Spatial Analysis and Modeling (GIST 4302/5302)

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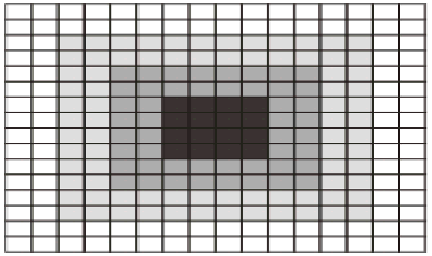
Outline of This Week

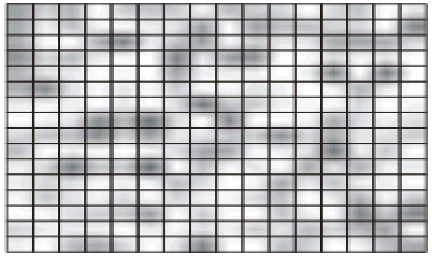
- Last week, we learned:
  - spatial point pattern analysis (PPA)
  - focus on location distribution of ‘events’
- This week, we will learn:
  - spatial autocorrelation
  - global measures of spatial autocorrelation
  - local measure of spatial autocorrelation

Spatial Autocorrelation

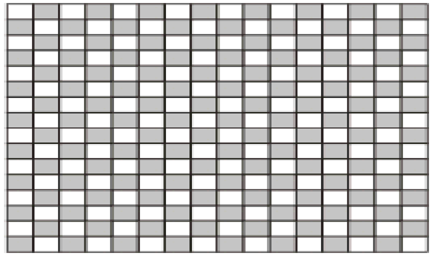
- Tobler's first law of geography
- Spatial auto/cross correlation



If like values tend to cluster together, then the field exhibits high **positive spatial autocorrelation**



If there is no apparent relationship between attribute value and location then there is **zero spatial autocorrelation**



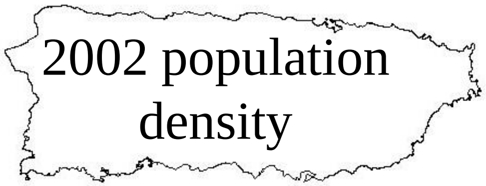
If like values tend to be located away from each other, then there is **negative spatial autocorrelation**

Spatial Autocorrelation

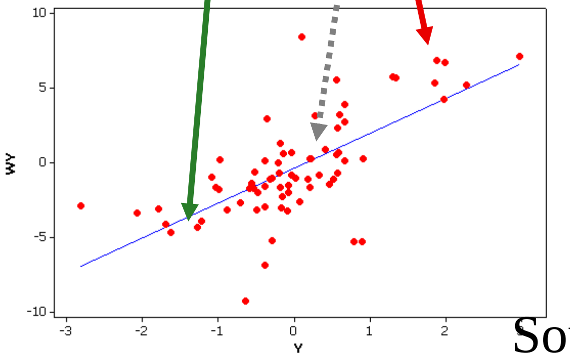
- Spatial autocorrelationship is everywhere
  - Spatial point pattern
    - K, F, G functions
    - Kernel functions
  - Areal/lattice (this topic)
  - Geostatistical data (next topic)

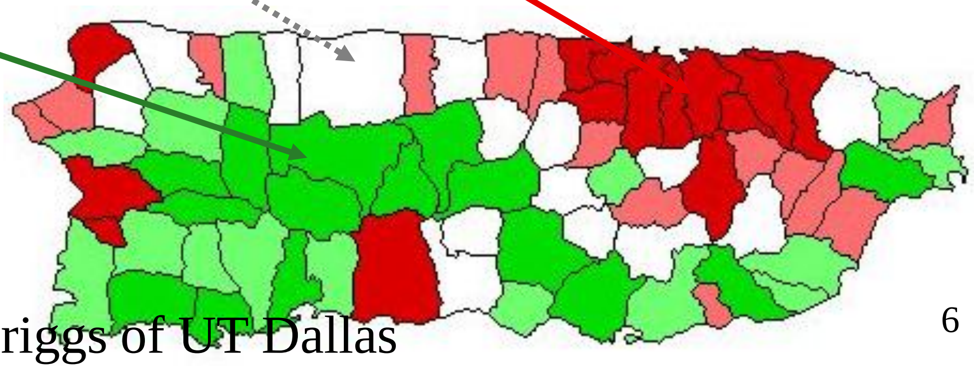
Spatial Autocorrelation of Areal Data

Positive spatial autocorrelation



- high values surrounded by nearby high values
- intermediate values surrounded by nearby intermediate values
- low values surrounded by nearby low values



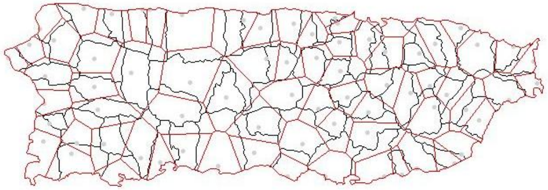


Source: Ron Briggs of UT Dallas

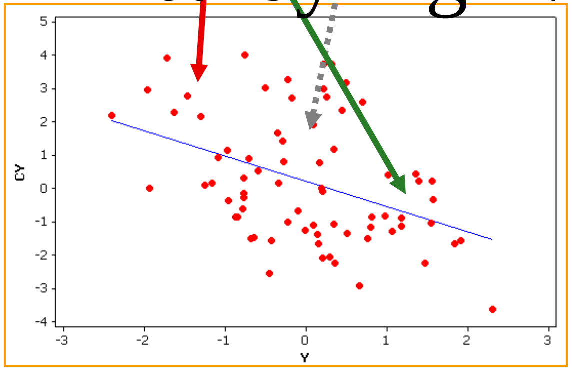
Negative spatial autocorrelation

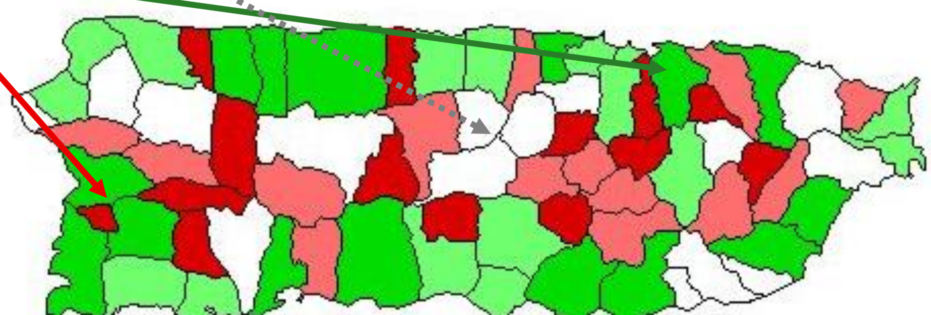
- high values surrounded by nearby low values
- intermediate values surrounded by nearby intermediate values
- low values surrounded by nearby high values

