 13 (Universe and Extended Signature)**.** A *universe* U defined by an answer set S of Π is $U = \{x : universe(x) \in S\}$. An *extended signature* $\langle C, O', H', F\rangle$ defined by an answer set S of Π is

- Extended Constants: $O' = \{o : extendedConstants(o) \in S\}$,

- Sort hierarchy $H' = (V, E)$ where $V = O' \cup C$, $E = \{(c_1, c_2) : link(c_1, c_2) \in S\} \cup \{(o, c) : link\_extendedConstant(o, c) \in S\}$.

□

Clearly, the extended signature defined by an answer set of Π is a sorted signature. Let U and $\Sigma' = \langle C, O', H', F \rangle$ be the universe and extended signature defined by an answer set of Π. Note that Σ' and Σ_U share the same object constants. The difference between Σ' and Σ_U is the difference between H' and H. Intuitively compared with Σ_U, Σ' includes information on which constant of U belongs to which sort in its sort hierarchy (the sort hierarchy of Σ_U contains information only on constants of O but not U).

Definition 14. The *static interpretation M defined by an answer set S* of Π is

- The universe of M is the one defined by S.
- The assignment M is defined as follows:
  - For every $c \in C_{ud}$, $M(c) = \{x : instance(x, c) \in S\}$.
  - For every o_1 such that $extendedConstants(o_1) \in S$, $M(o_1) = o_2$ such that $I\_extendedConstants(o_1, o_2) \in S$.[2]
  - Each static user defined function $f : c_1 \times \cdots \times c_n \to c_0 \in F_{ud}$ is mapped into the possibly partial function $M(f) : M(c_1) \times \cdots \times M(c_n) \to M(c_0)$ such that, $M(f)(M(t_1), \ldots, M(t_n)) = M(t)$ where $f(t_1, \ldots, t_n) = t \in S$.
  - The special function is_a is mapped to the function $M(is\_a)$: $M(is\_a)(o, c)$ is true iff $is\_a(o, c) \in S$.
  - The special function $link$ is mapped to function $M(link)$: $M(link)(c_1, c_2)$ is true iff $link(c_1, c_2) \in S$.
  - For every static function $f \in F_{ud}$, the special function $dom_f : c_1 \times \cdots \times c_n \to booleans$ is mapped to $M(dom_f)$ such that for every $\bar{x} \in M(c_0) \times \cdots \times M(c_n)$, $M(dom_f)(\bar{t})$ is *true* iff $dom_f(\bar{t}) \in S$.

[2] This is well defined: o_2 exists and unique.

- On predefined symbols, M is identified with the symbol's standard interpretation.

$\square$

Proposition 2. Let Σ be the signature defined by P and U the universe defined by an answer set S of Π. A static interpretation M defined by S is a static interpretation of Σ_U.

Proof. Let $\Sigma' = (C', O', H', F')$ be the extended signature defined by S. We will first show that M is a static interpretation of Σ', and then show that it is a static interpretation of Σ_U. Clearly M consists of only static information.

By the definition of interpretation (definition 1), M satisfies the clauses in definition 1 in a straightforward manner except the conditions on $M(is\_a)$ and $M(c) = \bigcup\{M(c_i) : c_i \text{ is a child of } c\}$. Here we use the new definition of interpretation by Gelfond [23].

1) We prove the condition on $M(is\_a)$. For is_a, we can verify $M(is\_a) : universe \times nodes \rightarrow boolean$. Note that $universe$ denotes U. We will show that for every $x \in universe$ and every sort $c \in nodes$, $M(is\_a)(x, c)$ is true iff c is a source node of H and $x \in M(c)$.

By definition 14, $M(is\_a)(x, c)$ is true iff $is\_a(x, c) \in S$. We have two rules of Π defining is_a:

$is\_a(O, C)$:- $link\_extendedConstant(O, C)$, $source(C)$.

$1\{is\_a(O, SubC) : subsort(SubC, C), source(SubC)\}1$:-

$link\_extendedConstant(O, C)$, $not\ source(C)$.

$\Longrightarrow$: Since $is\_a(x, c) \in S$, from the the two rules above, we have $source(c) \in S$ which implies that c is a source node (because of the axioms 2.7). Since $is\_a(x, c) \in S$, axioms 2.7 implies that $instance(x, c) \in S$ which implies that $x \in M(c)$ by definition 14. Done.

$\Longleftarrow$: Since $x \in M(c)$, by definition 14, $instance(x, c) \in S$. Since c is a source sort, there is no c_1 such that $link(c_1, c)$. Hence, $instance(x, c) \in S$ is supported by the ASP rule of the first axiom in equation 2.7.

Therefore, $is\_a(x, c) \in S$. Done.

2) On $M(c) = \bigcup\{M(c_i) : c_i \text{ is a child of } c\}$. We know $M(c) = \{x : instance(x, c)\}$ and for each child c_i of c, $M(c_i) = \{x : instance(x, c_i)\}$. Since c_i is a child of c in H', $link(c_i, c) \in \Pi$. By hierarchy axioms 2.7, $instance(,c)$ is only defined by $instance(*, c_i)$. Therefore, $M(c) = \bigcup\{M(c_i) : c_i \text{ is a child of } c\}$.

Finally, Σ_U differs from Σ' only by the sort hierarchy where the edges of latter is a super set of the former. Hence, an interpretation of Σ' is an interpretation of Σ_U. Therefore, M is a static interpretation of Σ_U. $\square$

Definition 15. A *static interpretation defined by a system description* P is a static interpretation defined by an answer set of the program defined by P. $\square$

Theorem 1. Given a system description P, let signature Σ and core BAT T be defined by P. Let S be an answer set of Π, U and M are the universe and static interpretation defined by S respectively. M is a pre-model of T with universe U.

Proof. Let Σ be (O, C, H, F) and M be a static interpretation of P. $\Sigma_U = (O \cup U, C, H, F)$. O is the set of declared constants and predefined constants. We can ignore predefined constants.

By definition 2 of pre-models, to show M is a pre-model of T with universe U, we show 1) M is a static interpretation of Σ_U, 2) $M(universe) = U$, and 3) for every object constant o of Σ_U that is not an object constant of Σ, $M(o) = o$.

1) holds by Proposition 2.

2) holds directly by the first clause of definition 14 of static interpretation defined by an answer set of Π.

3) for every object constant o of Σ_U, if $o \notin O$, we show $M(o) = o$. Since $o \notin O$, we do not have the following rule in Π

$link\_constantInTheory(o, c)$.

Since Π has only one rule defining $constantInTheory()$

$constantInTheory(X)$:- $link\_constantInTheory(X, C)$

we have $constantInTheory(o) \notin S$. (1)

Since $o \in O \cup U$, we have $o \in U$ and thus $universe(o) \in S$. Since Π has the following rule

$I\_extendedConstants(Id, Id)$:- $universe(Id)$

we have $I\_extendedConstants(o, o) \in S$. (2)

By Π, the rules defining $universe()$ are

$universe(X)$:- $link\_constantInStructure(X, C)$.

$universe(X)$:- $constantInTheory(X)$, *not* $consInTheoryDef(X)$.

Therefore, $link\_constantInStructure(o, C) \in S$ for some sort C because of (1). By the following rule of Π

$extendedConstants(X)$:- $link\_constantInStructure(X, C)$

we have $extendedConstants(o) \in S$, which, together with (2), implies that $M(o) = o$ by (the second subclause of the second clause of) definition 14. Now we complete the proof of 3) and thus the proof of this theorem. □

2.2.2.6 Semantics of a System Description

Definition 16. (Model of System Description)

Given a system description P, let Π be the program defined by P and T the core BAT defined by P, and U and M the universe and static interpretation defined by an answer set of Π. A *model* of P is a transition diagram T_M defined by pre-model M of T with universe U such that it has a non-empty collection of states. □

2.3 Design Of CALM

2.3.1 Background

2.3.1.1 $\mathcal{SPARC}$

$\mathcal{SPARC}$ is a variant of $\mathcal{ASP}$ where the signature of the logic program is explicitly described through providing a formal description of the sort hierarchy and providing the sorted signature of each predicate used in the rules of the program.

Sort Definitions The *sorts section* occurs at the top of a $\mathcal{SPARC}$ program, begins with the keyword **sorts** and is followed by a sequence of *sort definitions.* Each sort definition has the following form:

$\#sort\_name = sort\_expression$**.**

The complete syntax for describing sorts is available in the $\mathcal{SPARC}$manual. We provide here the subset of syntax we use in CALM. Basic sort expressions include enumerated sets of ground terms $\{id_1, id_2, ...\}$, integer ranges $number_1..number_2$, sort names $\#sort\_name$, and records $record\_name(\#sort_1, \#sort_2, ..., \#sort_n)$. Complex sort expressions incorporate set theoretic operators between simple sort expressions. In our case we employ the + operator to perform union operations. A sort must be defined before use in records and complex sort expressions.

Example Sort Section:

```
sorts
#fruits = {apples, bananas}.
#vegetables = {peas, carrots}.
#edibles = #fruits + #vegetables
             + pair(#fruits,#vegetables).
```

Predicate Declarations The *predicates section* follows the sorts section, begins with the keyword **predicates** and is followed by a sequence of *predicate declarations.* Each predicate declaration has the following form:

$predicate\_name(\#sort_1, \#sort_2, \ldots, \#sort_n)$

Example Predication Section:

```
predicates
prefer(#edibles, #edibles)
eat(#edibles)
available(#edibles)
satiated()
```

Program Rules The *rules section* follows the predicate section, begins with the keyword **rules** and is followed by a sequence of $\mathcal{ASP}$ rules over the predicates that have been declared in the predicates section. A rule has the following form:

$l_1 \text{ or } l_2 \text{ or} \ldots \text{or } l_k \leftarrow\ l_{k+1}, \ldots, l_m, not\ l_{m+1}, \ldots, not\ l_n.$

For $i \in [1..n]$, each l_i is either a predicate, its negation, or a relation over terms or variables that have occurred within a predicate in the same rule.

Example Rules Section:

```
rules
satiated :- eat(X).
eat(X) :- prefer(X,Y), available(X), not -eat(X).
-eat(X) :- prefer(Y,X), available(Y), available(X).
-eat(X) :- not available(X).
available(pair(X,Y)) :- available(X), available(Y).
prefer(apples, bananas). prefer(peas, carrots).
prefer(pair(apples,carrots), apples).
available(apples). available(bananas).
available(carrots).
```

A solver will ground rules, substituting values for variables in accordance with the predicate signatures and the sort definitions provided in their respective sections.

2.3.2 Translation

2.3.2.1 Algorithm

The steps of the algorithm are explained in detail in the following sections.

```
CALM Algorithm
  input: an ALM system description P
  output: a SPARC program Π_C

  1. Parse P to verify that it is syntactically and
     semantically correct and report any errors.
  2. Construct the Core BAT T as defined by P.
  3. Construct the static SPARC Program Π_M from T.
  4. Obtain the answer sets A_M of Π_M.
     4.1 There must be exactly one answer set in A_M.
  5. Construct SPARC program Π_C from Π_M, A_M, and T.
     5.1 Each sort c is defined by {X : instance(X,c) ∈ A_M}.
     5.2 The predicates section is copied from Π_M.
     5.3 The rules are copied from Π_M and extended with
         rules translated from the non-static axioms in T.
```

We break the creation of the output program Π_C into two parts. First we construct the static program Π_M which encodes the action signature Σ_M where M is the model defined by P. Π_M also contains all static axioms and static function definitions which may influence the definition of instances for sorts. If more or less than one answer set exists for Π_M, we report an error. If there is no answer set to the static program, then there is no possible transition diagram described by Π_C. It is more informative to know that the sub-program Π_M failed than trying to discern this fact from Π_C failing to have an answer set. Currently $\mathcal{CALM}$does not support system descriptions which define sort instance for non-source sorts in the hierarchy. Such programs have multiple answer sets for the static program Π_M.

From our calculation of the answer set A_M, we replace the definitions of the sorts in Π_C with an enumeration of the instances indicated in A_M. We believe the explicit enumeration of each sort with its calculated instances helps to reduce the size of the final ground program and saves some of the computation needed to calculate the sort instances in Π_C.

Part of our selection of $\mathcal{SPARC}$ as a target for translation is its ability to represent the sort hierarchy separate from the rules of the program. Its automated type checking of rules based on predicate signatures has proved useful in catching errors during development of $\mathcal{CALM}$. While the current implementation of $\mathcal{SPARC}$ translates to the DLV $\mathcal{ASP}$ solver, a future direct implementation of $\mathcal{SPARC}$ can utilize the sort definitions and predicate signatures to optimize grounding and solver execution. If happens, $\mathcal{CALM}$ will benefit from its use of $\mathcal{SPARC}$ over un-sorted $\mathcal{ASP}$ variants.

2.3.2.2 Translation of BAT to SPARC

Let a basic action theory $T = (\Sigma = \langle C, O, H, F\rangle, A)$ be given. We construct the SPARC Program Π_C in two stages. We first construct the SPARC Program Π_M derived from the signature and static rules of T.

Definition 17. Static Translation Π_M of a System Description P Let $T = (\Sigma = \langle C, O, H, F\rangle, A)$ be the core BAT of System Description P.

Each sort definition in the sort section of Π_M has the following structure:

$\texttt{\#sort = \#sort}_1\texttt{ + ... + \#sort}_n$

```
+ id₁(#sort_{1,1} ..., #sort_{1,n_1})
+ ...
+ id_k(#sort_{k,1}, ..., #sort_{k,n_k})
+ {const_1, ..., const_s}.
```

such that the following properties hold:

- All sorts used on the right hand side of the = have been previously defined above this sort definition.
- for $i \in [1..n]$, $\texttt{sort}_i \in \{\texttt{sort}_1 \ldots \texttt{sort}_n\}$ if and only if ($\texttt{sort}$, $\texttt{sort}_i$)$\in H$.
- for $j \in [1..k]$, $\texttt{id}_j$(#$\texttt{sort}_{j,1}, \ldots,$#$\texttt{sort}_{j,n_j}$) is in the sort definition if and only if there is a schema definition in the structure of P of the form

  $$\texttt{id}_j(V_{j,1},\ \ldots,\ V_{j,n_j})\ \texttt{in sort (where}\ l_1, \ldots, l_m\texttt{)?}$$
  $$f_1(\bar{t_1})\ =\ V_1$$
  $$\ldots$$
  $$f_{n_j}(\bar{t_{n_j}})\ =\ V_{n_j}$$

  such that the range sorts of $f_1 \ldots f_{n_j}$ are $\texttt{sort}_{j,1}, \ldots, \texttt{sort}_{j,n_j}$ respectively.
- $\texttt{const} \in \{\ \texttt{const}_1 \ldots \texttt{const}_s\}$ if and only if `const` is a ground term and either is a declared constant for `sort` in P, defines a declared constant for `sort` in P, or there is an instance definition in P of the form

  ```
  const in sort.
  ```

Each predicate definition in the predicate section of Π_M has the following form:

$$\texttt{fun(\#sort}_1\texttt{, ..., \#sort}_{n-1}\texttt{, \#sort}_n\texttt{)}$$

such that either $\texttt{fun:sort}_1\texttt{,...,sort}_{n-1} \rightarrow \texttt{sort}_n$ is a static function in F or $\texttt{fun:sort}_1\texttt{,...,sort}_{n-2} \rightarrow \texttt{sort}_{n-1}$ is a fluent function in F and $\texttt{sort}_n$ is the time step sort.

The rules section of Π_M include all static auxiliary rules (2.2, 2.3, 2.7) and all the static user defined axioms from the theory and the structure translated to $\mathcal{SPARC}$ rules.

$\square$

2.3.2.3 Correctness Of Translation

Definition 18. Temporal Interpretation

Given a BAT signature Σ, let I be an interpretation of Σ and $T \in \mathcal{N}$. The *temporal interpretation I(T)* is the set of fluents and statics:

$$\{f(\bar{t}, T) = V |\ \mathit{fluent}\ f(\bar{t}) = V \in I\} \bigcup \{f(\bar{t}) = V |\ \mathit{static}\ f(\bar{t}) = V \in I\}$$

$\square$

Definition 19. Predicate Representation of an Interpretation

Let I be an interpretation for a BAT signature Σ and $T \in \mathcal{N}$. The *predicate representation of* I is the set of predicates:

$$\{f(\bar{t}, V) | f(\bar{t}) = V \in I\} \bigcup \{\neg f(\bar{t}, V') | f(\bar{t}) = V \in I \ \mathit{and}\ V \neq V'\}$$

where V and V' are different instances in the range sort of f.

$\square$

Definition 20. State Defined By The $\mathcal{SPARC}$ Program Π_C

Given a System Description P such that Π_A, the $\mathcal{ASP}$ program defined by P, has only one answer set S, let Σ and M be the core BAT and static interpretation defined by P respectively. Let Π_C be the output of $\mathcal{CALM}$ on P. An interpretation σ of Σ with static part M is a *state defined by the $\mathcal{SPARC}$ program* Π_C if and only if $\sigma'(0)$, the predicate representation of the temporal interpretation $\sigma(0)$, is the only answerset of the program $\Pi_C \cup K$ where K is $\sigma'(0)$ with the defined literals removed.

□

Definition 21. Transition Defined By The $\mathcal{SPARC}$ Program Π_C

Given a System Description P such that Π_A, the $\mathcal{ASP}$ program defined by P, has only one answer set S, let Σ and M be the core BAT and static interpretation defined by P respectively. Let Π_C be the output of $\mathcal{CALM}$ on P. Let σ_0 and σ_1 be states defined by Π_C and $a \subset M(actions)$. Let $\sigma_0'(0)$ and $\sigma_1'(1)$ be the predicate representations of the temporal interpretations $\sigma_0(0)$ and $\sigma_1(1)$ respectively. Let $A = \sigma_0'(0) \bigcup \{occurs(x,0) | x \in a\} \bigcup \sigma_1'(1)$. The triple $\langle \sigma_0, a, \sigma_1 \rangle$ is a *transition defined by the $\mathcal{SPARC}$ program* Π_C if and only if A is the only answer set of the program $\Pi_C \cup K$ where K is $\sigma_0'(0) \bigcup \{occurs(x,0) | x \in a\}$ with the defined literals removed.

□

Definition 22. State Transition Diagram Defined By The $\mathcal{SPARC}$ Program Π_C

Given a System Description P such that Π_A, the $\mathcal{ASP}$ program defined by P, has only one answer set, let Π_C be the output of $\mathcal{CALM}$ on P. The *Transition Diagram defined by* Π_C is the transition diagram defined by the set of states and transitions defined by Π_C if the set of states is non-empty.

□

Proposition 3. Given an $\mathcal{ALM}$ system description P, let Π_C be the output of $\mathcal{CALM}$ on input P. ST is a transition diagram defined by Π_C if and only if ST is a model of P.

Proof

Let Π be the $\mathcal{ASP}$ program defined by P and T the core BAT defined by P, and U and M the universe and static interpretation defined by an answer set of Π.

Assumption: ST is a transition diagram defined by Π_C

We will show that ST is a model of P by showing that σ is a state of ST if and only if σ is a state of T_M and that $\langle \sigma_0, a, \sigma_1 \rangle$ is a transition of ST if and only if $\langle \sigma_0, a, \sigma_1 \rangle$ is a transition of T_M.

Assumption: σ is a state of ST.

$\sigma'(0)$, the predicate representation of the temporal interpretation $\sigma(0)$ with the defined literals removed, is the only answerset of the program $\Pi_C \cup \sigma'(0)$ with the defined literals removed.

Note that $\sigma'(0)$ contains no occurrences of actions.

The rules within Π_C that have been translated from dynamic causal laws and executability conditions do not have their bodies satisfied.

The ground $\mathcal{SPARC}$ program Π_C with the dynamic causal laws and executability conditions removed is equivalent to the logic program S_M.

Since $\sigma'(0)$ is the predicate representation of the temporal interpertation $\sigma(0)$,

σ is the only answerset of the logic program S_σ.

σ is a state of T_M.

Assumption: σ is a state of T_M.

σ is the only answerset of the program S_σ.

The ground $\mathcal{SPARC}$ program Π_C with the dynamic causal laws and executability conditions removed is equivalent to the logic program S_M.

$\sigma'(0)$, the predicate representation of the temporal interpretation $\sigma(0)$ with the defined literals removed, is the only answerset of the program $\Pi_C \cup \sigma'(0)$ with the defined literals removed.

σ is a state of ST.

Therefore σ is a state of ST if and only if σ is a state of T_M.

Assumption: $\langle \sigma_0, a, \sigma_1 \rangle$ is a transition of ST

Let $A_{ST} = \sigma'_0(0) \bigcup \{occurs(x, 0) | x \in a\} \bigcup \sigma'_1(1)$ where $\sigma'_0(0)$ and $\sigma'_1(1)$ are the predicate representations of the temporal interpretations $\sigma_0(0)$ and $\sigma_1(1)$ respectively.

A_{ST} is the only answer set of the program $\Pi_C \cup K$ where K is $\sigma'_0(0) \bigcup \{occurs(x, 0) \mid x \in a\}$ with the defined literals removed.

Program $P(M, \sigma_0, a)$ has an answer set A such that $f(t_1, \ldots, t_n) = t \in \sigma_1$, iff $f(t_1, \ldots, t_n) = t \in A$ when f is an attribute or static and $f(t_1, \ldots, t_n, 1) = t \in A$ when f is a fluent.

$\langle \sigma_0, a, \sigma_1 \rangle$ is a transition of T_M.

Assumption: $\langle \sigma_0, a, \sigma_1 \rangle$ is a transition of T_M

program $P(M, \sigma_0, a)$ has an answer set A such that $f(t_1, \ldots, t_n) = t \in \sigma_1$, iff $f(t_1, \ldots, t_n) = t \in A$ when f is an attribute or static and $f(t_1, \ldots, t_n, 1) = t \in A$ when f is a fluent.

$\langle \sigma_0, a, \sigma_1 \rangle$ is a transition of ST

Therefore $\langle \sigma_0, a, \sigma_1 \rangle$ is a transition of ST if and only if $\langle \sigma_0, a, \sigma_1 \rangle$ is a transition of T_M

Since $ST = T_M$ and ST has a non-empty set of states,

ST is a transition diagram defined by pre-model M of T with universe U such that it has a non-empty collection of states.

Conclusion: ST is a model of P

$\square$

2.3.3 Semantic Errors

One of the primary contributions of $\mathcal{CALM}$is the integration of syntax and semantic error detection in the compilation process of translating a System Description to the $\mathcal{SPARC}$program Π_C. In this section we encode the understanding of a Well Defined System Description. We detail the semantic errors in context of their relevant grammatical elements from the syntax of a system description.

When evaluating a grammatical element for semantic errors it is evaluated with respect to the partial construction of the Basic Action Theory (BAT) $\langle C, O, H, F \rangle$ defined by processing module dependencies and previously processed grammatical elements.

2.3.3.1 Theory

The following semantic requirements must be satisfied for locally defined theories:

- Within a System Description there is exactly one theory defined or imported.
- An imported theory must indicate the library from which it is being imported and the library and theory name must resolve to a file on disk.

- Within a library file, there is exactly one theory defined and the theory name matches the containing file.

Module Declarations The following semantic requirements must be satisfied for module declarations and import statements.

- Module import statements must indicate the name of the containing library and theory and the name of the module being imported.
- Module import statements must resolve to a library and theory file on disk which contains a locally defined module of the same name being imported.
- The names of modules which are locally defined or directly imported must be unique within the context of the containing theory.
- A module dependency declaration must resolve to either a locally defined module or directly imported module in the containing theory.

Sort Declarations For a sort declaration statement of the form

$$\begin{aligned} &id_1, \ldots, id_n :: sort_1, \ldots, sort_m \\ &\qquad attr_f : c_1 \times \cdots \times c_k \to c_0 \end{aligned}$$

The following semantic requirements must be satisfied:

- For $j \in [1..n]$, each $sort_j$ must have been previously declared and exists as a node in H. (Note that this is sufficient to guarantee a path exists in H from the node labeled by $sort_j$ to the node labeled by $universe$.)
- For $i \in [1..n], j \in [1..m]$, each id_i and $sort_j$ must be a unique string.
- For $i \in [1..n], j \in [1..m]$, each id_i must not occur along any path from the node labeled by $sort_j$ to the node labeled by $universe$ in H.

- For $i \in [1..n], j \in [1..m]$, if a node labeled by id_i exists in H, there must not already be an arc in H from the node labeled by id_i to the node labeled by $sort_j$.
- For $i \in [1..n], j \in [1..m]$, add id_i to C and nodes labeled by id_i to H if they do not exist, and add arcs from id_i to $sort_j$ in H.
- For $l \in [0..k]$, there must be a node labeled by c_l in H.
- For $i \in [1..n]$, there must not already be a function $attr_f : id_i \times c_1 \times \cdots \times c_k \rightarrow c_0$ in F.
- For $i \in [1..n]$, add the function $attr_f : id_i \times c_1 \times \cdots \times c_n \rightarrow c_0$ to F with markers indicating the function is static and basic.

Constant Declarations For a constant declaration statement of the form

$id_1, \ldots, id_n :: sort_1, \ldots, sort_m$

The following semantic requirements must be satisfied:

- For $i \in [1..n]$, id_i must not be a string in either C or O.
- For $j \in [1..m]$, $sort_j$ must be in C.
- For $i \in [1..n]$, add id_i to O and add a node labeled by id_i to H.
- For $i \in [1..n], j \in [1..m]$, add an object constant arc from the node labeled by id_i to the node labeled by $sort_j$ in H.

Function Declarations For a function declaration statement of the form

`[total]` $f : sort_1, \ldots, sort_n \rightarrow sort_0$

The following semantic requirements must be satisfied:

- If f is a defined function $sort_0$ must be $booleans$.
- For $i \in [0..n]$, $sort_i$ must be in C.
- $f : sort_1, \ldots, sort_n \rightarrow sort_0$ must not already be a function in F.
- Add $f : sort_1, \ldots, sort_n \rightarrow sort_0$ to F with markings indicating whether or not the function is total, static or fluent, and basic or defined.
- Add $dom_f : sort_1, \ldots, sort_n \rightarrow booleans$ to F with markings indicating whether or not the function is total, static or fluent, and basic or defined.

Dynamic Causal Laws For a dynamic causal law of the form

$$occurs(a) \;\; causes \;\; f(x_1, \ldots, x_n) = o \;\; if \;\; instance(a, c), \;\; cond$$

The following semantic requirements must be satisfied:

- a must be a variable.
- c must be in C and a subsort of *actions* according to H.
- f must be a basic fluent function in F.
- o must be either a variable occurring in the conditions, a constant in O, or a function whose inferred sort is compatible with the range sort of f.
- For $i \in [1..n]$, x_i must be either a variable, a constant in O, or a function and the inferred sort of x_i must be compatible with the i_{th} domain sort in the signature of f.
- The type inferred for each variable in this axiom must resolve to a sort in the sort hierarchy.

- All functions occurring in the conditions of this axiom must have a unique signature in F and have agreement between their signature and the inferred sort for the syntactic elements appearing as domain arguments and the range value of the function.

Executability Conditions   For an executability condition of the form

$$impossible\ \ occurs(a)\ \ if\ \ instance(a, c),\ \ cond$$

The following semantic requirements must be satisfied:

- a is a variable.
- c must be in C and a subsort of *actions* according to H.
- The type inferred for each variable in this axiom must resolve to a sort in the sort hierarchy.
- All functions occurring in the conditions of this axiom must have a unique signature in F and have agreement between their signature and the inferred sort for the syntactic elements appearing as domain arguments and the range value of the function.

State Constraints   For state constraints of the form

$$f(x_1, \ldots, x_n) = o\ \ if\ \ cond$$

The following semantic requirements must be satisfied:

- f must be a basic function in F.
- If f is a static function then only static functions may occur in the conditions of the state constraint.

- o must be either a variable occurring in the conditions, a constant in O, or a function whose inferred sort is compatible with the range sort of f.
- For $i \in [1..n]$, x_i must be either a variable, a constant in O, or a function and the inferred sort of x_i must be compatible with the i_{th} domain sort in the signature of f.
- The type inferred for each variable in this axiom must resolve to a sort in the sort hierarchy.
- All functions occurring in the conditions of this axiom must have a unique signature in F and have agreement between their signature and the inferred sort for the syntactic elements appearing as domain arguments and the range value of the function.

Function Definitions   For function definitions of the form

$f(x_1, \ldots, x_n) \ if \ cond$

The following semantic requirements must be satisfied:

- f must be a defined function in F.
- If f is a static function then only static functions may occur in the conditions of the function definition.
- For $i \in [1..n]$, x_i must be either a variable, a constant in O, or a function and the inferred sort of x_i must be compatible with the i_{th} domain sort in the signature of f.
- The type inferred for each variable in this axiom must resolve to a sort in the sort hierarchy.

- All functions occurring in the conditions of this axiom must have a unique signature in F and have agreement between their signature and the inferred sort for the syntactic elements appearing as domain arguments and the range value of the function.

2.3.3.2 Structure

All semantic requirements of a structure are related to how its syntactic expression extend the core BAT of the theory.

Constant Definitions For constant definitions of the form

$const = groundterm$

The following semantic requirements must be satisfied:

- *const* must be a declared constant in O.
- *const* must not have been defined by another constant definition statement.
- This constant definition is recorded for later reference.
- The *groundterm* is recorded as an instance of every sort c for which there is an object constant arc from the node *const* to a node labeled by c in H.

Instance Definitions For instance definitions of the form

$groundterm\ \ in\ \ c$

The following semantic requirements must be satisfied:

- c must be a sort name in C and a label of a node in H.
- The *groundterm* is recorded as an instance of c.

For instance definitions of the form

$$id(V_1, \ldots, V_n) \;\; in \;\; c \;\; where \;\; l_1, \ldots, l_m$$
$$f_1(t_{1,1}, \ldots, t_{1,k_1}) = V_1$$
$$\ldots$$
$$f_n(t_{n,1}, \ldots, t_{n,k_n}) = V_n$$

The following semantic requirements must be satisfied:

- c must be a sort name in C and a label of a node in H.
- id must not be the name of a function in F or a declared constant in O.
- For $i \in [1..n]$, V_i is a variable
- For $i \in [1..n]$, the signature of f_i is an attribute function $f_i : c'_i \times c_{i,1} \times \cdots \times c_{i,k_i} \rightarrow c_{i,0}$ in F such that c is a subsort of c'_i in H.
- For $i \in [1..n]$, the inferred sort of V_i is $c_{i,0}$, c must not be a subsort of $c_{i,0}$, and c is marked as dependent on $c_{i,0}$ in the BAT for later reference.
- The type inferred for other variables in this instance schema definition must resolve to a sort in the sort hierarchy.
- All functions occurring in this instance schema definition must have a unique signature in F and have agreement between their signature and the inferred sort for the syntactic elements appearing as domain arguments and the range value of the function.

Static Function Definitions Static function definitions in the structure have the same semantic requirements as static function definitions in the theory.

2.3.3.3 Type Checking

For simplicity, our discussion focuses on the type checking of the theory of a system description.

For example, consider an example in the travel domain we discussed earlier. Assume function *location* : *agents* $\rightarrow$ *locations*, and a state constraint

```
location(C) = P if
    holding(A,C), location(A) = P.
```

We know function *holding* : *agents* $*$ *carriables* $\rightarrow$ *booleans*. So, the literal $holding(A, C)$ means that the variable C belongs to sort *carriables*. However, the literal $location(C) = P$ implies that C is of sort *agents* by function *location*. We also know that the sort names *carriables* and *agents* are not related in terms of *subsort*. So, we spot an error using sort (type) information of the theory.

We will define below type compliance of a BAT theory. Naturally, the definition is recursive. We first introduce sort compatibility and then start the definition of compliance from terms.

The sorts $S_1, ..., S_n$ of an action signature are *compatible* if they have a common descendant in the directed graph of the sort hierarchy of the signature.

Given a BAT, a term t of form $f(t_1, ..., t_n)$ and a variable occurrence X,

- if X is t_i, let the sort of i^{th} parameter of f be s, s is the *inferred sort* of X from t,
- otherwise, let X occur in t_i and s be the inferred sort of X from t_i, s is the *inferred sort* of X from t.

The inferred sorts of variable X from a term t is the set of the inferred sort of every occurrence of X from t.

We can obtain the inferred sort of a constant occurrence (and constant respectively) from the definition of the inferred sort of a variable occurrence (and variable respectively) by replacing variables with constants. For simplicity, we assume no constant contains a functor.

Given a BAT, a term t of form $f(t_1, ..., t_n)$ is *compliant with sort* S if

- the range of f is compatible with S,
- for every $t_i (i \in 1..n)$, t_i is compliant with s_i which is the sort of the i^{th} parameter of f, and
- for every constant or variable x of t, the inferred sorts of x from t are compatible.

In the following, $occurs(a)$, where a is either a variable or a constant, is also called a *literal*.

Given a BAT, a literal l and a variable occurrence X,

- if l is of the form $X = f(t_1, ..., t_n)$ or $f(t_1, ..., t_n) = X$ and range of f is s, s is the *inferred sort* of X from l,
- if l is of the form $f(t_{11}, ..., t_{1m}) = g(t_{21}, ..., t_{2m})$, X occurs in $f(t_{11}, ..., t_{1m})$ (or $g(t_{21}, ..., t_{2m})$ respectively) and the inferred sort of X from $f(t_{11}, ..., t_{1m})$ (or $g(t_{21}, ..., t_{2m})$ respectively), s is the *inferred sort* of X from l,
- if l is of the form $occurs(X)$, the special sort *actions* is the *inferred sort* of X from l, and
- if l is of the form $instance(X, s)$ where s is a sort name, s is the *inferred sort* of X from l.

The inferred sorts of variable X from a literal is the set of the inferred sorts of every occurrence of X from l.

Similarly, we define the inferred sorts of constant and constant occurrence from a literal.

Give a BAT, a literal l is *type compliant* if

- when l is of form $t_1 = t_2$, or $t_1 \neq t_2$, where $t_1 = f(t_{11}, ..., t_{1m})$ and $t_2 = g(t_{21}, ..., t_{2n})$ are terms, the ranges of f and g are compatible, t_1 is compliant with range of f and t_2 is compliant with range of g,

- when l is of form $X = f(t_1, ..., t_n)$ or $f(t_1, ..., t_n) = X$, $f(t_1, ..., t_n)$ is compliant with the range of f,

- for every constant or variable x, the inferred sorts of x from l are compatible.

Given a BAT, an axiom is *type compliant* if

- every literal of the axiom is type compliant, and

- for every constant or variable x, the inferred sorts of x from the literals of the axiom are compatible.

A theory is *type compliant* if its every axiom is type compliant.

Note that due to space limitations, the type checking definition does not include arithmetic relations.

2.3.4 Reasoning System

2.3.4.1 Histories and Trajectories

Definition 23. A Trajectory Of The State Transition Diagram Defined by Π_C

A sequence $\langle \sigma_0, a_0, \sigma_1 \rangle, \langle \sigma_1, a_1, \sigma_2 \rangle, \ldots, \langle \sigma_{n-1}, a_{n-1}, \sigma_n \rangle$ of transitions defined by Π_C is called a *trajectory of the transition diagram of* Π_C.

□

Note that a trajectory is not required to define range values for every domain value of a basic fluent. Law of inertia for dom_f will preserve the undefined range value until the range value is causally determined by a dynamic causal law.

Definition 24. A History for A System Description

Given an $\mathcal{ALM}$ System Description P, a *History* Π_H *for* P is a collection of ground facts defined as follows:

- if $observed(f(\bar{x}), v, t) \in \Pi_H$ then $f(\bar{x})$ is a ground instance of a user defined fluent in P, v is a ground instance in the range of f, and t is a positive integer time step,
- if $happened(a, t) \in \Pi_H$ then a is a ground instance of a subsort of *actions* in P and t is a positive integer time step,
- there are no other facts in Π_H.

□

Definition 25. The Complete History Encoding of a Trajectory

Given an $\mathcal{ALM}$ System Description P, and a trajectory $\langle \sigma_0, a_0, \sigma_1 \rangle$, $\langle \sigma_1, a_1, \sigma_2 \rangle$, $\ldots$, $\langle \sigma_{n-1}, a_{n-1}, \sigma_n \rangle$ of the transition diagram of the $\mathcal{SPARC}$ program Π_C produced by $\mathcal{CALM}$ on P, the *complete history encoding of the trajectory* is the program Π_H defined as follows:

- $observed(f(\bar{x}), v, t) \in \Pi_H$ if and only if $f(\bar{x}) = v \in \sigma_t$ and f is a basic fluent.
- $happened(a, t) \in \Pi_H$ if and only if $a \in a_t$.

□

Definition 26. Model Of A History

Given an $\mathcal{ALM}$ System Description P with $\mathcal{SPARC}$ program Π_C produced by $\mathcal{CALM}$ on P and A History Π_H for P, a trajectory $T = \langle \sigma_0, a_0, \sigma_1 \rangle$, $\langle \sigma_1, a_1, \sigma_2 \rangle$, $\ldots$, $\langle \sigma_{n-1}, a_{n-1}, \sigma_n \rangle$ of the transition diagram of Π_C is called a *model* of Π_H if there exists a program Π_{CH} such that the following properties hold:

- Π_{CH} is the complete history encoding of T,
- If $observed(f(\bar{x}), v, t) \in \Pi_H$ then $observed(f(\bar{x}), v, t) \in \Pi_{CH}$,
- $happened(a, t) \in \Pi_H$ if and only if $happened(a, t) \in \Pi_{CH}$.

□

Note that this definition of model of a history is consistent with the definition in the original $\mathcal{ALM}$ paper[24].

2.3.4.2 Temporal Projection

Definition 27. Temporal Projection Problem

Given an $\mathcal{ALM}$ System Description P with output of $\mathcal{CALM}$ Π_C and a history Π_H for P, a *temporal projection problem* for P is the $\mathcal{SPARC}$ program Π_{TP} such that

- The sort section of Π_{TP} is the sort section of Π_C.
- The predicate section of Π_{TP} is an extension of the predicate section of Π_C with the following declarations:
  - `observed(#universe, #universe, #timeStep)`
  - `happened(#action, #timeStep)`
- The rules section of Π_{TP} is the rules section of Π_C extended by Π_H and the following rules:
  - `occurred(A, I):- happened(A, I).`
  - `f(`$\bar{X}$`, V, 0) :- observed(f(`$\bar{X}$`), V, 0).`

    for each $observed(f(\bar{x}), v, 0) \in \Pi_H$.
  - `:- not f(`$\bar{X}$`, V, I), observed(f(`$\bar{X}$`), V, I).`

for each $observed(f(\bar{x}), V, I) \in \Pi_H$.

$\square$

Definition 28. Solution of Temporal Projection Problem Π_{TP}

Given a temporal projection problem Π_{TP} for an $\mathcal{ALM}$ System Description P, let Π_C be the output of $\mathcal{CALM}$ on P and Π_H be the history, the collection of facts, added to Π_C to create Π_{TP}. Each model of Π_H is called a *solution of* Π_{TP}.

$\square$

Each answer set produced by the $\mathcal{SPARC}$ solver on a temporal projection problem Π_{TP} encodes a model for the given history.

2.3.4.3 Planning

Definition 29. Planning Problem

Given an $\mathcal{ALM}$ System Description P and a temporal projection problem Π_{TP}, let t be the maximum time step in the facts of Π_H, Let Π_S be a collection of observations of fluent values at $t' > t$, i.e. $\Pi_S \subset \{observed(f(\bar{x}), v, t') | f \ is \ a \ basic \ fluent \ in \ P\}$. The pair $\langle \Pi_{TP}, \Pi_S \rangle$ is called a *planning problem* for P. Π_S is called the *goal state* of the planning problem.

$\square$

Definition 30. Solutions of Planning Problem $\langle \Pi_{TP}, \Pi_S \rangle$

Given a planning problem $\langle \Pi_{TP}, \Pi_S \rangle$ for an $\mathcal{ALM}$ System Description P, let Π_C be the output of $\mathcal{CALM}$ on P and Π_H be the history of facts in Π_{TP} not in Π_C, let t be the maximum time step in Π_H and t' be the time step of the facts in Π_S. Let Π_A be a subset of $\{happened(A, I)|$ where A is a ground instance of an action in P and $t < I < t'\}$. Π_A is called a *solution of the planning problem* $\langle \Pi_{TP}, \Pi_S \rangle$ if the history $\Pi_H \cup \Pi_A \cup \Pi_S$ has a model and the following properties hold:

- There does not exist a time k such that Π_A has no action occurring at time k but has an action occurring after k.

- There is no proper subset Π'_A of Π_A such that $\Pi_H \cup \Pi'_A \cup \Pi_S$ has a model.

$\square$

Our method of solving planning problems is provided in section 2.4.4.3. We follow method provided in section 4.2.1 of the original $\mathcal{ALM}$ paper[24]. Consistency restoring rules are used to generate actions after the last time step in the history while the goal state is not achieved by the last time step allowed in the horizon. Each answer set of the $\mathcal{SPARC}$ program implementing a planning problem encodes a model of an extension of the provided history with a schedule of actions that achieves the goal state within the horizon.

2.4 Implementation of $\mathcal{CALM}$

Java was selected as the implementation language for its cross-platform portability as a compiled jar.

2.4.1 Parsing System Descriptions and Tasks

2.4.1.1 ANTLR4 Parser

ANTLR4 was selected for its ability to generate a syntax parser with hooks for adding semantic processing to successfully parsed non-terminals. The ANTLR4 grammar for $\mathcal{ALM}$ is in Appendix A. Instructions on how to modify the grammar and change the semantics of $\mathcal{ALM}$ are provided in Appendix B. The output of the parser is an object model for the abstract syntax tree of a system description. The syntax tree of a system description is processed in two passes. The first pass builds a hierarchy of module dependency which may include references to external libraries and theories. The second pass through the syntax processes each module referenced into a symbol table modeling the BAT structure the module encodes. During this second pass, each $\mathcal{ALM}$ statement is evaluated for semantic errors before being added to the symbol table modeling the BAT structure. Special auxiliary and user defined axioms are added to a collection of $\mathcal{ASP}\{f\}$ rules specifying constraints and requirements of the ontology and state transition diagram. If semantic errors are discovered, they are added to an error report displayed to the user to indicate where in the syntax errors are occurring. If no syntax or semantic errors are generated, the BAT and collected $\mathcal{ASP}\{f\}$ axioms are translated to $\mathcal{SPARC}$.

2.4.2 Modeling a Basic Action Theory

Proper semantic evaluation of a System Description and translation to a BAT $(\Sigma = \langle C, O, H, F\rangle, A)$ requires modeling the hierarchy of module dependencies. Our implementation uses a hierarchical symbol table to build a modular BAT representation.

2.4.2.1 Symbol Table

Our symbol table is primarily implemented as several Java based key-value maps. For simple identifiers such as names of constants and sorts, the keys are Strings and the values assigned are the instances of classes designed to model relevant relationships and properties.

To model the set of sort names C we map the string name of the sort to a SortEntry class which has a parent reference and set of child references to model the tree structure of the sort hierarchy H. To model links from object constants to sort nodes in H, each SortEntry has an initially empty set of instance objects that are populated by instance definitions when parsing the structure of a system description. Initial $universe$ and *actions* sort entries are created and added to the map.