competition for space



Grocery store density





Source: Ron Briggs of UT Dallas

Spatial Weight Matrix

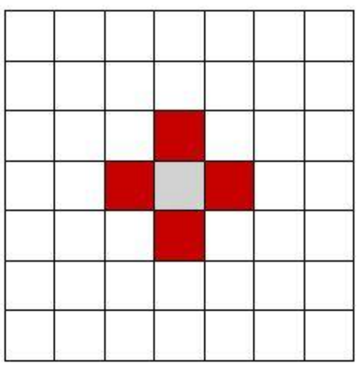
- **Core** concept in statistical analysis of areal data
- Two steps involved:
  - define which relationships between observations are to be given a nonzero weight, i.e., define spatial neighbors
  - assign weights to the neighbors
- Making the neighbors and weights is not easy as it seems to be
  - Which states are near Texas?

Spatial Neighbors

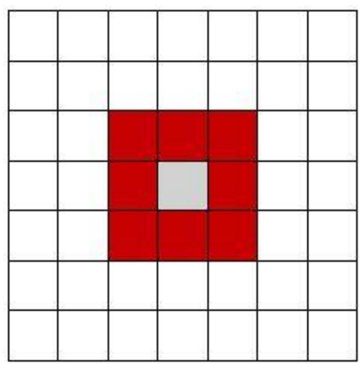
- **Contiguity-based neighbors**
  - Zone *i* and j are neighbors if zone i is contiguity or adjacent to zone *j*
  - But what constitutes contiguity?
- **Distance-based neighbors**
  - Zone i and j are neighbors if the distance between them are less than the threshold distance
  - But what distance do we use?

Contiguity-based Spatial Neighbors

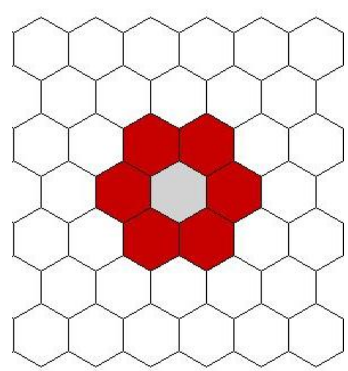
- Sharing a border or boundary
  - Rook: sharing a border
  - Queen: sharing a border or a point



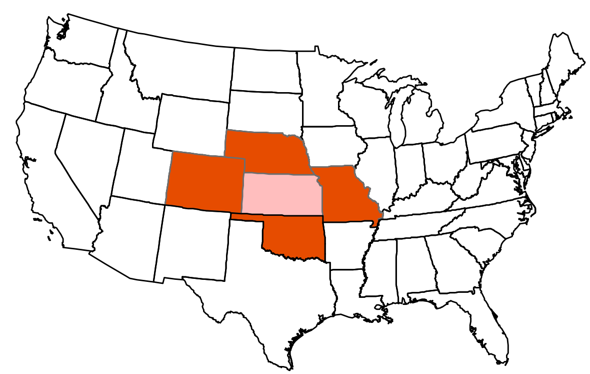
rook



queen



Hexagons



Irregular

Which use?