The set of object constants O in the core BAT is modeled as a map from string names of constants to instances of a ConstantEntry class which contains a field to indicate any definition of the constant and a set of SortEntry elements to indicate which sorts the constant belongs to. The extended object constants in O of the extended BAT signature are modeled by the set of instances within each SortEntry.

The set of function signatures F is modeled as two maps. The first is a map from function names to sets of normal function signature entries for the name. The second map is from normal function signatures to the related dom_f signature for the function.

The axioms of A are stored in $\mathcal{ASP}\{f\}$form and are organized into thematic sections. The sections are ordered when they are translated to $\mathcal{SPARC}$. The auxiliary axioms required to model the sort hierarchy and other BAT constructs are added their own section before parsing a system description.

2.4.2.2 Module Hierarchies

In order to properly evaluate the semantic soundness of a module with respect to its module dependencies, a symbol table per module is created. The symbol table for one module has references to the symbol tables of the modules that are included in dependency declarations. If the use of a sort or function in this module does not have a local declaration within the module, they are looked up in the dependent symbol tables recursively. If the use of a sort, function or constant cannot be resolved against declarations through module dependency, semantic errors are reported.

Before translation, all symbol tables are flattened by taking the union of the different maps across the symbol tables in the module hierarchy.

2.4.3 Error Checking and Reporting

An ErrorReport singleton class is accumulates syntax errors generated by the parser and passed into the call-back handler for semantic evaluation of successfully parsed non-terminals in a system description. Error messages are generated for both syntax and semantic errors which indicate file, line and column number of the syntax generating the error.

2.4.3.1 Message Structure

When the ANTLR4 Parser encounters an unexpected symbol in the grammar, it invokes a call-back method we provide to collect syntax errors. The parser provides the location of the error and a RecognitionException object which contains the offending token and a set of expected tokens for that location. We preserve these reported instances in a list of SyntaxError objects collected in the ErrorReport to be displayed after parsing has completed.

Semantic Errors have the following four parts:

1. A unique string called an ErrorId,
2. The message template to display,
3. An explanation of the semantic error,
4. And a recommendation for how to fix the error.

Example:

```
ErrorID: CND008
Message: Term  [graspers(A) at (temp-test.alm:48:32)]
    has not been declared as a function term or a
    constant for any sort.
Explanation: Non pre-defined and non-variable terms
    must be declared before they are used.
Recommendation: Either add a function definition or a
    constant definition for the term, or replace it
```

```
with a declared term.
```

The semantic error text is stored in a pre-defined table indexed by the unique ErrorId. The message template of each error contains numbered positions that can be filled in dynamically with the syntax tokens that are causing the semantic error to occur.

2.4.3.2 Semantic Error Checking

When non-terminal in the grammar is completed successfully, the ANTLR4 parse calls a method to semantically process the grammatical elements in the non-terminal. Within each non-terminal handler function the grammatical elements are first evaluated for any semantic errors. If a semantic error is found, new semantic error object is created in the ErrorReport with the appropriate ErrorId and a sequence of offending tokens in the grammar to fill in the message template. If no semantic errors are created in the non-terminal handler, the grammatical elements are entered into the appropriate parts of the symbol table.

When displaying semantic errors at the end of parsing the system description, each message template is filled by the sequence of syntax tokens provided at the time of error creation. Each numbered entry in the template is replaced with the corresponding token's text and file name, line and column number of where the offending token occurs.

2.4.3.3 Type Checking

Each axiom in the theory and each instance definition and static function definition in the structure must pass type checking. Type checking is passed when every variable in the generated logic rules has a concrete inferred type and there is agreement between the expected sorts defined by a function signature and the inferred sorts of the domain arguments and range value of the function.

For example consider the function signature $f : c_1 \times \cdots \times c_n \rightarrow c_0$ and the occurrence of a literal $f(x_1, \ldots, x_n) = x_0$. The expected sort of x_1 is c_1 from the signature of f. If x_1 is a constant or instance, then it has a set of sorts $s_1, \ldots, s_m$ for which it has been declared. As long as one of $s_1, \ldots, s_m$ is a subsort of c_1, then type checking passes for x_1 in the context of f. If x_1 is a variable, then we add the requirement that the inferred sort of variable x_1 be a subsort of c_1.

Variable type checking is implemented as follows: For each occurrence of a variable, the required sort is determined as the intersection of all the inferred sorts for the variable's occurrences. Intersection here is determined as the greatest common subsort of all inferred sorts for the variable.

For example, consider a dynamic causal law for the form:

$$occurs(A) \;\; causes \;\; f(x_1, \ldots, x_n) = o \;\; if \;\; instance(A, c_1), \;\; instance(A, c_2), \;\; cond$$

The first occurrence of A has an inferred sort of $actions$. The second occurrence of A has an inferred sort of c_1. The third occurrence of A has an inferred sort of c_2. The required sort from these occurrence is $actions \cap c_1 \cap c_2$, i.e. $\bigcup c_3$ where c_3 is a subsort of $actions$, c_1, and c_2. If no common subsort exists, then the required sort is labeled as the $EMPTY$ type. If the variable is free and has no inferred sort, it is labeled with the ANY type.

The emergent type system here is analogous to expressions in set-calculus. Expressions are reducible under the following reduction rules:

1. $\cap$ and $\cup$ are commutative operators and $\cap$ binds more tightly than $\cup$ as an infix operator.

2. $c_1 \cap c_2 = c_1$ if c_1 is a subsort of c_2

3. $c_1 \cap c_2 = EMPTY$ if neither is a subsort of the other.

4. $c_1 \cap EMPTY = EMPTY$

5. $c_1 \cap ANY = c_1$

6. $c_1 \cup c_2 = c_1$ if c_2 is a subsort of c_1.

7. $c_1 \cup ANY = ANY$

8. $c_1 \cup EMPTY = c_1$

9. $c_1 \cup \cdots \cup c_n = c$ if $c_1, \ldots, c_n$ is a list of all direct subsorts of c.

Variable type checking passes when every variable in a rule has a reduced type that is not ANY or $EMPTY$.

2.4.4 Translation to $\mathcal{SPARC}$

The implementation of translation from BAT to $\mathcal{SPARC}$ is performed through creating statements that populate the Sorts, Predicates and Rules section of the produced $\mathcal{SPARC}$ program.

2.4.4.1 Sorts Section

The sorts section of the produced $\mathcal{SPARC}$ program is produced by recursively defining each sort in the sort hierarchy. Begining with the *universe* sort, before a sort is defined, its immediate subsorts and its dependent sorts must first be defined in the output program. The produced $\mathcal{SPARC}$ sort definitions have the following form:

$$\texttt{\#}c = \texttt{\#}c_1 + \cdots + \texttt{\#}c_n + s_1(\texttt{\#}c_{1,1}, \ldots, \texttt{\#}c_{1,m_1}) + \cdots + s_k(\texttt{\#}c_{k,1}, \ldots, \texttt{\#}c_{k,m_k}) + \{g_1, \ldots, g_l\}$$

where $c_1, \ldots, c_n$ are the names of sorts that are direct subsorts of c in the sort hierarchy, $s_1(\#c_{1,1}, \ldots, \#c_{1,m_1}), \ldots, s_k(\#c_{k,1}, \ldots, \#c_{k,m_k})$ are derived from schema instance definitions where each $c_{i,j}$ is the inferred sort of the j_{th} variable in the i_{th} schema definition for sort c, and $g_1, \ldots, g_l$ are ground instances defined for sort c. The $\mathcal{SPARC}$ sort definition for $c_1, \ldots, c_n$ and each $c_{i,j}$ in schema pattern for s_i must occur before the $\mathcal{SPARC}$ sort definition for c.

Special and predefined sort definitions such as ranges of integers and the time steps allowed in trajectories are defined as enumerations of ground terms.

$\#timeStep = \{0, 1, 2, 3, \ldots, n\}$

2.4.4.2 Predicates Section

The predicate section is populated by the predicate signatures modeling the function signatures in the set of functions F in the BAT. Let $f : c_1 \times \cdots \times c_n \rightarrow c_0$ be a function signature in F. If f is a static function, the predicate signature has the following form:

$prefix\_f(\#c_1, \ldots, \#c_n, \#c_0)$

If f is a fluent function, the predicate signature has the following form:

$prefix\_f(\#c_1, \ldots, \#c_n, \#c_0, \#timeStep)$

In both cases the prefix is determined by whether or not the function is qualified by additional namespace requirements and whether the function is a special domain function. In the current version of $\mathcal{CALM}$ the only functions which are namespace qualified are attribute functions. For example if the function is a domain function for an attribute function f of sort c, the predicate signature would have the following form:

$dom\_c\_f(\#c, \#c_2, \ldots, \#c_n, \#c_0)$

Future versions of $\mathcal{CALM}$ may add additional namespace modeling to allow different modules to re-use the same function names locally but have them resolve to different functions globally in the translated program.

2.4.4.3 Rules Section

After parsing the grammatical elements of an $\mathcal{ALM}$ System Description, the rules modeling the the semantics of the statements are stored in $\mathcal{ASP}\{f\}$ form in the symbol table. These rules along with auxiliary rules for modeling the BAT structure and modeling functions as predicates must be translated from $\mathcal{ASP}\{f\}$ to their $\mathcal{SPARC}$ equivalent.

Normalization and Function Translation All rules, when translated from $\mathcal{ASP}\{f\}$ to their $\mathcal{SPARC}$ equivalent must go through a process of normalization and translation from functions to predicates. $\mathcal{CALM}$ supports nested function terms in the syntax of ALM System Descriptions. $\mathcal{SPARC}$ does not support functions or the ability to nest predicates within each other.

We explain the process of normalization by an example of nested fluent functions:

$foo(A, bar(B)) = baz(C)$.

The result of normalization is that nested function are replaced by new variables and new literals are added to the end of the body of the rule that equate the variable with the function it replaced. The normalization of the above literal is the following:

$foo(A, Z_1) = Z_2, bar(B) = Z_1, baz(C) = Z_2$.

After normalization, the functions can be translated to their predicate equivalent form. Since these functions are fluents, they have a time variable added to their domain.

$foo(A, Z_1, Z_2, T), bar(B, Z_1, T), baz(C, Z_2, T)$.

Special Auxiliary Rules There are many rules needed in addition to the explicit user defined axioms provided in the System Description. In order to model the semantics of BAT, the Sort Hierarchy H must be modeled along with domain functions dom_f in F. The axioms for law of inertia on fluents (and their domain functions) and closed world assumption on defined functions must also be added. Since we are modeling functions as predicates, for every function f in F we must add uniqueness constraints on value assignments to the function. Example:

$-f(\bar{t}, V_2)$ `:-` $f(\bar{t}, V_1),\ \ V_1 \neq V_2.$

All of these auxiliary rules are added to the produced $\mathcal{SPARC}$ program in their own sections prior to translating any of the user defined axioms in the system description.

Static Program Rules The user defined static state constraints and static function definitions from the theory and static function definitions from the structure are added to the $\mathcal{SPARC}$
program in their respective sections.

Once all the static user defined rules are added, we send the static $\mathcal{SPARC}$ program to the solver to verify that it has a unique answer set. Static programs with multiple answer sets are not currently supported by $\mathcal{CALM}$. If there is no answer set, translation halts and a semantic error is reported to indicate that the static program needs to be corrected before fluent rules can be added.

In the current implementation of $\mathcal{CALM}$ we use the resulting answer set A to replace the sort definitions in the sort section of the $\mathcal{SPARC}$ program with enumerations from the instance literals in the answer set.

`#`$c = \{x|\ \ instance(x, c) \in A\}$

Our thoughts on this initially is that it would be an optimization step on the grounder to not have to recalculate the sorts in the fluent program. Doing this optimization, however, removes the encoding of the sort hierarchy from the final $\mathcal{CALM}$ program, effectively removing the capability of extending the final program manually by adding new instances to source sorts and having super sorts inherit the new instances. It is planned that future versions of $\mathcal{CALM}$ will support multiple answer sets as a result of the static program, and in that case the sort hierarchy would need to be preserved in the final program.

Fluent Program Rules The user defined fluent axioms are normalized, translated and added to the static program to create the program Π_C that defines the $\mathcal{SPARC}$ encoding of the transition diagram that is the output of $\mathcal{CALM}$ on the input system description. For dynamic causal laws with literals in the head of rules, the time variable added to these literals is $T+1$ while the time variable added to fluent literals in the body is T. This step in translation models the transition between states in response to action occurrences. Fluent state constraints have T added to the literal in the head to enforce the state constraint within the same time indexed state.

History Rules In the current implementation of $\mathcal{CALM}$ the parser will accept System Descriptions which have been extended with the specification of a task and an accompanying history at the end of the $\mathcal{ALM}$ program. The history specifies known values of a trajectory through the transition diagram. A history is composed of observations of values assigned to fluents and action occurrences at various points in time. Syntax:

$observed(fluent(\bar{t}), v, n)$.
$happened(action\_instance(\bar{t}), n)$.

Fluent observations at time 0 specify the initial state of the trajectory. Observations at this time are translated to predicate facts in the program: Example initial state:

$fluent(\bar{t}, v, 0)$.

Fluent observation at time $T > 0$ specify state constraints on the remainder of the trajectory, that it must be consistent with observations, but that the observations do not cause the trajectory to match the observed values assigned to the fluents. The action occurrences must cause the fluent values to be assigned. Example rule based state constraint:

$:-\ \ -fluent(\bar{t}, v, T)\,.$

Action occurrences are added as facts to the program at the time indicated. Example:

$occurs(action\_instance(\bar{t}), n)\,.$

If any observation or action occurrence is incompatible with the user defined axioms governing the definition of state and transitions for the transition diagram, no answer set will result. Currently $\mathcal{CALM}$ does not support any facilities for debugging $\mathcal{ALM}$ system descriptions and histories which produce no answer set. To localize the problem in the trajectory, the best approach is to start with the initial state and check if an answer set is produced. If so, incrementally extend the trajectory with time steps until the offending state transition is added. If the initial state is incompatible with the transition diagram, one knows the issue is with the definition of allowed states.

Task Execution Rules The execution of a temporal projection task requires no additional axioms or rules apart from the translation of the history into facts of actions occurrences, initial state at time step 0 and the added constraints that trajectory derived from the initial state through action occurrences must be consistent with the observed fluent states.

The execution of a planning problem requires additional axioms to model the desired goal state and to ensure that new actions are only taken after the history has ended. Let n be the last time step recorded in the history. Let m be the horizon at which the goal state must be reached. Let $f_1(\bar{t_1}) = v_1, \ldots, f_k(\bar{t_k}) = v_k$ be the fluent assignments in the goal state. The following rules are added to the program:

- Indicate the time steps during which the goal is achieved.

plan_goal(I) :- $f_1(\bar{t_1}, v_1, I), \ldots, f_k(\bar{t_k}, v_k, I)$.

- Indicate the current time, the next time step after the last time in the history.

```
current_time(n+1).
```

- Indicate when plan actions are allowed, at or after the current time.

```
plan_allow_actions(I) :- current_time(I2),I>=I2,I<=m.
```

- The plan is successful when there exists a time when actions are allowed and the goal is achieved.

```
plan_success :- plan_goal(I), plan_allow_actions(I).
```

- It is impossible to not have a successful plan.

```
:- not plan_success.
```

- Indicate when an action occurrence is a plan action.

```
plan_action(I) :- occurs(A, I), plan_allow_actions(I).
```

- It is impossible for a plan to skip time steps in its plan of actions.

```
:- not plan_action(I), plan_action(I+1), I+1<=m,
   plan_allow_actions(I).
```

- Generate actions while planning allows actions and not goal state reached.

```
occurs(A, I) :+ instance(A,actions), not plan_goal(I),
   plan_allow_actions(I).
```

- It is impossible for planning to allow actions and not generation an action or be in the goal state.

```
:- not plan_action(I), not plan_goal(I),
   plan_allow_actions(I).
```

2.4.5 System Usage

The $\mathcal{CALM}$ executable can be obtainable from the link: https://goo.gl/NvXAZq. Download the whole folder to your computer. Examples can be found in the sub-folder `examples/`.

To compile an ALM system description in file `f1.alm`, command

`java -jar calm.jar f1.alm`

will output a SPARC program to standard output if there are no errors. For temporal projection in file `f2.tp` or planning problem in file `f3.p`, we have the following commands respectively:

`java -jar calm.jar f2.tp`

`java -jar calm.jar f3.p`

will output the answer sets that contain solutions to the problems.

If there is any difference between the usage above and that in the `readme.txt`, it is recommended to follow the instructions in `readme.txt`.

CHAPTER 3
ICLP

Notation In This Part

$\square$ denotes when a value is not relevant.

Notation For Sequences

Let $A = \langle a_1, \ldots, a_n \rangle$ and $B = \langle b_1, \ldots, b_m \rangle$ be sequences.

The longest common prefix between two sequences is :
$prefix(A, B) = \langle a_1, \ldots, a_k \rangle$ when $a_i = b_i$ for $i \in [1..k]$ and $a_{k+1} \neq b_{k+1}$.

The length of a sequence is indicated as normal: $|A| = n$.

The symbol $\in$ denotes membership. $a_1 \in A$ and $a_1 \notin B$.

The brackets $A[i]$ denotes the i_{th} element of A. $A[i] = a_i$.

The concatenation of two sequences is $A \frown B = \langle a_1, \ldots, a_n, b_1, \ldots, b_m \rangle$.

$\langle\rangle$ denotes the empty sequence.

Notation For Graphs and Trees

A *graph* G is the pair $\langle V, E \rangle$ where V is a set of objects, and E is a subset of $\{\{v_1, v_2\} | v_1 \in V \wedge v_2 \in V\}$. The elements of V are called *vertices* or *nodes*. The elements of E are called *edges*.

Given a graph G, $V(G)$ denotes the vertices of G and $E(G)$ denotes the edges of G.

Given a graph G, a sequence of the form $\langle v_0, e_1, v_1, e_2, v_2, \ldots, e_n, v_n \rangle$ is called a *path from* v_0 *to* v_n *in* G when for all $i \in [1..n]$, $v_i \in V(G)$, $e_i = \{v_{i-1}, v_i\} \in E(G)$ and for all $i, j \in [1..n]$, if $j \neq i$ then $e_j \neq e_i$.

Given a graph G, G is called *connected* when for all $u, v \in V(G)$ where $u \neq v$, there exists a path from u to v in G.

Given a graph G, let $P = \langle v_0, e_1, v_1, e_2, v_2, \dots, e_n, v_n\rangle$ be a path in G, P is called a *cycle* or *cyclic path* when $v_0 = v_n$.

$G = \langle\langle V, E\rangle, L_V, L_E\rangle$ is called a *labeled graph* when $\langle V, E\rangle$ is a graph, L_V is a function from V to a set of objects, and L_E is a function from E to a set of objects.

Given a graph G, if G is connected and there are no cyclic paths in G, then G is called a *tree*.

Given a tree T, for every pair of nodes $u, v \in V(T)$ where $u \neq v$, there is exactly one path from u to v.

Proof by contradiction: Suppose there was more than one path from u to v in T. Let $\langle u = v_0, e_1, v_1, \dots, e_n, v_n = v\rangle$ be one of the paths and $\langle u = u_0, f_1, u_1, \dots, f_k, u_k = v\rangle$ be a different path. Since $v_0 = u_0$ and $v_n = u_k$:

- let u_i and v_i be such that $u_i = v_i$ and $u_{i+1} \neq v_{i+1}$,
- let m and h be the least integers such that, $u_m = v_h$, $m > i$, $h > i$, and for all $a \in [i+1..m-1]$ and $b \in [i+1..h-1]$, $u_a \neq v_b$.
- The path $\langle u_i, f_{i+1}, u_{i+1}, \dots, u_{m-1}, f_m, u_m, e_h, v_{h-1}, \dots, v_{i+1}, e_{i+1}, v_i\rangle$ is a cycle in T which contradicts with T being a tree.

There cannot be more than one path from any node u to any node v in T. There is at least one path since T is a connected graph. Thus there is exactly one path from u to v in T when $u \neq v$. $\square$

Given a tree T, for all $v \in V(T)$, let $E_v = \{e | e \in E(T) \wedge v \in e\}$, if $|E_v| = 1$ then v is called a *leaf of* T.

A *rooted tree* is a tree with a designated vertex called the *root*.

Given a rooted tree T, for all $v \in V(T)$, let $P = \langle v_0, e_1, v_1, e_2, v_2, \dots, e_n, v_n\rangle$ be the path from the root node v_0 to $v_n = v$ in T. The node v_{n-1} in P is called the *parent of* v *in* T and $v_1, \dots, v_{n-1}$ are called *ancestors of* v *in* T.

Given a rooted tree T, for all $v, u \in V(T)$, if v is the parent of u in T then u is called a *child of* v *in* T, if v is an ancestor of u in T then u is called a *descendent of* v *in* T.

$T = \langle\langle V, E\rangle, L_V, L_E\rangle$ is called a *labeled tree* when $\langle V, E\rangle$ is a tree and T is a labeled graph.

3.1 Constraint Logic Programming (CLP)

We review constraint logic programming ($\mathcal{CLP}$)[32] here.

3.1.1 CLP Program

3.1.1.1 Terms

A *term* is defined recursively:

- A variable X is a term.
- A string constant t is a term
- The Herbrand function $f(t_1, \ldots, t_n)$ is a term when f is a function symbol of arity n and $t_1, \ldots, t_n$ are terms.

A term containing no variables is called a *ground term*.

$s(X)$ – is an example of a term where $s/1$ is a function symbol of arity 1.

$s(s(s(1)))$ – is an example of a ground term.

3.1.1.2 Predicates

$P(t_1, \ldots, t_n)$ is called an *Atom* or *Predicate* of arity n when P is a predicate symbol and $t_1, \ldots, t_n$ are terms.

A predicate is called a *ground predicate* when it contains no variables.

$father(X, Y)$ – is an example of an atom where $father/2$ is a predicate symbol of arity 2.

$father(abraham, isaac)$ – is an example of a ground predicate.

3.1.1.3 Literals

A *literal* l is either an atom or a primitive constraint from some constraint domain $\mathcal{D}$.

$X < Y * 3.14$ – is a primitive constraint where X and Y are variables from $\mathcal{R}$ and $<$ is the less-than relation from $\mathcal{R}$.

$lessthan(X, Y * 3.14)$ – is a predicate representation of the above primitive constraint.

3.1.1.4 CLP Rules

A *CLP rule* r is of the form:

$$l_0 \leftarrow l_1, \ldots, l_n$$

where

- l_0 is a predicate, called a *user-defined constraint*
- $l_1, \ldots, l_n$ is a possibly empty sequence of arbitrary literals.

l_0 is called the *head* of the rule. $l_1, \ldots, l_n$ are called the *body* of the rule. Rules which have a head but the body is empty are called *facts* and are written as l_0 without the $\leftarrow$ symbol.

3.1.1.5 CLP Program

Let Π be an ordered collection of CLP rules. Π is called a *constraint logic program* (CLP Program).

For this writing we assume that Π, an arbitrary CLP program, has been given.

Let the rules in Π be totally ordered with respect to each other. We indicate that rule r_1 is less than rule r_2 in Π with the notation $r_1 < r_2$.

The total order on rules of Π creates a lexicographical ordering on sequences of rules from Π. We denote this total ordering on sequences of rules from Π as ω and use the notation $A <_\omega B$ to indicate that the sequence of rules A is less than the sequence of rules B with respect to ω.

Note that for a set of ordered elements L, we refer to the least element in L as the *first element in L*

3.1.2 Queries and Derivation Trees

3.1.2.1 Query

An arbitrary sequence of literals $\langle l_1, \ldots, l_n \rangle$ from Π is called a *query*.

3.1.2.2 CLP State

The pair $\langle Q|C \rangle$ is called a *CLP State* where Q is a query and C is either $fail$ or a consistent set of primitive constraints. C is called a *constraint store*.

The state $\langle \langle\rangle|C \rangle$ is called a *success CLP state* when C is consistent.

In the following writing we often use Q to denote the CLP state $\langle Q|\emptyset \rangle$.

3.1.2.3 Constraint Transition

Given CLP states S and S' and a primitive constraint c, $\langle S, c, S' \rangle$ is a constraint transition when

- $S = \langle Q|C \rangle$ where C is a consistent set of constraints.
- $Q = \langle l_1, l_2, \ldots, l_n \rangle$ where $l_1 = c$.

- $Q' = \langle l_2, \ldots, l_n \rangle$.
- $C' = C \cup \{l_1\}$ if it is consistent, otherwise $C' = fail$.
- $S' = \langle Q' | C' \rangle$.

We define $con(\langle Q|C \rangle)$ to be $\langle Q'|C' \rangle$ when $\langle \langle Q|C \rangle, c, \langle Q'|C' \rangle \rangle$ is a constraint transition.

We define $con^*(\langle Q|C \rangle)$ to be the closure of applying constraint transitions.
$con^*(\langle Q|C \rangle) = con(\ldots con(\langle Q|C \rangle) \ldots)$.

3.1.2.4 Resolution Transition

A literal l is *resolvable with a rule* r if the literal in the head of r has the same predicate name and arity as l.

A CLP state $\langle Q|C \rangle$ is *resolvable with a rule* r if the first literal in Q is resolvable with r.

Given CLP states S and S' and a rule r from a program Π, $\langle S, r, S' \rangle$ is a *resolution transition* when

- S is some CLP state $\langle Q|C \rangle$ where C is a consistent set of constraints.
- Q is of the form $\langle l_1 = p(t_1, \ldots, t_m), \ldots, l_n \rangle$.
- r is of the form $p(s_1, \ldots, s_m) \leftarrow B_1, \ldots, B_k$.
- $Q' = \langle B_1, \ldots, B_k, l_2, \ldots, l_n \rangle$.
- $C' = C \cup \{t_1 = s_1, \ldots, t_m = s_m\}$ if it is consistent, otherwise $C' = fail$.
- $S' = \langle Q' | C' \rangle$.

We define $res(\langle Q|C \rangle, r)$ to be $\langle Q'|C' \rangle$ when $\langle \langle Q|C \rangle, r, \langle Q'|C' \rangle \rangle$ is a resolution transition.

3.1.2.5 SLD-Derivation Tree

Given a CLP state $\langle Q|C\rangle$ where $C \neq fail$, an *SLD-derivation tree for* $\langle Q|C\rangle$ with respect to Π is a labeled rooted tree $T = \langle\langle V, E\rangle, L_V, L_E\rangle$ such that L_V is a function from V to CLP states, L_E is a function from E to rules in Π, and T is the minimal labeled tree satisfying the following properties:

1. For the root node a in T, $L_V(a) = con^*(\langle Q|C\rangle)$.

2. For every node $b \in V$ and for every rule $r \in \Pi$, if $L_V(b)$ resolves with r then there exists a node $c \in V$ such that $e = \{b, c\} \in E$ and $L_V(c) = con^*(res(L_V(b), L_E(e)))$.

Given an SLD-derivation tree $T = \langle\langle V, E\rangle, L_V, L_E\rangle$ for a CLP state $\langle Q|C\rangle$, for every $b \in V$, let $P_b = \langle v_0, e_1, v_1, \ldots, e_n, v_n\rangle$ be the path from the root node v_0 to $b = v_n$ in T, the sequence of rules $P_r = \langle L_E(e_1), \ldots, L_E(e_n)\rangle$ is called the *path of derivation for* b *in* T. P_b is called the *corresponding path in* T *for* P_r. If $L_V(b)$ is of the form $\langle\langle\rangle|C' \neq fail\rangle$ then P is called a *successful path of derivation*, C' is called the *solution of* P, and the pair $\langle C', P\rangle$ is called an *annotated solution for* $\langle Q|C\rangle$.

Given a CLP state $\langle Q|C\rangle$, the annotated solutions for $\langle Q|C\rangle$ are totally ordered in the following way: let $\langle C_1, P_1\rangle$ and $\langle C_2, P_2\rangle$ be two different annotated solutions for $\langle Q|C\rangle$, then $\langle C_1, P_1\rangle <_\omega \langle C_2, P_2\rangle$ **iff** $P_1 <_\omega P_2$. [1]

Given an SLD-derivation tree T for a CLP state $\langle Q|C\rangle$, let $P_1, \ldots, P_n$ be all the successful paths of derivation in T, in order with respect to ω. For $i \in [1..n]$, let C_i be the solution of P_i in T, C_i is called the i_{th} *solution of* $\langle Q|C\rangle$ and $\langle C, P\rangle$ is called the i_{th} *annotated solution of* $\langle Q|C\rangle$.

[1] We abuse notation for $<_\omega$ to indicate when objects are ordered by components containing sequences of rules.

3.2 The Incremental Query Problem

3.2.1 Incremental Query

We refer to a sequence of literals from Π as a *simple query* to contrast it with the notion of an incremental query.

Given simple queries $Q_1, \ldots, Q_n$, the sequence $I = \langle Q_1; \ldots; Q_n\rangle$ is called an *incremental query*. For every $k \in [1..n]$ the prefix $I_k = \langle Q_1; \ldots; Q_k\rangle$ is also an incremental query, called a *sub-incremental query* of I. I is a sub-incremental query to itself.

Given an incremental query $I = \langle Q_1; \ldots; Q_n\rangle$, let $Q_n = \langle l_{n,1}, \ldots, l_{n,k_n}\rangle$. I represents the simple query $Q_1 \frown Q_2 \frown \ldots \frown Q_n = \langle l_{1,1}, \ldots, l_{1,k_1}, \ldots, l_{n,1}, \ldots, l_{n,k_n}\rangle$.

$flatten(I)$ denotes the simple query represented by incremental query I.

Given an SLD-Derivation Tree T for $flatten(I)$, the i_{th} solution to $flatten(I)$ is called the *i_{th} solution to I*.

An annotated solution for $flatten(I)$ is called an *annotated solution for I*

3.2.1.1 IQ Sequence

An *IQ sequence* is a sequence of incremental queries $\langle I_1, \ldots, I_n\rangle$ where one of the following properties holds between I_k and I_{k+1} for $k \in [1..n-1]$:

1. $I_i = \langle Q_1; \ldots; Q_m\rangle$, $I_{i+1} = \langle Q_1; \ldots; Q_m; Q_{m+1}\rangle$
   (I_i is a sub-incremental query of I_{i+1})

2. $I_i = \langle Q_1; \ldots; Q_m\rangle$, $I_{i+1} = \langle Q_1; \ldots; Q_{m-1}\rangle$
   (I_{i+1} is a sub-incremental query of I_i)

3. $I_i = I_{i+1}$
   (I_i and I_{i+1} are sub-incremental queries of each other.)

3.2.1.2 IQ Commands

An *IQ command* is either $dec()$, $next()$, or $inc(Q)$ where Q is a non-empty simple query. The command $inc(Q)$ is called a *query increment* and $dec()$ is called a *query decrement.*

Given a sequence of IQ commands $C = \langle C_1, \ldots, C_n \rangle$, *the IQ sequence defined by* C is $I = \langle I_0 = \langle\rangle, I_1, \ldots, I_n \rangle$ when the following properties hold:
For $i \in [1..n]$, let $I_{i-1} = \langle Q_1; \ldots; Q_k \rangle$:

- if $c_i = inc(Q)$ then $I_i = \langle Q_1; \ldots; Q_k; Q \rangle$.
- if $c_i = dec()$ and $k > 1$ then $I_i = \langle Q_1; \ldots; Q_{k-1} \rangle$.
- if $c_i = dec()$ and $k \leq 1$ then $I_i = \langle\rangle$.
- if $c_i = next()$ then $I_i = I_{i-1}$.

Given a sequence of IQ commands $C = \langle C_1, \ldots, C_n \rangle$, let $I = \langle I_0, I_1, \ldots I_n \rangle$ be the IQ sequence defined by C, for each $j \in [1..n]$ *the position of the related increment query command for* I_j is the maximum $k \in [1..j]$ such that $C_k = inc(Q)$ and $I_k = I_j$. If $|I_j| = 0$ then the position of the related increment query command is undefined. *The position of the related increment query command for* C_j is the position of the related increment query command for I_j when $C_j \neq dec()$ otherwise it is the position of the related increment query command for I_{j-1}.

Given a sequence of IQ commands $C = \langle C_1, \ldots, C_n \rangle$, let $I = \langle I_0, I_1, \ldots I_n \rangle$ be the IQ sequence defined by C, for each $j \in [1..n]$ where $C_j = inc(Q)$, the *expiring position of* C_j is the least integer $m \in [j+1..n]$ such that $|I_m| < |I_j|$

Given a sequence of IQ commands $C = \langle C_1, \ldots, C_n \rangle$, let $I = \langle I_0, \ldots, I_n \rangle$ be the IQ sequence defined by C. For each $j \in [1..n]$ where $C_j = inc(Q)$, *the positions of* *solution requests with respect to* C_j is $L = \{k|k \in [j..h], |I_k| = |I_j|\}$ where h is defined as follows: If the expiring position of C_j exists and it is m then $h = m - 1$, otherwise $h = n$.

3.2.2 The Incremental Query Problem And Solution

Given a sequence of IQ commands $C = \langle C_1, \ldots, C_n \rangle$, the pair $\langle \Pi, C \rangle$ is called an *incremental query problem* (IQ problem).

Given an IQ problem $\langle \Pi, C \rangle$, let $C = \langle C_1, \ldots, C_n \rangle$ be the sequence of IQ commands, let $I = \langle I_0 = \langle \rangle, I_1, \ldots, I_n \rangle$ be the IQ sequence defined by C. A sequence $S = \langle S_1, \ldots, S_n \rangle$ is called an *IQ Solution to* $\langle \Pi, C \rangle$ when for each $i \in [1..n]$, S_i satisfies the following properties:

- when $C_i = inc(Q)$, if no solution exists for I_i, then $S_i = fail$, otherwise S_i is the first annotated solution for I_i.
- when $C_i = next()$, if $|I_i| = 0$ then $S_i = \Box$. If $|I_i| > 0$, then let h be the position of the related increment query command for C_i, let L be the positions of solution requests with respect to C_h, let $k = |\{p \in L | p \leq i\}|$. If I_i has at least k solutions then S_i is the k_{th} annotated solution for I_i, otherwise $S_i = fail$.
- when $C_i = dec()$ then $S_i = \Box$.

3.3 Records Of Computation

3.3.1 Computation Trees and Paths Of Computation

Given a CLP state S, let R be the set of rules from Π which resolve with S. A *Choice Frame* has the form $\langle S, cr, L \rangle$ such that cr is either $\emptyset$ or $\{r\}$ where $r \in R$ and $L \subset R \setminus cr$. $L = \emptyset$ when $cr = \emptyset$. L is called the *unused resolution rules* of the choice frame and r is called the *chosen rule* of the choice frame.

Given a SLD-derivation tree $T = \langle\langle V, E\rangle, L_V, L_E\rangle$ for a CLP state $\langle Q|C\rangle$, the labeled rooted tree $T' = \langle\langle V' = V \cup \Delta_V, E' = E \cup \Delta_E\rangle, L'_V, L'_E\rangle$, where L'_V is a function from V' to CLP states or *success* and L'_E is a function from E' to choice frames, is called the *computation tree defined by* T when the following properties are satisfied:

1. $|\Delta_V| = |\Delta_E|$ is the number of successful paths of derivation in T.
2. The root node of T is the root node of T'.
3. For every $b \in V$ if $L_V(b)$ is a success CLP state, then there exists a node $c \in \Delta_V$ and edge $\{b, c\} \in \Delta_E$.
4. for all $b \in V'$, $L'_V(b) = L_V(b)$ when $b \in V$ otherwise $L'_V(b) = \mathit{success}$
5. for all $e = \{b, c\} \in E'$ if $c \in \Delta_V$ then $L'_E(e) = \langle L_V(b), \emptyset, \emptyset\rangle$
6. for all $e = \{b, c\} \in E'$ such that b is the parent of c and $c \notin \Delta_V$, then $L'_E(e) = \langle L_V(b), L_E(e), L_c\rangle$ where $L_c = \{L_E(\{b, d\})|d$ is a child of b in T, and $L_E(e) < L_E(\{b, d\})\}$

Intuitively, if T' is the computation tree defined by an SLD-derivation tree T, then every edge $\{b, c\}$ in T is labeled by a choice frame $\langle S, cr, L\rangle$ in T' where S is the CLP state labeling b, cr is the rule in Π labeling $\{b, c\}$ in T, and L contains all the rules labeling edges in T to the children of b which occur after the node c.

Given an SLD-derivation tree T for a CLP state $\langle Q|C\rangle$, the computation tree T' defined by T is called a *computation tree for* $\langle Q|C\rangle$.

Given a computation tree $T = \langle\langle V, E\rangle, L_V, L_E\rangle$ for a CLP state $\langle Q|C\rangle$, for every node b in T, let $\langle v_0, e_1, v_1, \ldots, e_n, v_n\rangle$ be the path in T from the root node v_0 to $b = v_n$, the sequence of choice frames $P = \langle L_E(e_1), \ldots, L_E(e_n)\rangle$ is called the *path of computation to* b in T. Let S be the CLP state of $L_E(e_n)$. The path of computation

$\langle L_E(e_1), \ldots, L_E(e_{n-1})\rangle$ is called a *path of computation to* S in T. If S is a success CLP state then P is called a *successful path of computation for* $\langle Q|C\rangle$.

Given the computation tree T' defined by an SLD-derivation tree T for a CLP state $\langle Q|C\rangle$, let $P = \langle F_1, \ldots, F_n\rangle$ be the path of computation to a node b in T'. If b is in T then $rules(P)$ denotes the path of derivation to b in T. If b is not in T then let c be the parent node of b in T', c is in T and $rules(P)$ denotes the path of derivation to c in T. Note that the sequence of chosen rules in the choice frames of P correspond to the sequence of rules in $rules(P)$.

Given a computation tree T for some CLP state $\langle Q|C\rangle$, let P be a successful path of computation in T. The *annotated solution derived from* P is $\langle K, rules(P)\rangle$ where K is the constraint store of the last choice frame on P.

Given a computation tree T for some CLP state $\langle Q|C\rangle$ and a simple query Q', let P be a path of computation in T. The *expansion of* P *by* Q' is the sequence of choice frames $\langle F'_1, \ldots, F'_n\rangle$ where for all $i \in [1..n]$, $F_i = \langle\langle Q_i|C_i\rangle, cr_i, L_i\rangle$ and $F'_i = \langle\langle Q_i \frown Q|C_i\rangle, cr_i, L_i\rangle$. $exp(P, Q')$ denotes the expansion of P by Q'.

Note that if P is a path of computation in a computation tree for some CLP state $\langle Q|C\rangle$ then

- $exp(P, Q')$ is a path of computation in a computation tree for CLP state $\langle Q \frown Q'|C\rangle$.
- $exp(exp(P, Q_1), Q_2) = ext(P, Q_1 \frown Q_2)$.
- $rules(exp(P, Q)) = rules(P)$.

Given a simple query Q and paths of computation P, P', and P'' such that $P = exp(P', Q) \frown P''$, P is called an *extension of* P' *with respect to* Q.

Given an incremental query $I = \langle Q_1; \ldots ; Q_n \rangle$ and a successful path of computation P for $flatten(I)$, let $I' = \langle Q_1; \ldots ; Q_m \rangle$ be a sub-incremental query to I. A successful path of computation P' for $flatten(I')$ is called the *precedent path of* P *for* I' when P is an extension of P' with respect to $Q_{m+1} \frown \ldots \frown Q_n$. $precedent(P, I')$ denotes the precedent path of P for I'.

Note that $rules(precedent(P, I'))$ is a prefix of $rules(P)$.

Note that $precedent(precedent(P, I'), I'') = precedent(P, I'')$ when there exists an incremental query I such that I' is a sub-incremental query of I, I'' is a sub-incremental query of both I and I', and P is a path of computation for $flatten(I)$.

Given a computation tree T' defined by an SLD-derivation tree T for a CLP state $\langle Q|C \rangle$, let $P_1, \ldots , P_n$ be the paths of computation to all leaf nodes in T'. The sequence $\langle P_1, \ldots , P_n \rangle$ is a *representation of* T' when for all $i, j \in [1..n]$ such that $i < j$, $rules(P_i) <_\omega rules(P_j)$.

From this point forward we represent computation trees as sequences of paths of computation.

Given a computation tree T for $\langle Q|C \rangle$, the maximum subsequence of successful paths of computation in T is called a *complete success tree for* $\langle Q|C \rangle$.

Given a complete success tree T for $\langle Q|C \rangle$, any prefix of T is called a *partial success tree for* $\langle Q|C \rangle$. The empty sequence $\langle \rangle$ is a trivial partial success tree for $\langle Q|C \rangle$.

3.3.2 Record Of Computation

Given an incremental query $I = \langle Q_1; \ldots ; Q_n \rangle$, for $i \in [1..n]$, let $I_i = \langle Q_1; \ldots ; Q_i \rangle$ and let T_i be a partial success tree for $flatten(I_i)$. The sequence $R = \langle T_1, \ldots , T_n \rangle$ is called a *record of computation for* I when the following properties hold:

1. For $i \in [2..n]$, for every $P \in T_i$, $precedent(P, I_{i-1}) \in T_{i-1}$

2. Every path of computation in $T_1, \ldots, T_n$ is marked as *closed* or *open* under the following restrictions:

   - all paths in T_n are marked as *open*.
   - for $i \in [1..n-1]$, for all paths $P_i \in T_i$, if P_i is marked as closed then T_{i+1} contains all the successful paths of computation for $flatten(I_{i+1})$ which are extensions of P_i.

3.4 IQ Transition Diagram

We now describe a state transition diagram of sound paths of computation which a solver may take for computing IQ solutions to arbitrary IQ problems. We call the transition diagram the *Incremental Query Transition Diagram for program* Π, denoted as $IQTD(\Pi)$.

The state description of the solver includes the structure of the current incremental query, a record of computation containing annotated solutions to the incremental query and its sub-incremental queries, the current CLP state and path of exploration, and what mode the search procedure is in.

3.4.1 IQ State Of Computation

An *IQ state* is of the form:

$$\langle\langle I_A, I_D\rangle, R, N, P, \langle Q|C\rangle, M\rangle$$

where:

I_A and I_D are sequences of simple queries.

$I_A \frown I_D = \langle I_1, \ldots, I_n\rangle$ is called the *incremental query of the IQ state*, I_A is called the *active sub-incremental query*, and I_D is called the *dormant queries* of $I_A \frown I_D$.

$R = \langle T_1, \ldots, T_n \rangle$ is a record of computation for $I_A \frown I_D$.

$N = \langle N_1, \ldots, N_n \rangle$ is a sequence of integers with the following properties:

- $1 \leq N_i \leq |T_i|$ for $i \in [1..n-1]$
- $1 \leq N_n \leq |T_n| + 1$

A record of computation may contain more solutions than what have been requested through IQ commands. N keeps track of how many solutions have been requested by IQ commands for each sub-incremental query of the current query.

$\langle Q|C \rangle$ is either a CLP state or $\langle \square | \square \rangle$.

P is either a path of computation or $\square$.

M is either *proceed*, *backtrack*, or *halt*.

- when $M = halt$ then $P = \square$, $\langle Q|C \rangle = \langle \square | \square \rangle$.
- when $M = backtrack$ then $P \neq \square$, $\langle Q|C \rangle = \langle \square | \square \rangle$.
- when $M = proceed$ then $P \neq \square$, $\langle Q|C \rangle \neq \langle \square | \square \rangle$.