Example

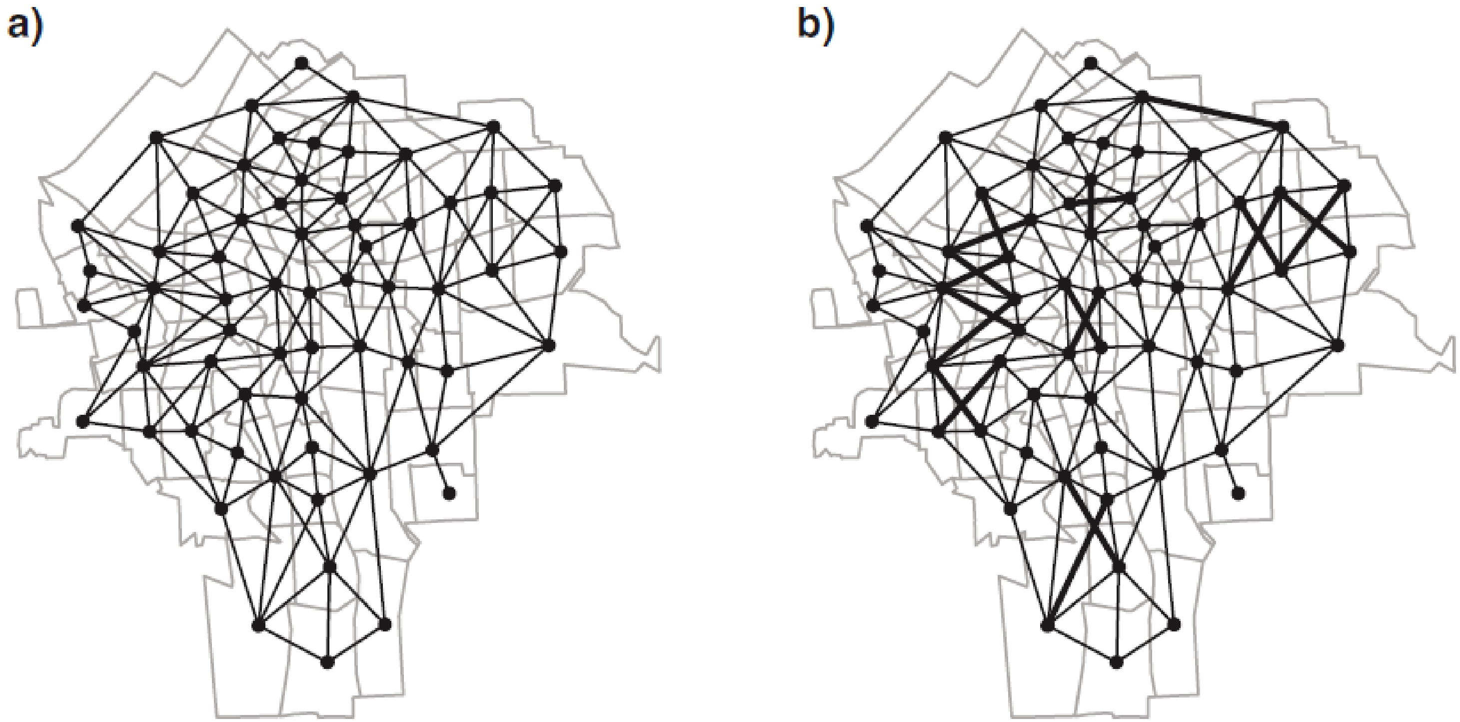
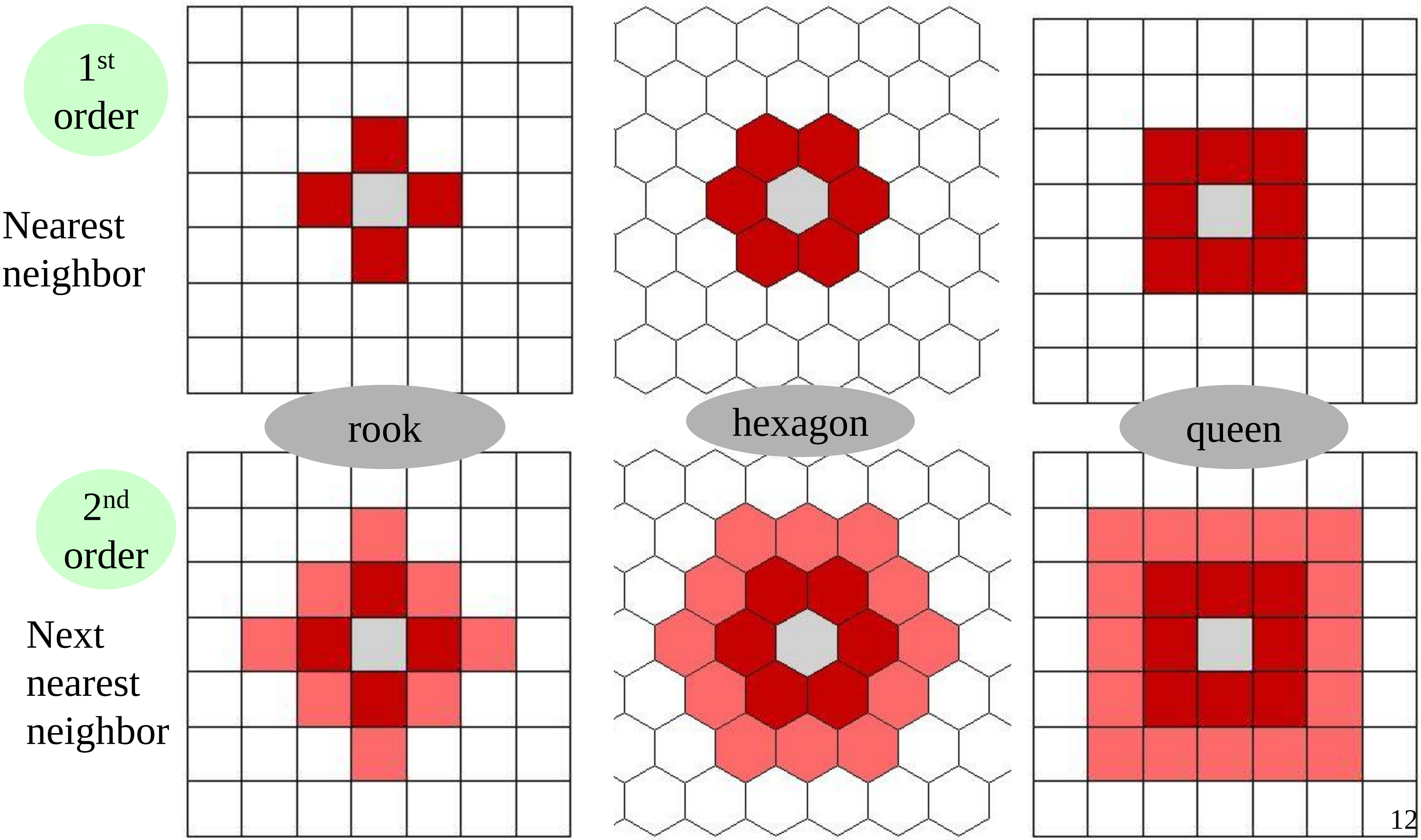


Fig. 9.3. (**a**) Queen-style census tract contiguities, Syracuse; (**b**) Rook-style contiguity differences shown as thicker lines

Source: Bivand and Pebesma and Gomez-Rubio

Higher-Order Contiguity



Distance-based Neighbors

- How to measure distance between polygons?
- Distance metrics
  – 2D Cartesian distance (projected data)
  – 3D spherical distance/great-circle distance (lat/long data)
    - Haversine formula

Haversine formula:

$$a = \sin^2(\Delta\varphi/2) + \cos(\varphi_1).\cos(\varphi_2).\sin^2(\Delta\lambda/2)$$

$$c = 2.\mathrm{atan2}(\sqrt{a}, \sqrt{(1-a)})$$

$$d = R.c$$

where φ is latitude, λ is longitude, R is earth's radius (mean radius = 6,371km)

Distance-based Neighbors

- k-nearest neighbors

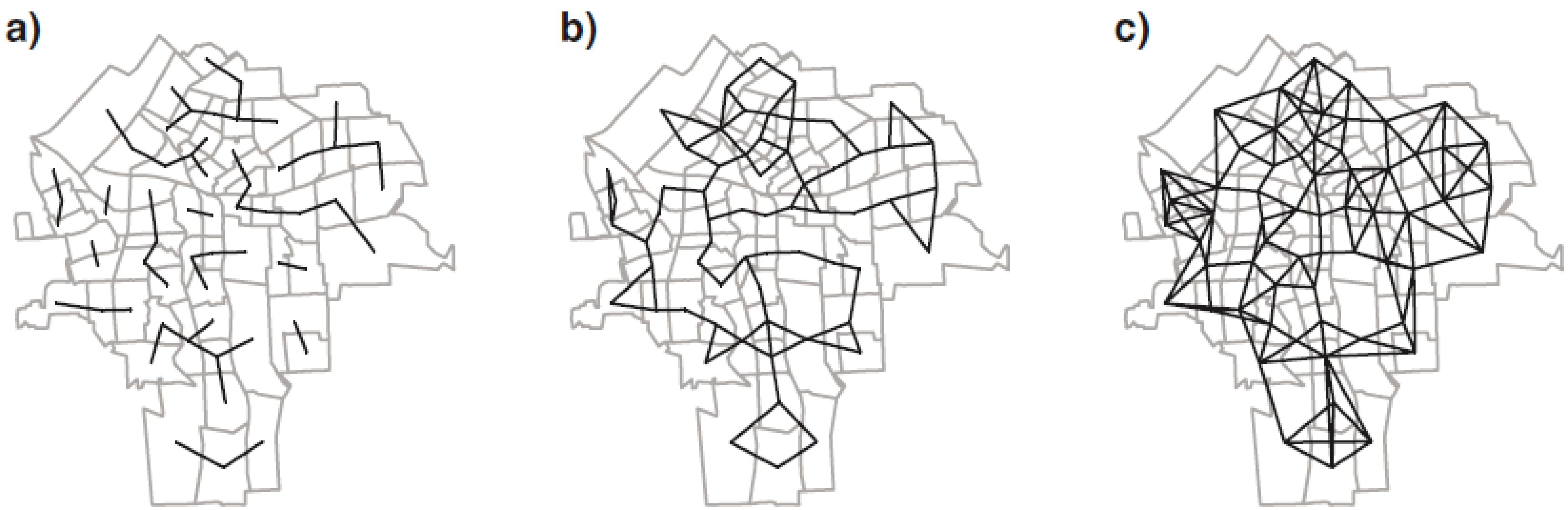


Fig. 9.5. (**a**) $k = 1$ neighbours; (**b**) $k = 2$ neighbours; (**c**) $k = 4$ neighbours

Source: Bivand and Pebesma and Gomez-Rubio

Distance-based Neighbors

- thresh-hold distance (buffer)

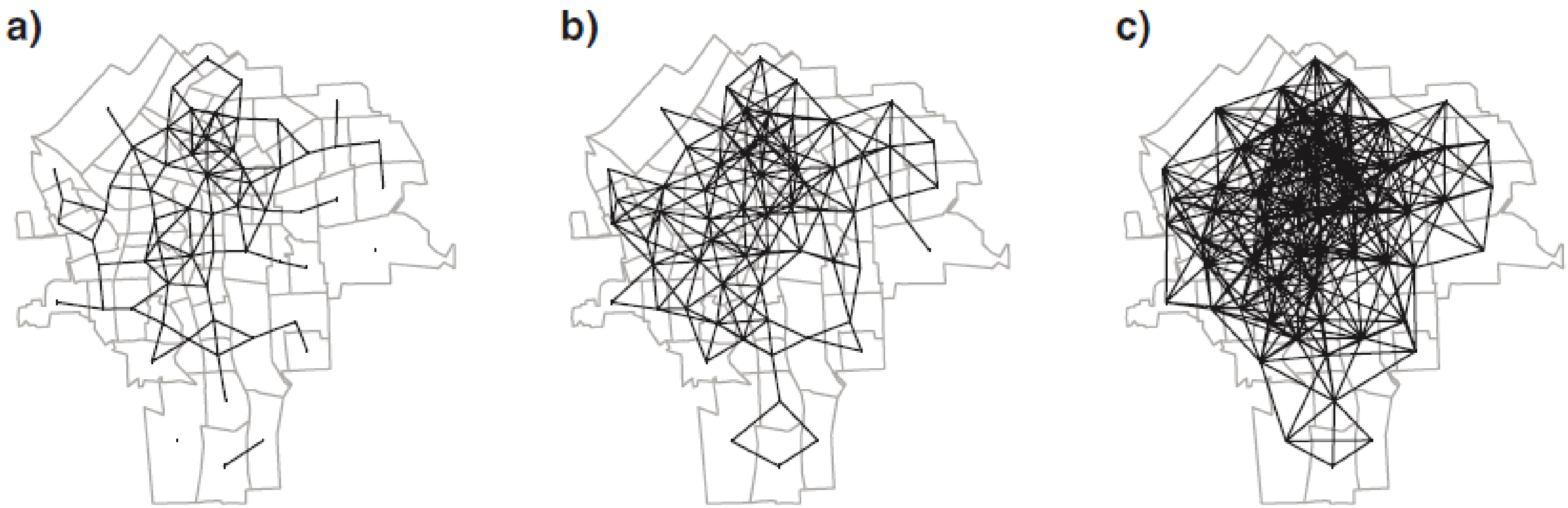
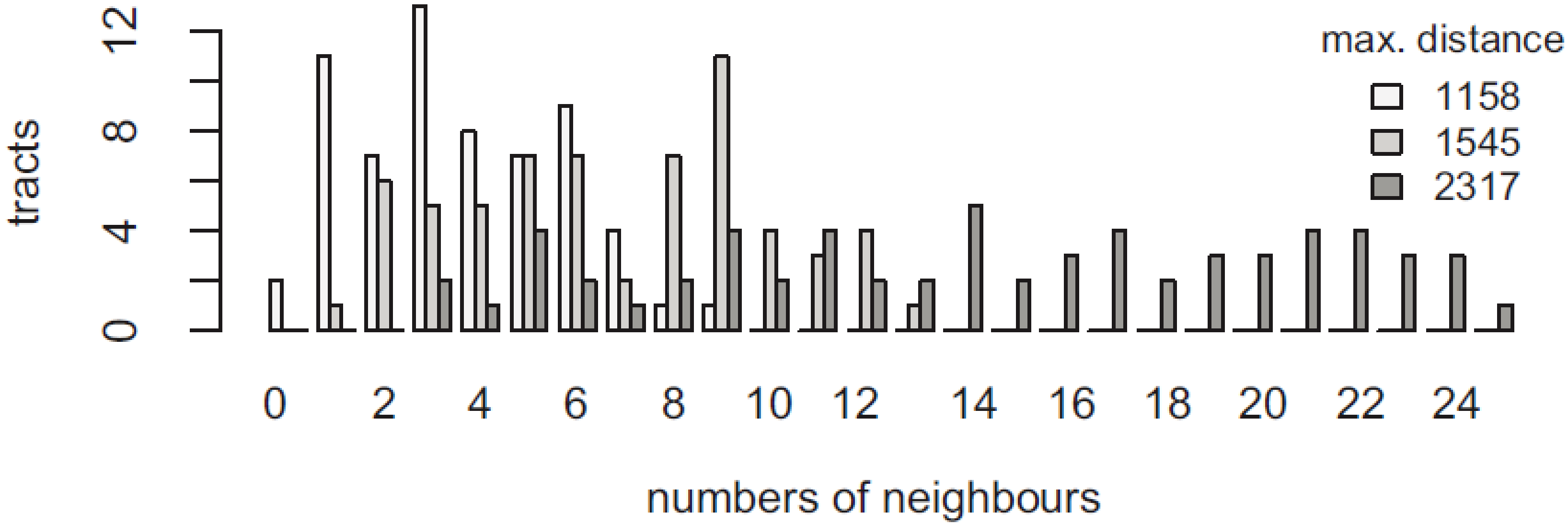


Fig. 9.6. (**a**) Neighbours within 1,158 m; (**b**) neighbours within 1,545 m; (**c**) neigh
bours within 2,317 m