When $\langle I_A, I_D \rangle = \langle \langle Q_1; \ldots; Q_k \rangle, \langle Q_{k+1}; \ldots; Q_n \rangle \rangle$
Let C_{k-1} be the solution of the first open path in T_{k-1}.

- when M is *proceed* or *backtrack* and $|P| > 0$ then the CLP state of the first choice frame in P is $\langle Q_k | C_{k-1} \rangle$
- when M is *proceed* and $P = \langle \rangle$ then $\langle Q|C \rangle = \langle Q_k | C_{k-1} \rangle$
- when M is *proceed* and $|P| > 0$, let $\langle \langle Q_P | C_P \rangle, \{r\}, L \rangle$ be the last choice frame of P, then $\langle Q|C \rangle$ the result of $con^k(res(\langle Q_P | C_P \rangle, r))$ where con^k is 0 or more applications of constraint resolution.

The *initial IQ state* is of the form:

$$\langle\langle I_A = \langle\rangle, I_D = \langle\rangle\rangle, N = \langle\rangle, R = \langle\rangle, P = \Box, \langle\Box|\Box\rangle, halt\rangle$$

A *success IQ state* is of the form:

$$\langle\langle I_A, I_D = \langle\rangle\rangle, R = \langle T_1, \dots, T_n\rangle, N = \langle N_1, \dots, N_n\rangle, \Box, \langle\Box|\Box\rangle, halt\rangle$$

where $|T_n| = N_n$

A *fail IQ state* is of the form:

$$\langle\langle I_A, I_D = \langle\rangle\rangle, R = \langle T_1, \dots, T_n\rangle, N = \langle N_1, \dots, N_n\rangle, \Box, \langle\Box|\Box\rangle, halt\rangle$$

where $|T_n| < N_n$

The initial, success and fail IQ states are called *halted states* and are the only IQ states where $M = halt$.

3.4.2 IQ State Transitions

The state transitions are organized around what mode the search procedure is in. From a halted state, the solver can receive IQ commands which modify the incremental query and record of computation. Searching for the next solution to an incremental query $\langle Q_1; \dots; Q_n\rangle$ is done in the context of saved solutions to the immediate sub-incremental query $\langle Q_1; \dots; Q_{n-1}\rangle$. If all saved solutions have been exhausted, it becomes necessary to search for new solutions to the sub-incremental query. State transitions model the book keeping about which sub-incremental query is being answered and which solutions have been used in the record of computation. Backtracking must be explicitly modeled to account for re-use of the information saved in the record of computation.

An *IQ state transition* has the form

$$\langle S, e, S' \rangle$$

where

- S and S' are IQ states
- $e \in \{inc(Q), dec(), nexp(), c, r, save, backtrack, fail\}$ where Q is a simple query, c is a primitive constraint, and r is either $\Box$ or a rule from Π.

Transitions are divided into 3 categories:

- *command transitions* - process IQ commands
- *proceed transitions* - model "forward" computation searching an SLD-Derivation tree
- *backtrack transitions* - carry saved information "backwards" during backtrack operations.

3.4.2.1 Command Transitions

Increment Query Command Transition

Let $inc(Q)$ be an IQ command. $\langle S, inc(Q), S' \rangle$ is a *command transition* when:

$$S = \langle \langle I_A, I_D = \langle \rangle \rangle, R, N, \Box, \langle \Box | \Box \rangle, halt \rangle$$
$$S' = \langle \langle I'_A, I_D = \langle \rangle \rangle, R', N', P', \langle Q' | C' \rangle, M' \rangle$$

where

- $I_A = \langle Q_1; \ldots; Q_n \rangle$
- $I'_A = \langle Q_1; \ldots; Q_n; Q \rangle$
- $R = \langle T_1, \ldots, T_n \rangle$
- $R' = \langle T_1, \ldots, T_n, \langle\rangle \rangle$
- $N = \langle N_1, \ldots, N_n \rangle$
- $N' = \langle N_1, \ldots, N_n, 1 \rangle$
- If S is the initial IQ state then $P' = \langle\rangle$, $Q' = Q$, $C' = \emptyset$, and $M' = proceed$.
- If $|T_n| = 0$ then $P' = \Box$, $\langle Q'|C' \rangle = \langle \Box|\Box \rangle$ and $M' = halt$.
- Otherwise
  - Every path in T_n is marked as *open*.
  - P_n is the first path in T_n,
  - $P' = \langle\rangle$, $Q' = Q$, C' is the solution of P_n and $M' = proceed$.

Next Solution Command Transition

If S is the initial IQ state, then $\langle S, next(), S \rangle$ is a *command transition.*

$\langle S, next(), S' \rangle$ is a *command transition* when:

$$S = \langle \langle I_A, I_D = \langle\rangle \rangle, R, N, \Box, \langle \Box|\Box \rangle, halt \rangle$$
$$S' = \langle \langle I_A, I_D = \langle\rangle \rangle, R, N', P', \langle \Box|\Box \rangle, M' \rangle$$

where

- $I_A = \langle Q_1; \ldots; Q_n \rangle$

- $R = \langle T_1, \ldots, T_n \rangle$
- $N = \langle N_1, \ldots, N_n \rangle$
- $N' = \langle N_1, \ldots, N_{n-1}, N_n + 1 \rangle$
- If $|T_n| \neq N_n$ then $P' = \Box$ and $M' = halt$
- If $|T_n| = N_n$ then let P_n be the last path in T_n, P' is such that $P_n = exp(P'', Q_n) \frown P'$ for some $P'' \in T_{n-1}$ and $M' = backtrack$,.

Decrement Query Command Transition

If S is the initial IQ state, then $\langle S, dec(), S \rangle$ is a *command transition*.

$\langle S, dec(), S' \rangle$ is a *command transition* when:

$$S = \langle \langle I_A, I_D = \langle \rangle \rangle, R, N, \Box, \langle \Box | \Box \rangle, halt \rangle$$
$$S' = \langle \langle I'_A, I_D = \langle \rangle \rangle, R', N', \Box, \langle \Box | \Box \rangle, halt \rangle$$

where

- $I_A = \langle Q_1; \ldots; Q_n \rangle$ $(n > 0)$
- $I'_A = \langle Q_1; \ldots; Q_{n-1} \rangle$
- $R = \langle T_1, \ldots, T_n \rangle$
- $R' = \langle T_1, \ldots, T_{n-1} \rangle$
- $N = \langle N_1, \ldots, N_n \rangle$
- $N' = \langle N_1, \ldots, N_{n-1} \rangle$
- all paths in T_{n-1} are marked as *open*.

3.4.2.2 Proceed Transitions

Proceed transitions attempt to make progress towards finding the $N[n]_{th}$ solution of $I_A \frown I_D$. The *Constraint Transition* and *Resolution Transition* model exploring a particular path of derivation to a new solution for I_A. The *Intermediate Save Transition* records the discovery of new solutions to I_A, when $I_D \neq \langle\rangle$. The *Final Save Transition* halts searching and records the $N[n]_{th}$ solution to I_A when $I_D = \langle\rangle$. If a Constraint Transition or Resolution Transition cannot make forward progress, then they will set the necessary state to initiate backtracking within the current active sub-incremental query.

Constraint Transition

Let c be a primitive constraint. $\langle S, c, S'\rangle$ is a proceed transition when:

$$S = \langle\langle I_A, I_D\rangle, R, N, P, \langle Q|C\rangle, proceed\rangle$$
$$S' = \langle\langle I_A, I_D\rangle, R, N, P, \langle Q'|C'\rangle, M'\rangle$$

where

- Q has the form $\langle c, l_2, \ldots, l_m\rangle$
- Let $\langle Q_c|C_c\rangle = con(\langle Q|C\rangle)$
- if $C_c = fail$ then $\langle Q'|C'\rangle = \langle\Box|\Box\rangle$ and $M' = backtrack$
- Otherwise $\langle Q'|C'\rangle = \langle Q_c|C_c\rangle$ and $M = proceed.$

Resolution Transition

Let r be a rule from Π or $\Box$. $\langle S, r, S'\rangle$ is a proceed transition when:

$$S = \langle\langle I_A, I_D\rangle, R, N, P, \langle Q|C\rangle, proceed\rangle$$
$$S' = \langle\langle I_A, I_D\rangle, R, N, P', \langle Q'|C'\rangle, M'\rangle$$

where

- The first literal in Q is a user defined constraint.
- L is the set of all rules from Π that resolves with $\langle Q|C\rangle$.
- If $L = \emptyset$ then $r = \Box$, $P' = P$, $\langle Q'|C'\rangle = \langle\Box|\Box\rangle$, and $M' = backtrack$.
- Otherwise
  - Let r be the first rule in L.
  - $P' = P \frown \langle\langle\langle Q|C\rangle, \{r\}, L \setminus \{r\}\rangle\rangle$
  - Let $\langle Q_r|C_r\rangle = res(\langle Q|C\rangle, r)$
  - if $C_r = fail$ then $\langle Q'|C'\rangle = \langle\Box|\Box\rangle$ and $M' = backtrack$
  - otherwise $\langle Q'|C'\rangle = \langle Q_r|C_r\rangle$ and $M = proceed$

Intermediate Save Transition

$\langle S, save, S'\rangle$ is a proceed transition when:

$$S = \langle\langle I_A, I_D\rangle, R, N, P, \langle\langle\rangle|C\rangle, proceed\rangle$$
$$S' = \langle\langle I'_A, I'_D\rangle, R', N, \langle\rangle, \langle Q'|C\rangle, proceed\rangle$$

where

- $I_A = \langle Q_1; \ldots; Q_k\rangle$ and $I_D = \langle Q_{k+1}; \ldots; Q_n\rangle$
- $I'_A = \langle Q_1; \ldots; Q_k; Q_{k+1}\rangle$ and $I'_D = \langle Q_{k+2}; \ldots; Q_n\rangle$
- $R = \langle T_1, \ldots, T_n\rangle$
- P_{k-1} is the first open path in T_{k-1}.
- $P_k = exp(P_{k-1}, Q_k) \frown P \frown \langle\langle\langle\emptyset|C\rangle, \emptyset, \emptyset\rangle\rangle$

- $R' = \langle T_1, \ldots, T_{k-1}, T_k \frown \langle P_k \rangle, T_{k+1}, \ldots, T_n \rangle$
- P_k is marked as *open* in R'.
- $Q' = Q_{k+1}$

Final Save Transition

$\langle S, save, S' \rangle$ is a proceed transition when:

$$S = \langle \langle I_A, I_D = \langle\rangle \rangle, R, N, P, \langle \langle\rangle | C \rangle, proceed \rangle$$
$$S' = \langle \langle I_A, I_D = \langle\rangle \rangle, R', N, \Box, \langle \Box | \Box \rangle, halt \rangle$$

where

- $I_A = \langle Q_1; \ldots; Q_n \rangle$.
- $R = \langle T_1, \ldots, T_n \rangle$.
- $R' = \langle T_1, \ldots, T_{n-1}, T_n \frown \langle P_n \rangle \rangle$ where P_{n-1} is the first *open* path in T_{n-1} and $P_n = exp(P_{n-1}, Q_n) \frown P \frown \langle \langle \langle \emptyset | C \rangle, \emptyset, \emptyset \rangle \rangle$.
- P_n is marked as *open* in T_n.

3.4.2.3 Backtrack Transitions

The traditional search of an SLD-Derivation tree does not formally model backtracking. When a state is returned to through backtracking, it is assumed that the information generated by exploring the sub-tree is no longer relevant. In our case we are saving any new solutions to sub-incremental queries we've encountered in the exploration of a failed sub-tree and we must formally model carrying this information back up the paths of derivation. Since we are searching for solutions of incremental queries in the context of solutions to their sub-incremental queries, backtracking must behave differently in the following cases:

- backtracking within $\langle Q_m | C_{m-1} \rangle$ when $I_A = \langle Q_1; \ldots; Q_m \rangle$ and C_{m-1} is a solution for $\langle Q_1; \ldots; Q_{m-1} \rangle$.

- backtracking to the next saved solution C'_{m-1} for $\langle Q_1; \ldots; Q_{m-1} \rangle$ to continue searching for a new solution to I_A. (The subtree below $\langle Q_m | C_{m-1} \rangle$ has been exhausted and no new solution was found).

- backtracking to find a new solution to $\langle Q_1; \ldots; Q_{m-1} \rangle$ when there are no more saved solutions C'_{m-1} to search under.

- Entering a fail state when no new solution could be found to $I_A \frown I_D$.

Backtrack Within Current Query Transition

$\langle S, \mathit{backtrack}, S' \rangle$ is a backtrack transition when:

$$S = \langle \langle I_A, I_D \rangle, R, N, P \neq \langle \rangle, \langle \square | \square \rangle, \mathit{backtrack} \rangle$$
$$S' = \langle \langle I_A, I_D \rangle, R, N, P', \langle Q' | C' \rangle, M' \rangle$$

where

- $P = \langle F_1, \ldots, F_m \rangle$.

- $F_i = \langle \langle Q_{F_i} | C_{F_i} \rangle, r_{F_i}, L_{F_i} \rangle$ is the last choice frame on P such that $L_{F_i} \neq \emptyset$.

- Let r be the first rule in L_{F_i}.

- $P' = \langle F_1, \ldots, F_{i-1} \rangle \frown \langle \langle \langle Q_{F_i} | C_{F_i} \rangle, \{r\}, L_{F_i} \setminus \{r\} \rangle \rangle$.

- $\langle Q_r | C_r \rangle = res(\langle Q_{F_i} | C_{F_i} \rangle, r)$

- if $C_r = fail$ then $\langle Q' | C' \rangle = \langle \square | \square \rangle$ and $M' = \mathit{backtrack}$.

- otherwise $\langle Q' | C' \rangle = \langle Q_r | C_r \rangle$ and $M = \mathit{proceed}$.

Next Existing Solution To Previous Query Transition

$\langle S, backtrack, S' \rangle$ is a backtrack transition when:

$$S = \langle \langle I_A, I_D \rangle, R, N, P, \langle \Box | \Box \rangle, backtrack \rangle$$
$$S' = \langle \langle I_A, I_D \rangle, R', N, \langle \rangle, \langle Q' | C' \rangle, proceed \rangle$$

where

- P has no choice frame with unused resolution rules.
- $I_A = \langle Q_1; \ldots; Q_k \rangle$ and $I_D = \langle Q_{k+1}; \ldots; Q_n \rangle$ with $k \geq 1$
- $R = \langle T_1, \ldots, T_n \rangle$
- There are 2 or more *open* paths in T_{k-1}
- Let P_1 and P_2 be the first and second paths marked as *open* in T_{k-1}.
- R' is R with P_1 marked as *closed*.
- $Q' = Q_k$ and C' is the solution of P_2.

Find New Solution To Previous Query Transition

$\langle S, backtrack, S' \rangle$ is a backtrack transition when:

$$S = \langle \langle I_A, I_D \rangle, R, N, P, \langle \Box | \Box \rangle, backtrack \rangle$$
$$S' = \langle \langle I'_A, I'_D \rangle, R', N, P', \langle \Box | \Box \rangle, backtrack \rangle$$

where

- P has no choice frame with unused resolution rules.
- $I_A = \langle Q_1; \ldots; Q_k \rangle$ and $I_D = \langle Q_{k+1}; \ldots; Q_n \rangle$ with $k \geq 2$

- $I'_A = \langle Q_1; \ldots; Q_{k-1}\rangle$ and $I'_D = \langle Q_k; Q_{k+1}; \ldots; Q_n\rangle$
- $R = \langle T_1, \ldots, T_n\rangle$
- There is only one open path in T_{k-1}, and let P_{k-1} be that path.
- $P_{k-2} = precedent(P_{k-1}, \langle Q_1, \ldots, Q_{k-2}\rangle)$
- P' is such that $P_{k-1} = exp(P_{k-2}, Q_{k-1}) \frown P'$
- R' is R with P_{k-1} marked as *closed* .

Fail Transition

$\langle S, fail, S'\rangle$ is a backtrack transition when:

$$S = \langle\langle I_A, I_D\rangle, R, N, P, \langle\Box|\Box\rangle, backtrack\rangle$$
$$S' = \langle\langle I'_A, I'_D = \langle\rangle\rangle, R, N, \Box, \langle\Box|\Box\rangle, halt\rangle$$

where

- P has no choice frame with unused resolution rules.
- either $k = 1$, or when $k > 2$ there is only 1 open path in T_{k-1} and no open paths in T_{k-2}.
- $I'_A = I_A \frown I_D$.

3.4.3 IQTD Path

Given a CLP program Π, we use the term $IQTD(\Pi)$ to denote the state diagram defined by the IQ state transitions.

Given the state diagram $IQTD(\Pi)$, let $P = \langle s_0, e_1, s_1, \ldots e_n, s_n\rangle$ be a sequence of labels on states and edges in $IQTD(\Pi)$ such that for each $i \in [1..n]$, $\langle s_{i-1}, e_i, s_i\rangle$ is an

IQ state transition, P is called a *path in* $IQTD(\Pi)$. If s_0 is the initial IQ state, then P is called a *sound path*. If s_n is a relevant IQ state, and P is a sound path, then P is called a *halted sound path*.

Given a CLP program Π and a halted path $P = \langle s_0, e_1, s_1, \ldots e_n, s_n \rangle$ from $IQTD(\Pi)$, let $C = \langle C_1, \ldots, C_k \rangle$ be the maximal subsequence of IQ commands labeling edges along P, C is called *the sequence of IQ commands defined by* P.

Given a CLP program Π, let P be a halted sound path in $IQTD(\Pi)$, let $C = \langle C_1, \ldots, C_n \rangle$ be the sequence of IQ commands defined by P and let $L = \langle L_0, \ldots, L_n \rangle$ be the sequence of halted IQ states along P.
For $i \in [0..n]$:

- let $L_i = \langle \langle I_{A_i}, \langle \rangle \rangle, R_i, N_i, \square, \langle \square | \square \rangle, halt \rangle$.
- let $R_i = \langle T_{i,1}, \ldots, T_{i,k_i} \rangle$.
- let $N_i = \langle N_{i,1}, \ldots, N_{i,k_i} \rangle$.
- let $T_{i,k_i} = \langle P_{i,1}, \ldots, P_{i,m_i} \rangle$.

The *IQ solution defined by* P is the sequence $S = \langle S_1, \ldots, S_n \rangle$ such that for $j \in [1..n]$:

- If $C_j = dec()$, then $S_j = \square$
- If $C_j = next()$ and $I_{A_j} = \langle \rangle$, then $S_j = \square$
- If $C_j \neq dec()$, $I_{A_j} \neq \langle \rangle$ and $|T_{j,k_j}| < N_{j,k_j}$, then $S_j = fail$.
- Otherwise let $h = N_{j,k_j}$, S_j is the annotated solution derived from $P_{j,h}$.

Theorem Given an IQ problem $\langle \Pi, C = \langle C_1, \ldots, C_n \rangle \rangle$, $S = \langle S_1, \ldots, S_n \rangle$ is an IQ solution to $\langle \Pi, C \rangle$ if and only if there exists a halted sound path P in $IQTD(\Pi)$ such that C is the sequence of IQ commands defined by P and S is the IQ solution defined by P.

The proof is found in the appendix C.4.

CHAPTER 4
CONCLUSION AND FUTURE WORK

4.1 $\mathcal{CALM}$ Conclusion

$\mathcal{CALM}$ is the first robust compiler for $\mathcal{ALM}$ supporting the full syntax and able to handle malformed system descriptions through providing semantic errors. $\mathcal{CALM}$ has already been used in the context of *Processing Narratives by Means of Action Languages*[39]. Without a robust compiler it is rare for a new programming language to be investigated for its effectiveness as a modeling language. We hope the existence of $\mathcal{CALM}$ will allow enable the $\mathcal{ALM}$ approach to action languages and modeling dynamic domains.

In the original $\mathcal{ALM}$ paper[24] it was suggested that an instance definition D for a non-source sort S with sub-sorts could be implemented by adding a disjunctive clause which would investigate D as being is_a of each source sub-sort of S. Due to limitations with $\mathcal{SPARC}$'s implementation of sorts, this is not possible. $\mathcal{SPARC}$ requires that the sort declarations be total and closed. There is no facility for the hypothetical consideration of elements in sub-sorts in the hierarchy.

The original paper did not explain the concept of a parameterized constant as used in the monkey banana problem. Implementation of constant definitions in the structure proved complex for non-ground constants. Our implementation only supports the definition of ground constants. Parameterized constant declarations in the theory are supported. Instances are treated as belonging to an anonymous subsort of the declared sort for the constants.

4.2 $\mathcal{CALM}$ Future Work

There are many basic extensions to the $\mathcal{ALM}$ syntax that would expand utility and expressiveness of the language. The following extensions could be implemented with the $\mathcal{SPARC}$ solver:

- Adding support for aggregate literals[41, 8, 13] in the body of axioms.
- Adding support for discrete time parameters in fluent axioms such as dynamic causal laws. This would allow the specification of the effects of actions to occur later than the next time step.
- Replacing the implementation of parameterized constants with support for parameterized sort declarations whose instances are automatically instantiated through schema definitions.
- Adding support for additional constraints on generated plans in planning problems.
- Adding support for diagnosis and explanation problems[9].

In order to expand capability of $\mathcal{ALM}$ to continuous constraint domains, translation to a constraint enabled $\mathcal{ASP}$ solver is required. One such solver is $\mathcal{AC}(\mathcal{C})$.

4.3 $\mathcal{ICLP}$ Conclusion

Our work in the development of an Incremental Query Transition Diagram ($\mathcal{IQTD}$) extends and generalized the work of Peter Stuckey[31] and Pascal Van Hentenryck[43]. Our transition diagram models the state of an sld-resolution based solver that is capable of saving the solution and path of computation for each sub-incremental query in the query stack when they are discovered. This transition diagram will form the basis for modifying the $\mathcal{CLAM}(\mathcal{R})$ solver with new instructions for saving incremental solutions. An Incremental $\mathcal{CLAM}(\mathcal{R})$ solver will allow for an implementation of the $ACSolver$ which does not have to restart the $\mathcal{CLP}$ solver each time a query is withdrawn during backtracking on guesses.