Source: Bivand and Pebesma and Gomez-Rubio

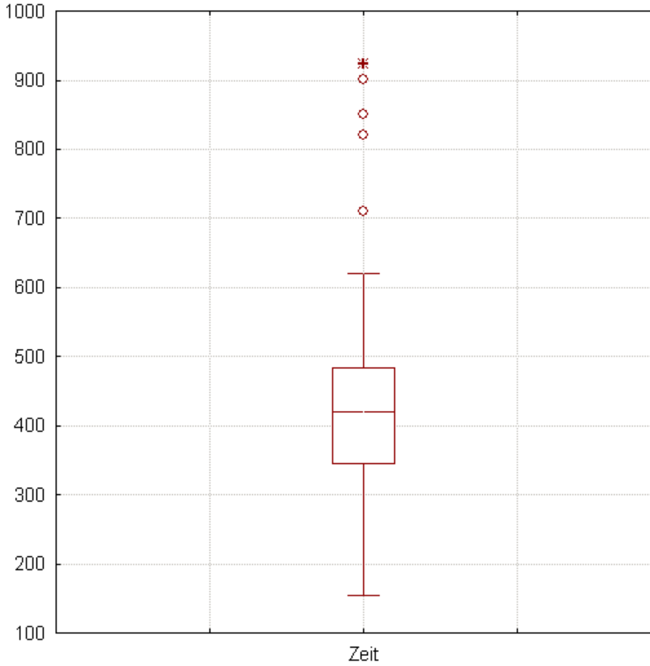
Neighbor/Connectivity Histogram



Source: Bivand and Pebesma and Gomez-Rubio

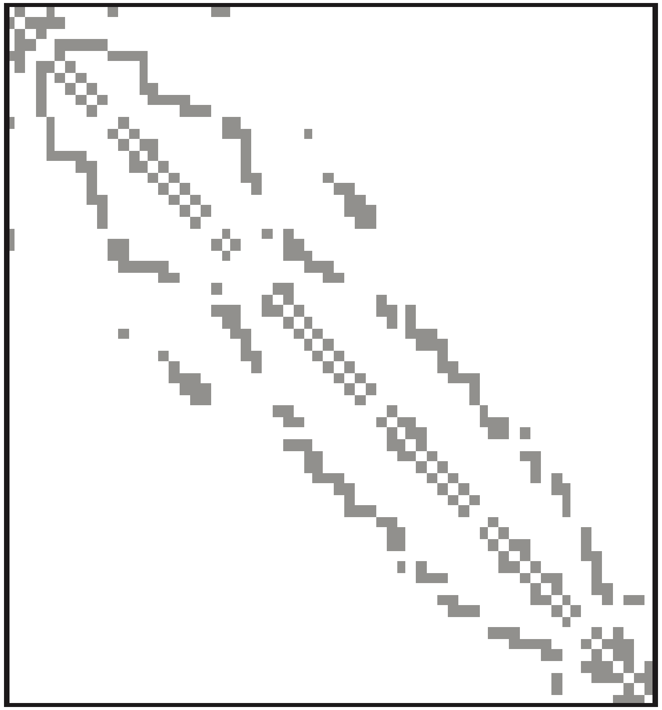
Side Note: Box-plot

- Help indicate the degree of dispersion and skewness and identify outliers
  - Non-parametric
  - 25%, 50%, 75% percentiles
  - end of the hinge could mean differently depending on implementation
  - Points outside the range are usually taken as outliers



Spatial Weight Matrix

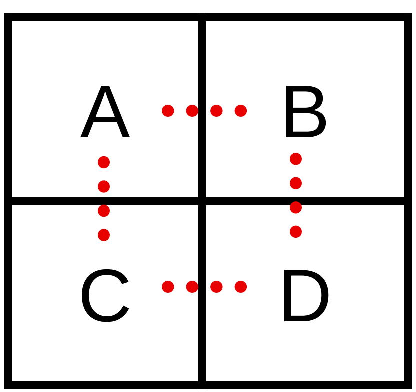
- Spatial weights can be seen as a list of weights indexed by a list of neighbors
- If zone j is not a neighbor of zone i, weights Wij will set to zero
  - The weight matrix can be illustrated as an image
  - Sparse matrix



A Simple Example for Rook case

- Matrix contains a:
  - 1 if share a border
  - 0 if do not share a border

4 areal units



Common border

4x4 matrix

W =	A	B	C	D
A	0	1	1	0
B	1	0	0	1
C	1	0	0	1
D	0	1	1	0

1 Washington		1				1																																												
2 Oregon	1		1		1	1																																												
3 California		1		1	1																																												
4 Arizona			1		1				1	1			1																																				
5 Nevada		1	1	1		1			1																																								
6 Idaho	1	1			1		1	1	1																																								
7 Montana						1		1								1	1																																
8 Wyoming						1	1		1					1		1	1																																	
9 Utah				1	1	1		1		1			1																																				
10 New Mexico				1					1		1	1	1																																			
11 Texas										1		1									1	1																											
12 Oklahoma										1	1		1	1						1	1																												
13 Colorado				1				1	1	1		1		1	1																																		
14 Kansas													1	1		1					1																													
15 Nebraska								1						1	1		1			1	1																													
16 South Dakota							1	1							1		1	1	1																														
17 North Dakota							1									1		1																															
18 Minnesota																	1	1		1									1																						
19 Iowa															1	1		1		1						1	1																						
20 Missouri												1		1	1				1			1			1	1	1																							
21 Arkansas											1	1								1		1	1	1																								
22 Louisiana											1										1		1																										
23 Mississippi																					1	1		1									1																
24 Tennessee																				1	1		1		1								1	1		1	1													
25 Kentucky																				1				1		1			1	1	1						1												
26 Illinois																			1	1					1		1		1																					
27 Wisconsin																		1	1							1		1																							
28 Michigan																											1		1	1																			
29 Indiana																									1	1		1		1																			
30 Ohio																									1			1	1		1												1							
31 West Virginia																									1					1							1	1						1							
32 Florida																																	1	1																	
33 Alabama																							1	1									1		1																	
34 Georgia																								1									1	1		1	1															
35 South Carolina																																		1		1															
36 North Carolina																								1										1	1		1														
37 Virginia																								1	1						1					1		1		1									
38 Maryland																														1						1		1	1		1							
39 Delaware																																							1			1	1							
40 District of Columbia																																					1	1											
41 New Jersey																																								1			1	1					
42 Pennsylvania																														1	1							1	1		1		1					
43 New York																																										1	1		1		1		1	
44 Connecticut																																												1		1	1			
45 Rhode Island																																													1		1			
46 Massachussets																																												1	1	1		1	1	
47 New Hampshire																																															1		1	1
48 Vermont																																												1			1	1		
49 Maine																																																1		

Sparse Contiguity Matrix for US States -- obtained from Anselin's web site (see powerpoint for link)										
Name	**Fips**	**Ncount**	**N1**	**N2**	**N3**	**N4**	**N5**	**N6**	**N7**	**N8**
Alabama	1	4	28	13	12	47				
Arizona	4	5	35	8	49	6	32			
Arkansas	5	6	22	28	48	47	40	29		
California	6	3	4	32	41					
Colorado	8	7	35	4	20	40	31	49	56	
Connecticut	9	3	44	36	25					
Delaware	10	3	24	42	34					
District of Columbia	11	2	51	24						
Florida	12	2	13	1						
Georgia	13	5	12	45	37	1	47			
Idaho	16	6	32	41	56	49	30	53		
Illinois	17	5	29	21	18	55	19			
Indiana	18	4	26	21	17	39				
Iowa	19	6	29	31	17	55	27	46		
Kansas	20	4	40	29	31	8				
Kentucky	21	7	47	29	18	39	54	51	17	
Louisiana	22	3	28	48	5					
Maine	23	1	33							
Maryland	24	5	51	10	54	42	11			
Massachusetts	25	5	44	9	36	50	33			
Michigan	26	3	18	39	55					
Minnesota	27	4	19	55	46	38				
Mississippi	28	4	22	5	1	47				
Missouri	29	8	5	40	17	21	47	20	19	31
Montana	30	4	16	56	38	46				
Nebraska	31	6	29	20	8	19	56	46		
Nevada	32	5	6	4	49	16	41			
New Hampshire	33	3	25	23	50					
New Jersey	34	3	10	36	42					
New Mexico	35	5	48	40	8	4	49			
New York	36	5	34	9	42	50	25			
North Carolina	37	4	45	13	47	51				
North Dakota	38	3	46	27	30					
Ohio	39	5	26	21	54	42	18			
Oklahoma	40	6	5	35	48	29	20	8		
Oregon	41	4	6	32	16	53				
Pennsylvania	42	6	24	54	10	39	36	34		
Rhode Island	44	2	25	9						
South Carolina	45	2	13	37						
South Dakota	46	6	56	27	19	31	38	30		
Tennessee	47	8	5	28	1	37	13	51	21	29
Texas	48	4	22	5	35	40				
Utah	49	6	4	8	35	56	32	16		
Vermont	50	3	36	25	33					
Virginia	51	6	47	37	24	54	11	21		
Washington	53	2	41	16						
West Virginia	54	5	51	21	24	39	42			
Wisconsin	55	4	26	17	19	27				
Wyoming	56	6	49	16	31	8	46	30		

Style of Spatial Weight Matrix

- Row
  - a weight of unity for each neighbor relationship
- Row standardization
  - Symmetry not guaranteed
  - can be interpreted as allowing the calculation of average values across neighbors
- General spatial weights based on distances

Row vs. Row standardization

A	B	C
D	E	F

Divide each number by the **row sum**

Total number of neighbors
--some have more than others

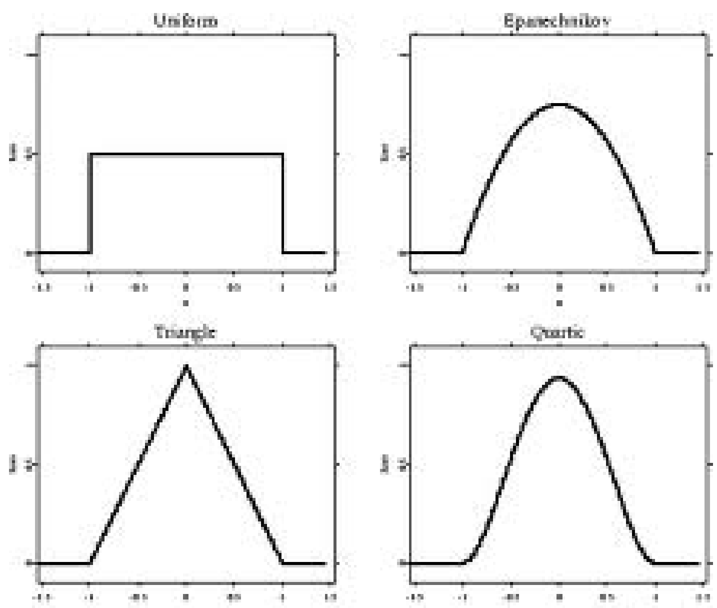
	A	B	C	D	E	F	Row Sum
A	0	1	0	1	0	0	2
B	1	0	1	0	1	0	***3***
C	0	1	0	0	0	1	2
D	1	0	0	0	1	0	2
E	0	1	0	1	0	1	***3***
F	0	0	1	0	1	0	2

Row standardized
--usually use this

	A	B	C	D	E	F	Row Sum
A	0.0	0.5	0.0	0.5	0.0	0.0	1
B	0.3	0.0	0.3	0.0	0.3	0.0	1
C	0.0	0.5	0.0	0.0	0.0	0.5	1
D	0.5	0.0	0.0	0.0	0.5	0.0	1
E	0.0	0.3	0.0	0.3	0.0	0.3	1
F	0.0	0.0	0.5	0.0	0.5	0.0	1

General Spatial Weights Based on Distance

- Decay functions of distance
  - Most common choice is the inverse (reciprocal) of the distance between locations i and j ($w_{ij} = 1/d_{ij}$)
  - Other functions also used
    - inverse of <u>squared</u> distance ($w_{ij} = 1/d_{ij}^2$), or
    - negative exponential ($w_{ij} = e^{-d}$ *or* $w_{ij} = e^{-d^2}$)