4.4 $\mathcal{ICLP}$ Future Work

The next step towards the development of an $ACSolver$ which utilizes an incremental $\mathcal{CLAM}(\mathcal{R})$ solver is to optimize the $\mathcal{IQTD}$ states to eliminate redundant representatation of common paths to solutions between nested sld-derivation trees. The successful paths of computation saved at level n should reference and extend successful paths of computation saved for level $n-1$. After this optimization, the successful paths of computation reflect the paths of computation saved in the $\mathcal{WAM}$ and $\mathcal{CLAM}(\mathcal{R})$. The semantics of the existing instruction set can be modified and augmented to facilitate saving and restoring both paths of computation and solutions in the constraint store.

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APPENDICES

Appendix A
ALM Grammar in ANTLR4

```
/**
 * This Grammar Follows the BNF for ALM as described in
 * Appendix A of "Modular Action Language ALM"
 * by Daniela Inclezan and Michael Gelfond.
 *
 * This document was created by Edward Wertz
 * Date: 7/2/2015
 * Copyright: Texas Tech University
 */

grammar ALM;

/* ABOUT THE GRAMMAR RULES THAT FOLLOW:
 * 1) Lexer Rules and TOKEN names start with capitol letters.
 * 2) parser rules start with lowercase letters
 *
 * Parser rules are clustered based on their "rank", "rank" is
 * the maximum distance from lexer tokens in the BNF grammer
 * The top-level rule will be the last rule in the grammar.
 */

/*
 * LEXER RULES
```

```
 */

// ORDER OF RULES IN FILE (TOP TO BOTTOM) matters
// ORDER OF DEFINITIONS (LEFT TO RIGHT) within
// NON-TERMINAL and TOKENS matters

COMMENT:    '%' ~[\r\n]* ('\r'? '\n' | EOF)  -> skip;

WhiteSpace: (' '|'\t'|'\r'|'\n') -> skip; //SKIP WHITESPACE

//EAGERLY CREATE THESE SPECIFIC TOKENS
MOD: 'mod';
EQ: '=';// Describes <eq>
NEQ: '!='; //Describes <neq>
ARITH_OP: '+' | '-' | '*' | '/' | 'mod' | '^'; //<arithmetic_op>
COMP_REL: '>' | '<' | '<=' | '>='; // <comparison_rel>
RIGHT_ARROW: '->'; //Used in describing Function signatures.
OCCURS: 'occurs'; //key word
INSTANCE: 'instance'; //key word
IS_A: 'is_a'; //key word
HAS_CHILD: 'has_child'; //special function
HAS_PARENT: 'has_parent'; //special function
LINK: 'link'; //special function
SOURCE: 'source'; //special function
SINK: 'sink'; //special function
SUBSORT: 'subsort'; //special function
DOM: 'DOM';  //key word
```

```
SORT: 'sort';
STATE: 'state';
CONSTRAINTS: 'constraints';
FUNCTION: 'function';
DECLARATIONS: 'declarations';
DEFINITIONS: 'definitions';
SYSTEM: 'system';
DESCRIPTION: 'description';
THEORY: 'theory';
MODULE: 'module';
IMPORT: 'import';
FROM: 'from';
DEPENDS: 'depends';
ON: 'on';
ATTRIBUTES: 'attributes';
OBJECT: 'object';
CONSTANT: 'constant';
STATICS: 'statics';
FLUENTS: 'fluents';
BASIC: 'basic';
DEFINED: 'defined';
TOTAL: 'total';
AXIOMS: 'axioms';
DYNAMIC: 'dynamic';
CAUSAL: 'causal';
LAWS: 'laws';
EXECUTABILITY: 'executability';
```

```
CONDITIONS: 'conditions';
CAUSES: 'causes';
IMPOSSIBLE: 'impossible';
IF: 'if';
FALSE: 'false';
TRUE: 'true';
STRUCTURE: 'structure';
IN: 'in';
WHERE: 'where';
VALUE: 'value';
OF: 'of';
INSTANCES: 'instances';
TEMPORAL : 'temporal';
PROJECTION : 'projection';
MAX: 'max';
STEPS: 'steps';
HISTORY : 'history';
OBSERVED: 'observed';
HAPPENED: 'happened';
PLANNING: 'planning';
PROBLEM: 'problem';
DIAGNOSTIC: 'diagnostic';
GOAL : 'goal';
SITUATION : 'situation';
WHEN : 'when';
NORMAL: 'normal';
ACTION: 'action';
```

```
ADDITIONAL: 'additional';
RESTRICTIONS : 'restrictions';
PERMISSIONS : 'permissions';
POSSIBLE: 'possible';
AVOID: 'avoid';
BOOLEANS: 'booleans';
INTEGERS: 'integers';
UNIVERSE: 'universe';
ACTIONS: 'actions';
CURRENT: 'current';
TIME: 'time';

// THESE TOKENS ARE MORE GENERAL AND LESS EAGERLY DETERMINED
ID: [a-z]([a-zA-Z0-9_\-])*;  //<identifier>
VAR: [A-Z]([a-zA-Z0-9_\-])*; //<variable>
POSINT: [1-9][0-9]*; // <positive_integer>
NEGINT: '-'[1-9][0-9]*;//<negative_integer>
ZERO: [0]+; //ZERO

/*
 * ALM BNF RULES subsumed by LEXER rules
 * ----------------------------
 * <boolean> -> BOOL
 * <non_zero_digit> -> POSINT and NEGINT
 * <digit> -> POSINT and NEGINT and ID and VAR
 * <lowercase_letter> -> ID and VAR
 * <uppercase_letter> -> ID and VAR
```

```
 * <letter> -> ID and VAR
 * <identifier> -> ID
 * <variable> -> VAR
 * <positive_integer> -> POSINT
 * <integer> -> ZERO and POSINT and NEGINT
 * <arithmetic_op> -> ARITH_OP
 * <comparison_rel> -> COMP_REL
 */


/*
 * BASIC PARSER RULES BUILT OUT OF SPECIAL LEXER TOKENS
 */

bool : TRUE | FALSE;

nat_num : ZERO | POSINT; //<natural_number>
integer : ZERO | POSINT | NEGINT; //<integer>
relation: EQ | NEQ | COMP_REL; //<arithmetic_rel>


// RECOVER KEYWORDS  && INTEGERS  INTO IDENTIFIER

id : OCCURS | INSTANCE | IS_A | HAS_CHILD | HAS_PARENT | LINK
   | SOURCE | SINK |  SUBSORT  | DOM | ID | MOD | SORT | STATE
   | CONSTRAINTS | FUNCTION | DECLARATIONS | DEFINITIONS | SYSTEM
   | DESCRIPTION | THEORY | MODULE | IMPORT | FROM | DEPENDS
```

```
    | ON | ATTRIBUTES | OBJECT | CONSTANT | STATICS | FLUENTS
    | BASIC | DEFINED | TOTAL | AXIOMS | DYNAMIC | CAUSAL | LAWS
    | EXECUTABILITY | CONDITIONS | CAUSES | IMPOSSIBLE | IF | FALSE
    | TRUE | STRUCTURE | IN | WHERE | VALUE | OF | INSTANCES
    | TEMPORAL  | PROJECTION | MAX | STEPS | HISTORY | OBSERVED
    | HAPPENED | PLANNING | PROBLEM | DIAGNOSTIC | GOAL
    | SITUATION | WHEN | NORMAL | ACTION | ADDITIONAL | RESTRICTIONS
    | PERMISSIONS | POSSIBLE | AVOID | BOOLEANS | INTEGERS | UNIVERSE
    | ACTIONS | CURRENT | TIME | integer;




alm_name : id | VAR;

/*
 * TERMS denote objects which populate sorts.
 * integers are terms .
 * true and false are terms.
 * variables are terms.
 * f(t1, ..., tn)  is a term where all ti are terms
 * and f is an identifier.
 * */

//The pattern for instance and constants of user defined sorts.
object_constant: (id ( '(' term (',' term)*   ')')? | integer);
```

```
//function_terms and  object_constants have the same syntax.
function_term: object_constant;

//terms denote values of sorts.
term: bool | VAR | id | integer | function_term | expression;

var_or_obj: (object_constant | VAR);

/* EXPRESSIONS
 * An expression is a mathematical entity which denotes an integer.
 * addition/subtraction
 * multiplication/division/modulus/exponent
 * factors participate in the above operations.
 */
expression: expression '+' arithmetic_term
   | expression '-' arithmetic_term | arithmetic_term;
arithmetic_term: arithmetic_term '*' factor
   | arithmetic_term '/' factor | arithmetic_term MOD factor
   | factor '^' factor | factor;
factor: VAR | '-' VAR  | integer | function_term
   | '-' function_term | '(' expression ')'
   | '-' '(' expression ')';
// ('-' integer) is not a factor, integer includes NEGINT
```

```
/* LITERALS */

//These these literals are handled per occurrence.
occurs_atom: OCCURS '(' var_or_obj ')';
instance_atom: INSTANCE '(' var_or_obj ',' sort_name ')';
is_a_atom: IS_A '(' var_or_obj ',' sort_name ')';
link_atom: LINK '(' sort_name ',' sort_name ')';
subsort_atom: SUBSORT '(' sort_name ',' sort_name ')';
has_child_atom: HAS_CHILD '(' sort_name ')';
has_parent_atom: HAS_PARENT '(' sort_name ')';
sink_atom: SINK '(' sort_name ')';
source_atom: SOURCE '(' sort_name ')';

//Pattern for any atom (positive occurrence of a predicate),
// including special atoms.
atom:  instance_atom | is_a_atom | link_atom | subsort_atom
   | has_child_atom | has_parent_atom | sink_atom | source_atom
   | function_term ;


//<literal> not including special literals
literal: atom | '-' atom |  term relation term ;
occurs_literal:  occurs_atom | '-' occurs_atom;



/* ALM FILE */
```

```
alm_file : (system_description | theory | module) EOF;

/* ALM SYSTEM DESCRIPTION */

library_name: alm_name;
sys_desc_name: alm_name;
system_description  : SYSTEM DESCRIPTION sys_desc_name theory
   (structure solver_mode?)? EOF;    //<system_description>


/* ALM THEORY */

theory_name: alm_name;
theory: (THEORY theory_name sequence_of_modules)
   | (IMPORT theory_name FROM library_name);//<theory>


/* ALM MODULE */

module_name: alm_name;
//<set_of_modules><remainder_modules>
sequence_of_modules: (module)+;
//<module>
module: (MODULE module_name module_body)
   | (IMPORT theory_name ('.' module_name)? FROM library_name);
module_body: module_dependencies? sort_declarations?
   constant_declarations? function_declarations? axioms?;
```

```
/* ALM MODULE DEPENDENCIES */

module_dependencies: DEPENDS ON one_dependency
   (',' one_dependency)*;
one_dependency: (theory_name '.')? module_name;

/* ALM SORT DECLARATIONS */

integer_range: integer '..' integer ;
predefined_sorts: BOOLEANS | INTEGERS | integer_range;
sort_name: predefined_sorts | UNIVERSE | ACTIONS | id ;
new_sort_name : id | integer_range;

//<sort_declaration><remainder_sort_declaration>
sort_declarations: SORT DECLARATIONS  (one_sort_decl)+ ;
//<one_sort_decl>,<sort_name>,<remainder_sort_names>,
//<remainder_sorts>
one_sort_decl: new_sort_name (',' new_sort_name)* '::'
   sort_name (',' sort_name)* attributes?;
//<attributes><remainder_attribute_declarations>
attributes: ATTRIBUTES (one_attribute_decl)+;
//<one_attribue_decl>,<arguments>,<remainder_args>
one_attribute_decl: id ':' (sort_name (',' sort_name )*
   RIGHT_ARROW)? sort_name;
```

```
/* ALM CONSTANT DECLARATIONS */

//<constant_declaraions><remainder_constant_declarations>
constant_declarations: CONSTANT DECLARATIONS (one_constant_decl)+;
//<one_constant_decl>,<const_params>,<remainder_const_params>
one_constant_decl:   object_constant (',' object_constant)*  ':'
   sort_name (',' sort_name)* attribute_defs?;



/* ALM FUNCTION DECLARATIONS */

function_name:id;
function_declarations: FUNCTION DECLARATIONS static_declarations?
   fluent_declarations?;
static_declarations: STATICS basic_function_declarations?
   defined_function_declarations?;
fluent_declarations: FLUENTS basic_function_declarations?
   defined_function_declarations?;
basic_function_declarations: BASIC (one_function_decl)+;
defined_function_declarations: DEFINED (one_function_decl)+;
one_function_decl: (TOTAL)? function_name ':' sort_name
   (('*' sort_name )* RIGHT_ARROW sort_name)?;
```

```
pos_fun_def: function_term EQ term | function_term
   | '-' function_term;
neg_fun_def: function_term NEQ term;
fun_def : (pos_fun_def | neg_fun_def);


/* ALM AXIOMS */

//<axioms>,<remainder_axioms>
axioms: AXIOMS (dynamic_causal_laws | executability_conditions
   | state_constraints | function_definitions)+ ;
dynamic_causal_laws: DYNAMIC CAUSAL LAWS
   (one_dynamic_causal_law)*;
executability_conditions: EXECUTABILITY CONDITIONS
   (one_executability_condition)*;
state_constraints: STATE CONSTRAINTS (one_state_constraint)*;
function_definitions: FUNCTION DEFINITIONS (one_definition)*;


/* DYNAMIC CAUSAL LAW */

//<dynamic_causal_law><body>
one_dynamic_causal_law: occurs_atom CAUSES pos_fun_def IF
    instance_atom (',' literal)* '.';

/* EXECUTABILITY CONDITION */
```

```
//<executability_condition>, <extended body>
one_executability_condition: IMPOSSIBLE occurs_atom IF
   instance_atom ( ',' ( occurs_literal| literal))* '.';

/* STATE CONSTRAINT */

one_state_constraint: fun_def '.' | (FALSE | fun_def) IF
   literal (',' literal)* '.';

 /* DEFINITION */

one_definition: function_term '.' |  function_term IF
   literal (',' literal)* '.';

/* ALM STRUCTURE */

structure_name: alm_name;
structure: STRUCTURE structure_name (constant_defs
   | instance_defs | statics_defs)*;


/* CONSTANT DEFINITIONS */

//<constant_defs><remainder_constant_defs>
constant_defs: CONSTANT DEFINITIONS (one_constant_def)+;
one_constant_def: object_constant '=' term;
```

```
/* INSTANCE DEFINITIONS */

//<instance_defs><remainder_instance_defs>
instance_defs: INSTANCES (one_instance_def)+;
//<one_instance_def>
one_instance_def: var_or_obj (',' var_or_obj)* IN sort_name
   (',' sort_name)* (WHERE literal (',' literal)* )? attribute_defs;
attribute_defs: (one_attribute_def)*;
one_attribute_def: function_term EQ term;

/* STATICS DEFINITIONS */

statics_defs:  VALUE OF STATICS (one_static_def)+ ;
one_static_def: fun_def (IF literal (',' literal)*)? '.';
//<one_static_literal><body>

/* SOLVER MODES */

solver_mode : (temporal_projection  | planning_problem
   | diagnostic_problem) added_constraints? action_conditions?;

/* SOLVER MODE COMMON PARTS */

max_steps : MAX STEPS  POSINT;
current_time: CURRENT TIME nat_num;
history : HISTORY (observed | happened)+;
```

```
observed : OBSERVED '(' function_term ',' term ','
   nat_num ')' '.' ;
happened : HAPPENED '(' object_constant ',' nat_num ')' '.';

/* SOLVER MODE ADDITIONAL CONSTRAINTS */

added_constraints: ADDITIONAL CONSTRAINTS (one_added_constraint)+;
one_added_constraint: (IMPOSSIBLE | AVOID) literal
   (',' literal)* '.';

action_conditions: ACTION (RESTRICTIONS | PERMISSIONS)
   (one_action_condition)+;
one_action_condition: (POSSIBLE | IMPOSSIBLE | AVOID)
   function_term  WHEN literal (',' literal)* '.';

/* TEMPORAL PROJECTION SPECIFIC */

temporal_projection : TEMPORAL PROJECTION max_steps history;

/* PLANNING PROBLEM SPECIFIC */

planning_problem : PLANNING PROBLEM max_steps current_time?
   history goal_state;
goal_state: GOAL EQ '{' literal (',' literal)* '}';

/* DIAGNOSTIC PROBLEM SPECIFIC */
```

```
diagnostic_problem : DIAGNOSTIC PROBLEM max_steps current_time?
   history normal_conditions? current_state;
normal_conditions: NORMAL CONDITIONS (one_normal_condition)+;
one_normal_condition : id ':'  literal ('when' literal
   (',' literal)*)?'.';
current_state: SITUATION EQ '{' literal (',' literal)* '}' '.';
```

Appendix B
CALM User and Developer Manual

B.1 Using CALM

B.1.1 Prerequisites

Required applications for executing CALM.

Java Java JDK version 8 (1.8) or higher.
https://www.oracle.com/technetwork/java/javase/downloads/index.html

- Download zip or executable.
- unzip binary into a directory.
- Set JAVA_HOME environment variable to the directory

B.1.2 CALM Distributable

Build the CALM Distributable from source by following the development instructions below or download the Windows distributable from the following url:
https://drive.google.com/open?id=1muof2vvDdJerEaaigVbOMQq6E_s8M7Ku

B.1.3 Commandline Usage

From the CALM\ directory, perform the following command:

```
java -jar calm.jar <path>\<file>
```

Where <path>\<file> is the path to a file containing an ALM System description and optional task. The output of the execution will be located in the directory CALM\output\<file>\

B.1.4 Example Applications

If the distributable was downloaded from the above url, an examples directory is included with sample System Descriptions to execute. If the distributable was manually assembled from the development instructions, examples can be copied from the following location:

`ALM-Compiler\src\test\resources\sysdesc\unittest\programs`

Execute the following from within the `CALM` directory:

`java -jar calm.jar ./examples/basicMotion.tp`

The output of execution will be located at the following directory:

`CALM\output\basicMotion.tp\`

B.2 CALM Development

B.2.1 Prerequisites

Required applications for compiling CALM.

Java Java JDK version 8 (1.8) or higher.

`https://www.oracle.com/technetwork/java/javase/downloads/index.html`

- Download zip or executable.
- If zip, unzip binary into a directory.
- Set JAVA_HOME environment variable to the directory

Maven Maven version 3.0 or higher.

`https://maven.apache.org/download.cgi`

- Download zip of binary.
- Unzip binary into a directory.

- Set MAVEN_HOME environment variable to the directory.
- Set M2_HOME environment variable to the directory.
- Add the bin sub-folder to the PATH environment variable.
- Windows:
  `https://howtodoinjava.com/maven/how-to-install-maven-on-windows/`
- Linux :
  `https://www.baeldung.com/install-maven-on-windows-linux-mac`

B.2.2 Compiling CALM

From a clean directory on your file system perform the following commands:

```
> git clone https://github.com/Topology/ALM-Compiler.git
> cd ./ALM-Compiler
> mvn clean build package
```

The compiled jar with all dependencies is in the *target* sub-directory.

B.2.3 Creating The CALM Distributable

After compiling the maven project (`mvn clean build package`) follow the following steps to create a distributable directory for the intended target operating system.

1. Create a new empty directory called `CALM\`.
2. Copy and rename the alm compiler jar containing its dependencies from
   `ALM-Compiler\target\alm-compiler-...-with-dependencies.jar`
   to the new directory: `CALM\calm.jar`
3. Create a subfolder `CALM\clingo\`.

4. Download and unzip the appropriate OS specific version of clingo from `https://sourceforge.net/projects/potassco/files/clingo/4.5.4/` into the `CALM\clingo\` directory.

5. Rename the clingo executable to `clingo` or `clingo.exe` as required by your operating system. The program `CALM\clingo\clingo` should be executable.

6. Create a new folder `CALM\sparc\`

7. Download the `sparc.jar` from `https://github.com/iensen/sparc/raw/master/sparc.jar` into the directory and verify that `java -jar CALM\sparc\sparc.jar` executes.

8. Create a subfolder `CALM\output\`

9. Create a subfolder `CALM\library\`

10. Add any libraries and theories under the following pattern: `CALM\library\<library_name>\<theory_name>.alm` where the file `<theory_name>.alm` contains an ALM Theory with the same name as the file name.

B.2.4 CALM Architecture

The organization of the source code is presented here through describing the contents of important directories and files.

ALM Grammar and Parser `ALM-Compiler\src\main\antlr4\ALM.g4`

- This file contains the ALM grammar encoded in ANTLR4's input language. After modifying the grammar, this file is given as input to the ANTLR4 tool to generate a new parser for the modified ALM grammar.

After modifying the ALM grammar, the parser is created in 3 steps:

1. Compile the project to generate the ANTLR4 parser from the grammar.
2. Copy the generated code (except ALMBaseListener.java) to the right location.
3. Update the previous ALMBaseListener.java implementation.

We recommend developing CALM using an IDE. Changes to the grammar will alter the generated parser in complex ways. Using an IDE will track changes between the new ALMListener.java interface and the old ALMBaseListener.java implementation of the interface.

`ALM-Compiler\target\generated-sources\antlr4\`

- Contains the generated source code for the ANTLR4 parser after the maven project has been compiled: `mvn clean build`.
  (Do not copy the ALMBaseListener.java.)

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\parser`

- Copy the generated ANTLR4 parser (except ALMBaseListener.java) to this location. Open the project in an IDE to edit the old copy of ALMBaseListener.java. The IDE will indicate the places where the old ALMBaseListener is incompatible with the new parser and ALMListener interface.

The Main Class and Compiler Configuration
`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\ALMCompiler.java`

- This file contains the main function which processes commandline arguments, configures the CALM settings, parses the input ALM System Description, and translates the system description to SPARC.

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\ALMCompilerSettings.java`

- This file contains the configuration of CALM and processing of commandline arguments.

Multi-Module BAT Hierarchy

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\ALMModuleManager.java`

- This file keeps track of the references to libraries and modules and resolves a module reference to the portion of the symbol table created by parsing the module definition. The `resolveModules()` function recursively imports modules from libraries on disk until all module references are resolved or there is a module resolution failure.

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\datastruct\sig\`

- This directory contains the SymbolTable.java class, which models a modular or hierarchical BAT signature. It also contains the class definitions needed to model elements within the BAT Signature.

Semantics and Error Checking

`ALM-...\src\main\java\edu\ttu\krlab\alm\parser\ALMBaseListener.java`

- This file contains the implementations of the `enter` and `exit` functions called before and after parsing each non-terminal in the ALM grammar. The `exit` function is provided the syntax tree produced by the ANTLR4 parser. Through implementing the non-terminal `exit` functions of key grammatical elements of ALM we perform error checking, construction of the BAT Symbol Table, and creation of $\mathcal{ASP}\{f\}$rules for user defined and auxiliary axioms.

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\datastruct\ALMTerm.java`

- This class assists with parsing and translating ANTLR4 syntax tree objects to terms in ALM. It also assists with type checking nested terms.

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\datastruct\err\`

- This directory contains the ErrorReport.java class for reporting errors during compilation and the ErrorMessageTable.java class which defines the messages to produce when reporting each error.

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\datastruct\type\`

- This directory contains the TypeChecker.java class which is used to track variable occurrences in $\mathcal{ASP}\{f\}$rules and check that a consistent type or sort can be inferred between all variable occurrences.

Translation to $\mathcal{SPARC}$

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\datastruct\sparc\`

- This directory contains SPARCProgram.java class and other elements needed to model the elements of a $\mathcal{SPARC}$Program.

`ALM-Compiler\src\main\java\edu\ttu\krlab\alm\ALMTranslator.java`

- This class contains the functions which translate the BAT Signature in the symbol table to the sorts section, translate the function signatures in the symbol table to the predicate section, and translate the $\mathcal{ASP}\{f\}$rules to $\mathcal{ASP}$rules in the rules section of the $\mathcal{SPARC}$program.

Interacting with the $\mathcal{SPARC}$Solver

`ALM-Compiler\src\main\java\edu\ttu\krlab\answerset\parser\`

- This directory contains classes and utilities needed to execute the SPARC solver and process the resulting answer sets.

Appendix C
IQTD Proofs

C.1 Extra Definitions

Given an IQ problem $\langle \Pi, C\rangle$, let $I = \langle I_0, I_1, \ldots, I_n\rangle$ be the IQ sequence defined by C. $\langle \Pi, C\rangle$ is called a *finite IQ problem* if for $i \in [1..n]$ the sld-derivation tree for $\langle flatten(I_i)|\emptyset\rangle$ is finite with respect to Π.

Given an IQ state $S = \langle\langle I_A, I_D\rangle, N, R, P, \langle Q|C\rangle, M\rangle$, let $n = |I_A \frown I_D|$, $m = |I_A|$, and $R = \langle T_1, \ldots, T_n\rangle$. The *rank of* S, denoted $rank(S)$, is defined as follows:

- If $M = halt$, let P' be the last path of computation in T_n. $rank(S) = rules(P')$
- If $M = proceed$, let P' be the first open path of computation in T_{m-1}. $rank(S) = rules(exp(P', I_A[|I_A|]) \frown P)$
- If $M = backtrack$, let P' be the first open path of computation in T_{m-1}. $rank(S)$ is the greatest path of derivation in the sld-derivation tree of $\langle flatten(I_A \frown I_D)|\emptyset\rangle$ which has $rules(exp(P', I_A[|I_A|]) \frown P)$ as a prefix.

Given a CLP Program Π, a path $P = \langle s_0, e_1, s_1, \ldots, e_z, s_z\rangle$ in $IQTD(\Pi)$ is called a *run* when s_0 and s_z are halted IQ states and for $i \in [1..z-1]$, s_i is not a halted IQ state.

Given a CLP Program Π and a run $P = \langle s_0, e_1, s_1, \ldots, e_z, s_z\rangle$ in $IQTD(\Pi)$, the *IQ Command of the run* is the IQ command labeling the first IQ State transition of P. Note that from the definition of the IQ State transitions, the first transition must be an IQ Command Transition and there can be no other IQ Command Transitions in the run.

Given a CLP Program Π and a halted sound path $P = \langle s_0, e_1, s_1, \ldots, e_z, s_z \rangle$ in $IQTD(\Pi)$, let $L = \langle L_0, \ldots, L_n \rangle$ be the halted IQ states along P where $s_0 = L_0$ and $S_z = L_N$, for $i \in [1..n]$ the i_{th} *run of* P is the path from L_{i-1} to L_i, the i_{th} *IQ Command of* P is the IQ Command C_i labeling the first IQ State Transition in the i_{th} run, and the sequence $\langle C_1, \ldots, C_n \rangle$ is called the Sequence of IQ Commands defined by P.

C.2 Deterministic Run Lemma

Given an IQ command C' and a halted sound path P in $IQTD(\Pi)$ where $C = \langle C_1, \ldots, C_n \rangle$ is the sequence of IQ commands defined by P and $L = \langle L_0, L_1, \ldots, L_n \rangle$ is the sequence of halted IQ states in P, if $\langle \Pi, C \frown \langle C' \rangle \rangle$ is a finite IQ problem then there exists a unique run $\langle s_0, e_1, s_1, \ldots, e_z, s_z \rangle$ in $IQTD(\Pi)$ such that $s_0 = L_n$ and $e_1 = C'$.

Proof

Assume C' is an IQ command and P is a halted sound path in $IQTD(\Pi)$. (3)

Let $C = \langle C_1, \ldots, C_n \rangle$ be the IQ commands along P, $L = \langle L_0, L_1, \ldots, L_n \rangle$ be the sequence of halted IQ states along P, and $I = \langle I_0, I_1, \ldots, I_n, I_{n+1} \rangle$ be the IQ sequence defined by $\langle C_1, \ldots, C_n, C' \rangle$.

Assume $\langle \Pi, C \frown \langle C' \rangle \rangle$ is a finite IQ problem. (4)

We construct a path P' in $IQTD(\Pi)$ as follows:

1. If $\langle L_n, C', s_1 \rangle$ is an IQ command transition in $IQTD(\Pi)$ then
   $P' := \langle L_n, C', s_1 \rangle$
2. Loop
   (a) P' is of the form $\langle L_n, C', s_1, \ldots, e_j, s_j \rangle$

(b) If there exists an IQ state transition $\langle s_j, e_{j+1}, s_{j+1}\rangle$ in $IQTD(\Pi)$ that is not an IQ command transition then
$P' := \langle L_n, C', s_1, \ldots, e_j, s_j, e_{j+1}, s_{j+1}\rangle$

(c) otherwise exit the loop

From the definition of all state transitions, each non-halted state s' along P' after L_n has a unique transition $\langle s', e, s''\rangle$.

Since the IQ problem is finite (4), there are a finite number of state transitions in P' and P' ends in a halted IQ state.

P' is the only run starting with the IQ command transition $\langle L_n, C', s_1\rangle$.

□

C.3 The Correspondence Lemma

The first element, $\langle I_A, I_D\rangle$, of a IQ state is called *the query* of the state.

Given a halted sound path $P = \langle s_0, e_1, \cdots, e_z, s_z\rangle$, for any IQ state s_i, we define $c(i)$ as the maximal number such that

- $c(i) \leqslant i$, and
- $C_{c(i)}$ is an IQ command.

We call $C_{c(i)}$ the *IQ command for* s_i.

For any state s_i and s_j $(j \geqslant i)$ of P, let $\langle I_A, I_D\rangle$ be the query of s_i. we say s_i is *useful* for s_j if for all $m \in [c(i), j]$, I_A is a prefix of I_m let I_m be the IQ sequence defined by $\langle C_1, \cdots, C_{c(m)}\rangle$. □

Given a halted sound path P in $IQTD(\Pi)$ let

- P be of the form $\langle s_0, e_1, s_i, \ldots, e_z, s_z\rangle$

- $C = \langle C_1, \ldots, C_n \rangle$ be the sequence of IQ commands defined by P.
- $I = \langle I_0, \ldots, I_n \rangle$ be the IQ Sequence defined by C,
- for $i \in [1..n]$ and $g \in [1..|I_i|]$, let $m_{i,g}$ be the number of solutions for incremental query $I_i[1..g]$
- $L = \langle L_0, \ldots, L_n \rangle$ be the sequence of halted states in P
- for $k \in [1..n]$, let $P_k = \langle s_{l_k} = L_{k-1}, e_{l_k+1} = C_k, s_{l_k+1}, \ldots, e_{l_k+i}, s_{l_k+i} = L_k \rangle$ be the k_{th} run of P
- for $i \in [0..z]$, let $\langle \langle I_{A_i}, I_{D_i} \rangle, R_i, N_i, P_i, \langle Q_i | C_i \rangle, M_i \rangle$ be the form of each IQ state s_i.

For $j \in [0..z]$, let k be the least integer such that run P_k contains s_j, then the following holds:

- If s_j is the first IQ state in P_k then $I_{A_j} \frown I_{D_j} = I_{k-1}$, otherwise $I_{A_j} \frown I_{D_j} = I_k$. **(CL.1)**
- For $i \in [1..|I_k|]$, for $g \in [1..|R_j[i]|]$, $R_j[i][g]$ is a successful path of computation for $\langle flatten(I_k[1..i]) | \emptyset \rangle$ and the annotated solution derived from $R_j[i][g]$ is the g_{th} annotated solution of incremental query $I_k[1..i]$ **(CL.2)**
- If $C_k = dec()$ then let $q = |I_k|$, otherwise let $q = |I_k - 1|$. For $g \in [1..q]$ if $N_j[g] < m_{k,g}$ then $N_j[g] \leq |R_j[g]| \leq m_{k,g}$ otherwise $|R_j[g]| = m_{k,g}$. **(CL.3)**
- If $C_k = inc(Q)$ and s_j is not the first IQ State of P_k, let $q = |I_k|$, then $N_j[q] = 1$. If s_j is the last IQ State in P_k and $m_{k,q} > 0$ then $|R_j[q]| = 1$ otherwise $|R_j[q]| = 0$. **(CL.4)**

- If $C_k = next()$, $I_k \neq I_0$, and s_j is not the first IQ State of P_k, let h be the position of the related increment query command for C_k, let r be the number of solution requests for I_k in $\langle C_h, \ldots, C_k \rangle$, let $q = |I_k|$, then $N_j[q] = r$. If $r > m_{k,q}$ then $|R_j[q]| = m_{k,q}$. If $r \leq m_{k,q}$ and s_j is the last IQ State of P_k then $r \leq |R_j[q]| \leq m_{k,q}$. If $r \leq m_{k,q}$ and s_j is not the last IQ State of P_k then $r - 1 \leq |R_j[q]| \leq m_{k,q}$. **(CL.5)**

- If $M_j \neq halt$, let $q = |I_{A_j}|$ and P be defined as follows: If $q = 1$ then $P = P_j$. If $q > 1$ then $P = exp(P_o, I_{A_j}[q]) \frown \langle P_j \rangle$ where P_o is the first open path of $R_j[q-1]$. Let $\langle F_1, \ldots, F_m \rangle$ be the sequence of choice frames along P. Let $T = \langle \langle V, E \rangle, L_V, L_E \rangle$ be the computation tree for $\langle flatten(I_{A_j}) | \emptyset \rangle$. For $a \in [1..m]$ there exists a node $b_a \in V$ such that $\langle F_1, \ldots, F_a \rangle$ is the path of computation to b_a in T and $L_V(b_a) = con^*(res(\langle Q_a | C_a \rangle, r_a))$ where $F_a = \langle \langle Q_a | C_a \rangle, \{r_a\}, L_a \rangle$. If $a = m$ and $M_j = proceed$ then $L_V(b_a) = con^*(\langle Q_j | C_j \rangle)$. **(CL.6)**

- If $j > 0$ and $\langle s_{j-1}, e_j, s_j \rangle$ is not an increment query or decrement query command transition, then $rank(s_j) \geq_\omega rank(s_{j-1})$. For $i \in [1..|I_k|]$ Let $T_{sld_i} = \langle \langle V_{sld_i}, E_{sld_i} \rangle, L_{V_{sld_i}}, L_{E_{sld_i}} \rangle$ be the sld-derivation tree for $\langle flatten(I_k[1..i]) | \emptyset \rangle$ and for all $b \in V_{sld_i}$ let D_b be the path of derivation to b in T_{sld_i}. If $D_b \leq_\omega rank(s_j)$ there exists s_x such that s_x is useful for s_j, $rank(s_x) = D_b$, and if D_b is a successful path of derivation then $\langle s_x, e, s_{x+1} \rangle$ is either a intermediate save transition or a final save transition. **(CL.7)**

- If $P_j \neq \square$ then for $i \in [1..|P_j|]$ let $\langle \langle Q_i | C_i \rangle, \{r_i\}, L_i \rangle = P_j[i]$ and let L_i^* be the set of rules from Π which resolve with $\langle Q_i | C_i \rangle$. The following properties hold: $r_i \in L_i^*$ and $\forall r^* \in L_i^*, r^* \in L_i$ **iff** $r^* >_\omega r_i$. **(CL.8)**

Proof

Let the premises of the correspondence lemma be given. We prove that all claims **(CL.1)** through **(CL.8)** hold through strong induction on the length of a prefix of $P = \langle s_0, e_1, \cdots, e_z, s_z \rangle$.

It is clear that all claims hold for s_0, the initial IQ halted state.

Assume for $n \in [1..z-1]$ that all claims hold for all states s_i such that $i \in [1..n]$, we show by cases that each claim holds for s_{n+1}. (5)

Proof of claim **(CL.1)**:

Suppose s_{n+1} is in the k_{th} run of P.

case 1: s_{n+1} is the first IQ state of P_k

s_{n+1} is the last IQ state of P_{k-1}

$I_{A_{n+1}} \frown I_{D_{n+1}} = I_{k-1}$ holds by (5) for claim **(CL.1)**

case 2: s_{n+1} is not the first IQ state of P_k

Let $\langle s_t, C_k, s_{t+1}$ be the first state transition of P_k.

by case 1 $I_{A_t} \frown I_{D_t} = I_{k-1}$

by the definition of all IQ Command Transitions, $I_{A_{t+1}} \frown I_{D_{t+1}} = I_k$

by the definition of all state transitions that are not IQ Command Transitions, for all states $s_i \in [s_{t+1}, \ldots, s_{n+1}]$ in P_k, $I_{A_i} \frown I_{D_i} = I_k$

therefore $I_{A_{n+1}} \frown I_{D_{n+1}} = I_k$

claim **(CL.1)** is proven.

Proof of claim **(CL.2)**:

case 1: $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is not a save or a command transition.

(CL.2) holds by strong inductive hypothesis for all states prior to s_{n+1}

for IQ state transitions that are not save or command transitions, the record of computation and the incremental queries of s_{n+1} and s_n are the same.

claim **(CL.2)** is satisfied in this case.

case 2: $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is an increment command transition

I_{k-1} is a proper sub-incremental query of I_k $|I_{k-1}| < |I_k|$

$R_{n+1} = R_n \frown \langle\rangle$

claim **(CL.2)** is satisfied in this case.

case 3: $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is a next command transition

The incremental query and record of computation remain unchanged between s_n and s_{n+1}

by the assumption of strong inductive hypothesis **(CL.2)** holds for s_n

claim **(CL.2)** for s_{n+1} is satisfied in this case.

case 4: $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is a decrement command transition

I_k is a proper sub-incremental query of I_{k-1}

R_{n+1} is a proper prefix of R_n

by the assumption of strong inductive hypothesis **(CL.2)** holds for s_n

claim **(CL.2)** holds for I_k and R_{n+1} in s_{n+1}

case 5: $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is a save transition

The incremental queries of s_n and s_{n+1} are the same.

R_{n+1} is the same as R_n except that $R_{n+1}[|I_{A_n}|] = R_n[|I_{A_n}|] \frown \langle\langle P'\rangle\rangle$ where P' is a successful path of computation to a solution of $flatten(I_{A_n})$.

Let g be such that $R_{n+1}[|I_{A_n}|][g] = P'$.

The rank or P' is greater than all other saved paths in R_{n+1} by the assumption of strong inductive hypothesis for claim **(CL.7)**

By the assumption of strong inductive hypothesis for claim **(CL.8)**, the rank of P' is less than all remaining unexplored paths.

By the definition of all state transitions, resolution always picks the least rule to resolve with the remaining query and no branch in the sld-derivation tree is skipped.

$R_{n+1}[|I_{A_n}|][g] = P'$ is the g_th annotated solution for $flatten(I_{A_n})$.

claim **(CL.2)** for s_{n+1} is satisfied in this case.

claim **(CL.2)** is proven.

Proof of claim **(CL.3)**:

by strong inductive hypothesis **(CL.3)** is satisfied for s_n.

by the definition of each state transition, **(CL.3)** is satisfied in each case.

claim **(CL.3)** is proven.

Proof of claim **(CL.4)**:

let P_k be the run containing the transition $\langle s_n, e_{n+1}, s_{n+1}\rangle$

By the deterministic run lemma and the fact that the IQ problem is finite, P_k is finite in length.

Assume C_k is an increment command.

For all states along P_k after the command transition $|R_{n+1}[|I_{A_{n+1}}|]| = 0$

P_k either ends in a final save transition and $|R_{n+1}[|I_{A_{n+1}}|]| = 1$ or ends in a fail transition and $|R_{n+1}[|I_{A_{n+1}}|]| = 0$

claim **(CL.4)** is proven.

Proof of claim **(CL.5)**:

let P_k be the run containing the transition $\langle s_n, e_{n+1}, s_{n+1}\rangle$

let $q = |I_k|$.

By the deterministic run lemma and the fact that the IQ problem is finite, P_k is finite in length.

Assume C_k is a next command and s_x is the first IQ state in P_k such that $x < n+1$.

For all states s_i along P_k after the command transition and before the final state $|R_i[q]| = |R_x[q]|$ and $N_i[q] = N_x[q] + 1$.

by the assumption of strong induction, claim **(CL.5)** holds for all states along P_k prior to s_{n+1}.

case 1: $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is not the final transition in P_k, claim **(CL.5)** holds.

case 2: $|R_n[q]| = m_{k,q}$ and $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is the final transition in P_k.

All solutions have been previously found for $flatten(I_k)$.

by the assumption of strong induction, all claims hold for all states prior to s_{n+1}.

from claim **(CL.7)** the paths of derivation in the sld-derivation tree are explored in a monotonically increasing manner.

since the last annotated solution has been previously recorded, all paths of derivation of higher rank do not lead to new solutions.

P_k ends in a fail transition.

$|R_{n+1}[q]| = m_{k,q}$ and $N_{n+1}[q] = N_n[q]$.

claim **(CL.5)** holds for s_{n+1}.

case 3: $|R_n[q]| < m_{k,q}$and $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is the final transition in P_k.

$\langle s_n, e_{n+1}, s_{n+1} \rangle$ is either a fail transition or a final save transition.

by the assumption of strong induction, all claims hold for all states prior to s_{n+1}.

from claim **(CL.7)** the paths of derivation in the sld-derivation tree are explored in a monotonically increasing manner.

from claim **(CL.8)** the paths of derivation are explored in order.

since not all solutions have been recorded, there are successful paths of derivation remaining to be explored.

$\langle s_n, e_{n+1}, s_{n+1} \rangle$ must be a final save transition.

From the definition of all transitions along P_k, $N_{n+1}[q] = N_n[q] = N_x[q] + 1$ and $|R_{n+1}[q]| = |R_x[q]| + 1$

since claim **(CL.5)** holds for s_x, claim **(CL.5)** holds for s_{n+1}.

claim **(CL.5)** is proven.

Proof of claim **(CL.6)**:

We prove claim **(CL.6)** by examining modes of the solver as it explores the sld-derivation tree and constructs paths of computation. This claim asserts that for every non-halted state of the solver, there is a well defined node in the computation tree for the active incremental query that the solver is visiting.

case 1: $M_{n+1} = \mathit{proceed}$

by assumption of strong inductive hypothesis, claim **(CL.6)** holds for s_n.

case 1.1: $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is a resolution transition.

Let b be the node in the computation tree of $\langle flatten(I_{A_n}), \emptyset \rangle$ for state s_n

Since $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is a resolution transition, there exists rules which resolve with the head of the active CLP query.

let r be the rule chosen for resolution by this transition.

let b_r be the node in the computation tree annotated by the choice frame reflecting resolution with r.

The existence of b_r satisfies claim **(CL.6)** in this case.

case 1.2: $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is a constraint transition.

Let b be the node in the computation tree of $\langle flatten(I_{A_n}), \emptyset \rangle$ for state s_n

Since $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is a constraint transition, b is the node in the computation tree satisfying claim **(CL.6)** in this case.

case 1.3: $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is an intermediate save transition.

Note that the active path of computation in s_{n+1} is an empty extension of the active path of computation in s_n.

Let b be the node in the computation tree of $\langle flatten(I_{A_n}), \emptyset \rangle$ for state s_n by strong inductive hypothesis.

b is the node in the computation tree satisfying claim **(CL.6)** in this case.

case 2: $M_{n+1} = backtrack$

Note that proceed transitions build the active path of computation one choice frame at a time.