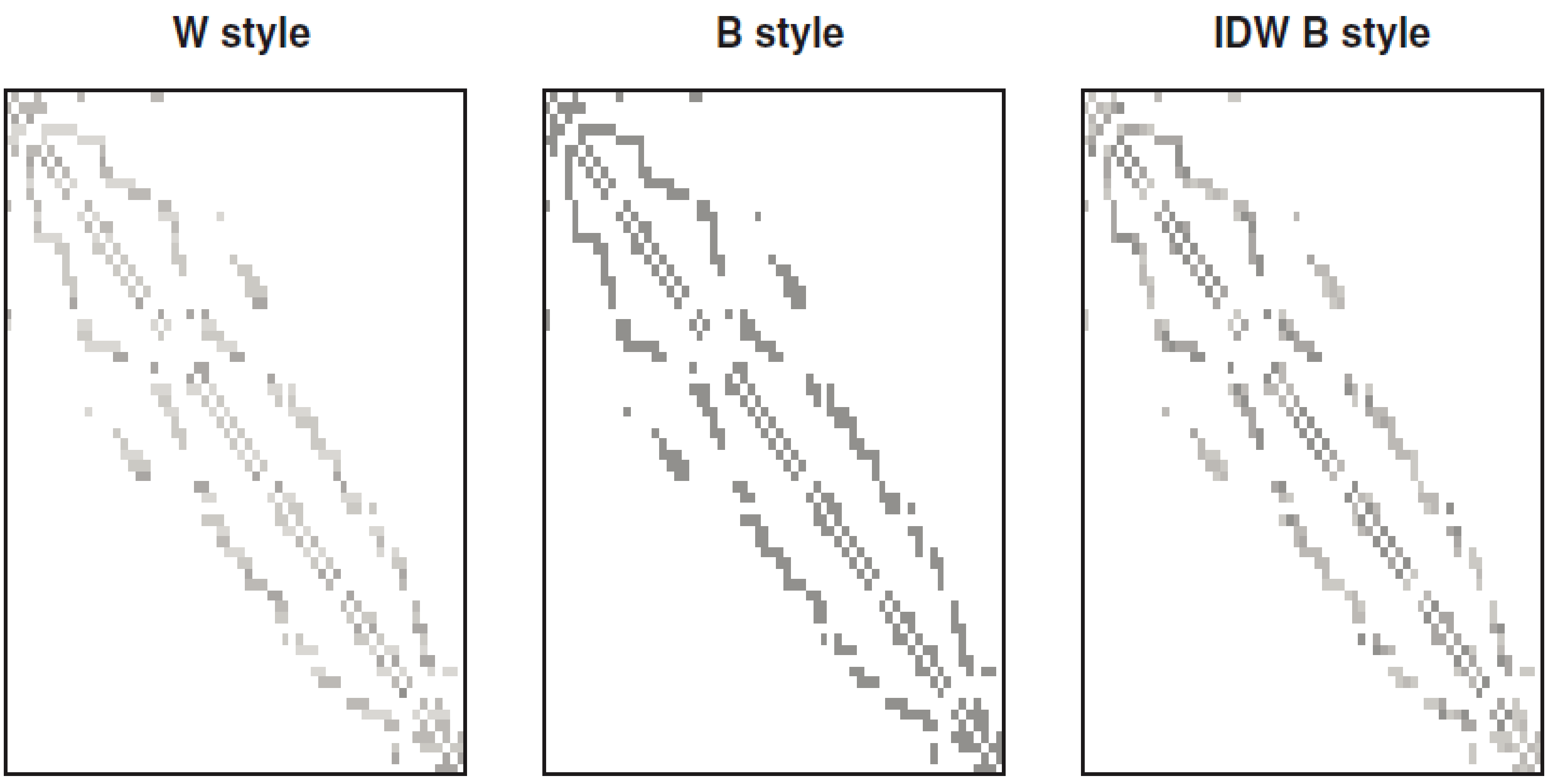
Due to the Federal government shutdown, NOAA.gov and most associated web sites are unavailable.

Only web sites necessary to protect lives and property will be maintained.

See Weather.gov for critical weather information or contact USA.gov for more information about the shutdown.