The sequence of choice frames along active path in a state whose mode is backtrack has been previously active in a proceed transition. Let s_x be the proceed transition prior to s_{n+1} which had the same active path.

by assumption in strong induction, claim **(CL.6)** holds for s_x.

claim **(CL.6)** holds for s_{n+1}

claim **(CL.6)** is proven.

Proof of claim **(CL.7)**:

The essence of this claim is that the sld-derivation trees are searched and paths of computation are constructed in a monotonically increasing order with respect to rank and that all solutions belonging to paths of derivation of lower rank have been saved in the record of computation.

We prove claim **(CL.7)** in cases of the $\langle s_n, e_{n+1}, s_{n+1} \rangle$ transition.

by assumption of strong inductive hypothesis, claim **(CL.6)** holds for all states s_i where $i < n$.

case 1: $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is a next command transition, constraint transition or save transition.

By the definition of rank and the definition of the transition, $rank(s_{n+1}) =_\omega rank(s_n)$

Since s_{n+1} is of the same rank as s_n and claim **(CL.7)** holds for s_n, claim **(CL.7)** holds for s_{n+1}.

case 2: $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is a resolution transition.

Let P'_n be the active path of computation for state s_n

Let P'_{n+1} be the active path of computation for state s_{n+1}

By definition of a resolution transition P'_{n+1} extends P'_n and $rules(P'_n) <_\omega rules(P'_{n+1})$

Since P'_n is a prefix of P'_{n+1} and resolution selected the least rule to resolve with, there does not exist a path of derivation P_b in the sld-derivation tree such that $rules(P'_n) <_\omega P_b <_\omega rules(P'_{n+1})$.

Since claim **(CL.7)** holds for s_n by strong inductive hypothesis, **(CL.7)** holds for s_{n+1} in this case

case 3: $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is any backtrack transition except for the fail transition.

Let P' be the active path of computation for state s_{n+1}

Let $x < n+1$ be the largest integer such that $\langle s_{x-1}, e_x, s_x\rangle$ is a proceed transition and P' is the active path of computation for state s_x

By definition of rank, $rank(s_x) = rank(s_{n+1})$

by assumption of strong inductive hypothesis, claim **(CL.7)** holds for s_x

claim **(CL.7)** holds for s_{n+1} in this case.

case 4: $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is any backtrack transition except for the fail transition.

Let P' be the active path of computation for state s_{n+1}

Let $x < n+1$ be the largest integer such that $\langle s_{x-1}, e_x, s_x\rangle$ is a proceed transition and P' is the active path of computation for state s_x

By definition of rank, $rank(s_x) = rank(s_{n+1})$

by assumption of strong inductive hypothesis, claim **(CL.7)** holds for s_x

claim **(CL.7)** holds for s_{n+1} in this case.

case 5: $\langle s_n, e_{n+1}, s_{n+1}\rangle$ is a decrement command transition or fail transition.

Let P' be the last saved successful path of computation in $R_{n+1}[|I_{A_{n+1}}|]$.

Let $x < n+1$ be the largest integer such that $\langle s_{x-1}, e_x, s_x\rangle$ is the final save transition which add P' to R_{n+1}.

By definition of rank, $rank(s_x) = rank(s_{n+1})$

by assumption of strong inductive hypothesis, claim **(CL.7)** holds for s_x

claim **(CL.7)** holds for s_{n+1} in this case.

case 6: $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is an increment command transition.

Let P' be the first saved successful path of computation in $R_{n+1}[|I_{A_{n+1}}| - 1]$.

Let $x < n+1$ be the largest integer such that $\langle s_{x-1}, e_x, s_x \rangle$ is the save transition which add P' to R_x.

By definition of rank, $rank(s_x) = rank(s_{n+1})$

by assumption of strong inductive hypothesis, claim **(CL.7)** holds for s_x

claim **(CL.7)** holds for s_{n+1} in this case.

claim **(CL.7)** is proven.

Proof of claim **(CL.8)**:

The essence of claim **(CL.8)** is that in each application of the resolution transition and the backtrack within current query transitions, the construction of the choice frame is such that the least available rule is chosen for resolution. This ensures that the sld-derivation tree is explored in order with respect to the $<_\omega$ ordering on paths of derivation.

This property is trivially satisfied by strong inductive hypothesis for all transitions $\langle s_n, e_{n+1}, s_{n+1} \rangle$ other than the resolution transition and the backtrack within the current query transition.

When $\langle s_n, e_{n+1}, s_{n+1} \rangle$ is either the resolution transition or the backtrack within the current query transition, this claim is satisfied by construction of the active path of computation in the definition of the transitions.

claim **(CL.8)** is proven.

By strong induction, The Correspondence Lemma holds for all states along every sound path in the IQTD transition diagram.

$\square$

C.4 Proof Of Theorem

Theorem Given a finite IQ problem $\langle \Pi, C \rangle$, S is an IQ solution to $\langle \Pi, C \rangle$ if and only if there exists a halted sound path P in $IQTD(\Pi)$ such that C is the sequence of IQ commands defined by P and S is the IQ solution defined by P.

Proof $\Leftarrow$ (uses Correspondence Lemma)

Claim: Given an IQ problem $\langle \Pi, C = \langle C_1, \ldots, C_n \rangle \rangle$, S is an IQ solution to $\langle \Pi, C \rangle$ if there exists a halted sound path P in $IQTD(\Pi)$ such that C is the sequence of IQ commands defined by P and S is the IQ solution defined by P.

Proof:

Assume P is a halted sound in $IQTD(\Pi)$ where C is the sequence of IQ commands in P

Let S be the IQ solution defined by P

Let $L = \langle L_0, \ldots, L_n \rangle$ be the sequence of halted IQ states along P, C be $\langle C_1, \ldots, C_n \rangle$, S be $\langle S_1, \ldots, S_n \rangle$, $I = \langle I_0, \ldots, I_n \rangle$ be the IQ Sequence defined by C, and for $i \in [1..n]$, let L_i be $\langle \langle I_{A_i}, I_{D_i} = \langle \rangle \rangle, R_i, N_i, \square, \langle \square | \square \rangle, halt \rangle$.

To prove that S is the IQ Solution for the IQ Problem $\langle \Pi, C \rangle$ **??**, for $i \in [1..n]$, we consider three cases of the IQ Command C_i in the i_{th} run $\langle L_{i-1}, C_i, \ldots, L_i \rangle$ of P which defines S_i.

Case 1: $C_i = inc(Q)$. We consider two possibilities for I_i in our proof of 16

Case 1.1: No solution exists for I_i has

From $C_i = inc(Q)$ we have that $C_i \neq dec()$. (6)

By the definition of IQ Sequence defined by C and $C = inq(Q)$, I_i contains the query Q and $|I_i| > 0$.

By the correspondance lemma (CL.1), since L_i is in the i_{th} run of P and not the first halted state in the run, $I_i = I_{A_i} \frown I_{D_i}$.

Since $I_{D_i} = \langle\rangle$ we have that $I_i = I_{A_i}$ and $|I_{A_i}| > 0$. (7)

By the correspondance lemma (CL.4), since L_i is in the i_{th} run of P and is not the first halted state of the run and $C_i = inc(Q)$ we have $N_i[k] = 1$.

By the correspondance lemma (CL.4), since I_i is the incremental query of L_i and I_i has 0 solutions, and $0 < N_i[k] = 1$, we conclude that $|R_i[k]| = 0$.

Therefore $|R_i[k]| < N_i[k]$ (8)

From the definition of IQ Solution defined by P, if $C_i \neq dec()$, $|I_{A_i}| = k > 0$ and $|R_i[k]| < N_i[k]$, then $S_i = fail$.

Since 6, 7 and 8 hold, we conclude that:

$S_i = fail$ (9)

Case 1.2: I_i has At least 1 solution

By the definition of IQ Sequence defined by C and $C_i = inc(Q)$, I_i contains the query Q and $|I_i| > 0$.

Let $|I_i| = k$ for some $k > 0$.

By the correspondance lemma (CL.1), since L_i is in the i_{th} run of P and not the first halted state in the run, $I_i = I_{A_i} \frown I_{D_i}$.

Since $I_{D_i} = \langle\rangle$ we have that $I_i = I_{A_i}$ and $|I_{A_i}| = k > 0$. (10)

From $C_i = inc(Q)$ we have $C_i \neq dec$. (11)

By the correspondance lemma (CL.4), since L_i is in the i_{th} run of P and is not the first halted state of the run and $C_i = inc(Q)$ we have $N_i[k] = 1$.

By 10 I_i is the incremental query of L_i. (12)

I_i has at least 1 solution and L_i is a halted IQ state and $1 \geq N_i[k] = 1$. (13)

By 12, 13 and correspondance lemma (CL4) we have $|R_i[k]| = N_i[k]$ and then we have $|R_i[k]| \geq N_i[k]$. (14)

By 11, 14 and from the definition of IQ Solution defined by P, S_i is the annotated solution derived from $R_i[k][N_i[k]] = R_i[k][1]$.

By the correspondance (Cl.2), the annotated solution derived from $R_i[k][1]$ is the k_{th} annotated solution of incremental query $I_i[1..k]$.

Since $k = |I_i|$ therefore $I_i[1..k]$ is I_i.

S_i is the first annotated solution for I_i. (15)

Combining 9 and 15 we have that when $C_i = inc(Q)$, if no solution exists for I_i, then $S_i = fail$, otherwise S_i is the first annotated solution for I_i. (16)

Case 2: $C_i = next()$. We consider two possibilities for $|I_i|$ in our proof of 35.

Case 2.1: $|I_i| = 0$

By the correspondance lemma (CL.1), since L_i is in the i_{th} run of P and not the first halted state in the run, $I_i = I_{A_i} \frown I_{D_i}$.

Since $I_{D_i} = \langle\rangle$ we have that $I_i = I_{A_i}$. (17)

By 17 and $|I_i| = 0$ we have $|I_{A_i}| = 0$. (18)

By 18, $C_i = next()$ and from the definition of IQ Solution defined by P, if $C_i = next()$ and $|I_{A_i}| = 0$ then $S_i = \square$.

$S_i = \square$ (19)

Case 2.2: $|I_i| > 0$. We consider two possibilities for I_i in our proof of 34.

Before start to prove sub cases of case 2.2, we want to prove 22 and 23.

By the correspondance lemma (CL.1), since L_i is in the i_{th} run of P and not the first halted state in the run, $I_i = I_{A_i} \frown I_{D_i}$.

Since $I_{D_i} = \langle\rangle$ we have that $I_i = I_{A_i}$. (20)

By 20 and $|I_i| > 0$ we have $|I_{A_i}| > 0$. (21)

By 20 I_i is the incremental query of L_i. (22)

Let r be the number of solution requests for I_i in $\langle C_h, ..., C_i\rangle$ where h is the position of the related increment query command for C_i.

Let $k = |I_i|$ for some $k > 0$.

By the correspondance lemma (CL.5) and because L_i is not the first IQ state of i_{th} run we have $N_i[k] = r$. (23)

Let m be the number of solutions for incremental query I_i.

Case 2.2.1: I_i has at least r solutions.

I_i has at least r solutions means that $m \geq r$. (24)

L_i is the last IQ state of i_{th} run and because of 24 By the correspondance lemma (CL.5) we have $r \leq |R_i[k]| \leq m$. (25)

By 23 and 25 we have $|R_i[k]| \geq N_i[k]$. (26)

By 26, 20, $C_i = next()$ and from the definition of IQ Solution defined by P, S_i is the annotated solution derived from $R_i[k][N_i[k]]$. (27) .

By 27 and 23 we have S_i is the annotated solution derived from $R_i[k][r]$. (28)

By the correspondance (Cl.2), the annotated solution derived from $R_i[k][r]$ is the r_{th} annotated solution of incremental query $I_i[1..k]$.

Since $|I_i| = k$ therefore $I_i[1..k]$ is I_i.

S_i is the r_{th} annotated solution for I_i (29)

Case 2.2.2: I_i has less than r solutions

I_i has at most r solutions means that $m < r$. (30)

By the correspondance lemma (CL.5) and 30 we have $|R_i[k]| = m$. (31)

By 23, 31 and 30 we have $N_i[k] > |R_i[k]|$. (32)

by 32, $C_i \neq dec()$, 21 and From the definition of IQ Solution defined by P, if $C_i \neq dec()$, $|I_{A_i}| > 0$ and $|R_i[k]| < N_i[k]$ then $S_i = fail$.

$S_i = fail$. (33)

From 29 and 33 If $|I_i| > 0$, then let h be the position of the related increment query command for C_i, let k be the number of solution requests for I_i in $\langle C_h, \ldots, C_i \rangle$. If I_i has at least r solutions then S_i is the r_{th} annotated solution for I_i, otherwise $S_i = fail$. (34)

Combining 19 and 34 we have that when $C_i = next()$, if $|I_i| = 0$ then $S_i = \square$. If $|I_i| > 0$, then let h be the position of the related increment query command for C_i, let k be the number of solution requests for I_i in $\langle C_h, \ldots, C_i \rangle$. If I_i has at least r solutions then S_i is the r_{th} annotated solution for I_i, otherwise $S_i = fail$. (35) .

Case 3: $C_i = dec()$

From the definition of IQ Solution defined by P, If $C_i = dec()$, then $S_i = \square$.

when $C_i = dec()$ then $S_i = \Box$ (36) .

By the definition of an IQ Solution to an IQ Problem and 16, 35, 36 S is an IQ solution to $\langle \Pi, C \rangle$.

$\Box$

Proof $\Rightarrow$ (uses Deterministic Run Lemma)

We must show that if S is an IQ solution to the IQ problem $\langle \Pi, C \rangle$ then there exists a halted sound path P in $IQTD(\Pi)$ such that C is the sequence of IQ commands defined by P and S is the IQ solution defined by P. 48

Assume S is an IQ solution to an IQ problem $\langle \Pi, C \rangle$. (37)

By the definition of an IQ problem, C is a sequence of IQ commands of the form $\langle C_1, \ldots, C_n \rangle$

Let $I = \langle I_0, I_1, \ldots, I_n \rangle$ be the IQ sequence defined by C.

From the definition of S being an IQ solution to $\langle \Pi, C \rangle$, $S = \langle S_1, \ldots, S_n \rangle$ we have that for $i \in [1..n]$, S_i satisfies the following properties: (38)

- when $C_i = inc(Q)$, if no solution exists for I_i, then $S_i = fail$, otherwise S_i is the first annotated solution for I_i.
- when $C_i = next()$, if $|I_i| = 0$ then $S_i = \Box$. If $|I_i| > 0$, then let h be the position of the related increment query command for C_i, let k be the number of solution requests for I_i in $\langle C_h, \ldots, C_i \rangle$. If I_i has at least k solutions then S_i is the k_{th} annotated solution for I_i, otherwise $S_i = fail$.
- when $C_i = dec()$ then $S_i = \Box$.

We construct a path P in $IQTD(\Pi)$ as follows:

Let $P' = \langle L_0 \rangle$ such that L_0 is the initial halted IQ state.

Repeat for $i \in [1..n]$ in order

- Let L_{i-1} be the last state in P'.
- By construction L_{i-1} is a halted IQ state.
- Let $\langle L_{i-1}, C_i, s', \dots, L_i \rangle$ be the unique run as determined by the deterministic run lemma.
- $P' := P' \frown \langle L_{i-1}, C_i, s', \dots, L_i \rangle$

$P = P'$ at the end of the above construction.

P is a halted sound path in $IQTD(\Pi)$ (39)

Let C' be the sequence of IQ Commands defined by P.

By construction of P, $C = C'$

C is the sequence of IQ commands defined by P. (40)

Let $S' = \langle S'_1, \dots, S'_n \rangle$ be the IQ solution defined by P.

By the proof of the theorem in the reverse direction, S' is the IQ solution of the IQ problem $\langle \Pi, C \rangle$.

We show 47 holds by proving $S = S'$.

Let $L = \langle L_0, L_1, \dots, L_n \rangle$ be the halted IQ states along P where L_0 is the initial IQ state and for $i \in [0..n]$ L_i is of the form $\langle \langle I_{A_i}, \langle \rangle \rangle, R_i, N_i, \Box, \langle \Box | \Box \rangle, halt \rangle$.

For $i \in [1..n]$ we consider the definitions of S_i and S'_i and the i_{th} run of P, $P_i = \langle L_{i-1}, e_{i,1} = C_i, s_{i,1}, e_{i,2}, s_{i,2}, \dots, e_{i,k_i}, L_i \rangle$.

Case 1: $C_i = dec()$:

From the definition of S being the IQ solution to the IQ problem $\langle \Pi, C \rangle$, when $C_i = dec()$, $S_i = \square$.

From the definition of S' being the IQ solution defined by P, when $C_i = dec()$, when $C_i = dec()$, $S'_i = \square$.

$S_i = S'_i$. (41)

Let $q_i = |I_i|$ and m_i be the number of solutions to I_i.

By the correspondence lemma (CL.1) and L_i not being the first IQ state in P_i, $I_i = I_{A_i} \frown I_{D_i}$.

From $I_{D_i} = \langle\rangle$ we have that $I_i = I_{A_i}$ and $|I_{A_i}| = q_i$.

Case 2: $C_i = inc(Q)$:

From the definition of S being the IQ solution to the IQ problem $\langle \Pi, C \rangle$, when $C_i = inc(Q)$, if no solution exists for I_i, then $S_i = fail$, otherwise S_i is the first annotated solution for I_i.

We consider two cases for m_i.

Case 2.1: $m_i = 0$

$S_i = fail$

By the correspondence lemma (CL.4), since L_i is not the first IQ state of P_i and $m_i = 0$ then $N_i[q_i] = 1$ and $|R_i[q_i]| = 0$.

$|R_i[q_i]| < N_i[q_i]$

By the definition of S' being the IQ solution defined by P, If $C_i \neq dec()$, $|I_{A_i}| = q_i > 0$ and $|R_i[q_i]| < N_i[q_i]$, then $S'_i = fail$.

$S'_i = fail$.

$S_i = S'_i$. (42)

Case 2.2: $m_i > 0$

S_i is the first annotated solution to I_i.

By the correspondence lemma (CL.4), since L_i is not the first IQ state of P_i and $m_i = 0$ then $N_i[q_i] = 1$ and $|R_i[q_i]| = 1$.

$|R_i[q_i]| \geq N_i[q_i]$

By the definition of S' being the IQ solution defined by P, If $C_i \neq dec()$, $|I_{A_i}| = q_i > 0$ and $|R_i[q_i]| \geq N_i[q_i]$, then S'_i is the annotated solution derived from $R_i[q_i][N_i[q_i] = 1]$.

By the correspondence lemma (CL.2), the annotated solution derived from $R_i[q_i][1]$ is the first annotated solution of incremental query $I_i[1..q_i]$.

$I_i = I_i[1..q_i]$

S'_i is the first annotated solution of I_i.

$S_i = S'_i$. (43)

Case 3: $C_i = next()$

let h be the position of the related increment query command for C_i and let k be the number of solution requests or I_i in $\langle C_h, \ldots, C_i \rangle$.

From the definition of S being the IQ solution to the IQ problem $\langle \Pi, C \rangle$, when $C_i = next()$, if $|I_i| = 0$ then $S_i = \square$, otherwise If $|I_i| > 0$ and I_i has at least k solutions then S_i is the k_{th} annotated solution for I_i, otherwise $S_i = fail$.

We consider the values for q_i, m_i and k.

Case 3.1: $q_i = 0$

$S_i = \square$

$|I_{A_i}| = |I_i| = q_i = 0$

By the definition of S' being the IQ solution defined by P, if $C_i = next()$ and $|I_{A_i}| = 0$, then $S'_i = \square$.

$S'_i = \square$ cl.7

$S_i = S'_i \quad (44)$

Case 3.2: $q_i > 0$ and $m_i < k$

$S_i = fail$

By the correspondence lemma (CL.5), since L_i is the last IQ state of P_i and $m_i < k$, then $N_i[q_i] = k$ and $|R_i[q_i]| = m_i$.

$|R_i[q_i]| < N_i[q_i]$.

By the definition of S' being the IQ solution defined by P, If $C_i \neq dec()$, $|I_{A_i}| = q_i > 0$ and $|R_i[q_i]| < N_i[q_i]$, then $S'_i = fail$.

$S'_i = fail$.

$S_i = S'_i. \quad (45)$

Case 3.3: $q_i > 0$ and $m_i \geq k$

S_i is the k_{th} annotated solution for I_i.

By the correspondence lemma (CL.5), since L_i is the last IQ state of P_i and $m_i \geq k$, then $N_i[q_i] = k$ and $k \leq |R_i[q_i]| \leq m_i$.

$|R_i[q_i]| \geq N_i[q_i]$.

By the definition of S' being the IQ solution defined by P, If $C_i \neq dec()$, $|I_{A_i}| = q_i > 0$ and $|R_i[q_i]| \geq N_i[q_i]$, then S'_i is the annotated solution derived from $R_i[q_i][N_i[q_i] = k]$.

By the correspondence lemma (CL.2), the annotated solution derived from $R_i[q_i][k]$ is the k_{th} annotated solution of incremental query $I_i[1..q_i]$.

$I_i = I_i[1..q_i]$

S'_i is the k_{th} annotated solution of I_i.

$S_i = S'_i$. (46)

For $i \in [1..n]$, in all cases 41, 42, 43, 44, 45, and 46 $S_i = S'_i$.

$S = S'$.

From S' being the IQ solution defined by P and $S = S'$, S is the IQ solution defined by P. (47)

From showing 39, 40, and 47 under the assumption of 37 we conclude that:

If S is an IQ solution to the IQ problem $\langle \Pi, C \rangle$ then there exists a halted sound path P in $IQTD(\Pi)$ such that C is the sequence of IQ commands defined by P and S is the IQ solution defined by P. (48)

□