Example

- Compare three different weight matrix in images



Measure of Spatial Autocorrelation

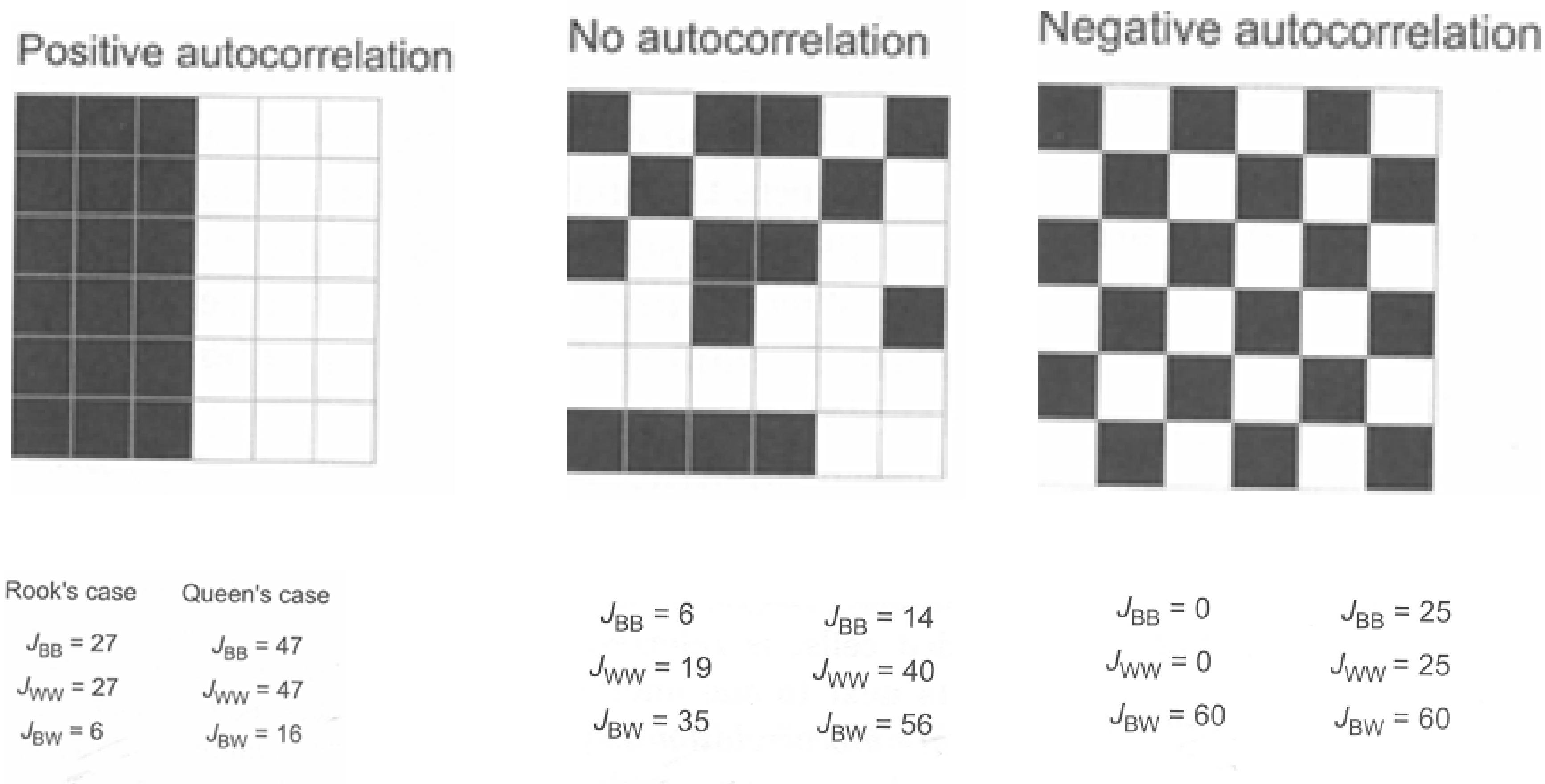
Global Measures and Local Measures

- Global Measures
  - A single value which applies to the entire data set
    - The same pattern or process occurs over the entire geographic area
    - An average for the entire area
- Local Measures
  - A value calculated for <u>each</u> observation unit
    - Different patterns or processes may occur in different parts of the region
    - A unique number for each location
- Global measures usually can be decomposed into a combination of local measures

Global Measures and Local Measures

- Global Measures
  - Join Count
  - Moran's I, Geary's C, Getis-Ord's G
- Local Measures
  - Local Moran's I , Geary's C, Getis-Ord's G

Join (or Joint or Joins) Count Statistic



– 60 for Rook Case
– 110 for Queen Case

Join Count: Test Statistic

Test Statistic given by: Z= $\frac{\text{Observed - Expected}}{\text{SD of Expected}}$

Expected = random pattern generated by tossing a coin in each cell.

Expected given by:

$$E(J_{BB}) = kp_B^2$$

$$E(J_{WW}) = kp_W^2$$

$$E(J_{BW}) = 2kp_Bp_W$$

Standard Deviation of Expected (standard error) given by:

$$E(s_{BB}) = \sqrt{kp_B^2 + 2mp_B^3 - (k+2m)p_B^4}$$

$$E(s_{WW}) = \sqrt{kp_W^2 + 2mp_W^3 - (k+2m)p_W^4}$$

$$E(s_{BW}) = \sqrt{2(k+m)p_Bp_W - 4(k+2m)p_B^2p_W^2}$$

Where: k is the total number of joins (neighbors)

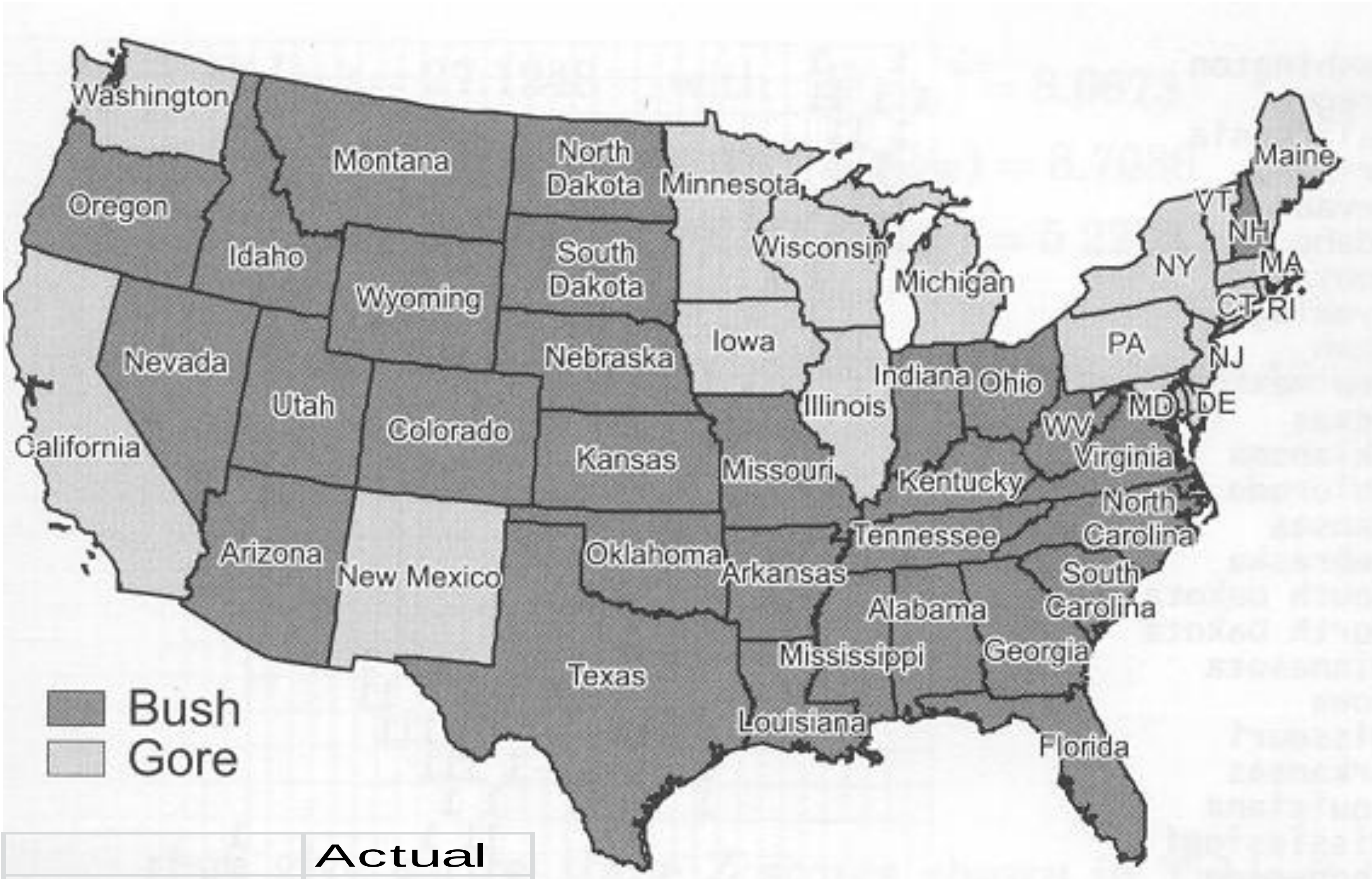
p_B is the expected proportion Black, if random

p_W is the expected proportion White

m is calculated from k according to:

$$m = \frac{1}{2}\sum_{i=1}^{n} k_i(k_i - 1)$$

Gore/Bush Presidential Election 2000



	Actual
Jbb	60
Jgg	21
Jbg	28
Total	109

Join Count Statistic for Gore/Bush 2000 by State

candidates	probability
Bush	0.49885
Gore	0.50115

	Actual	Expected	Stan Dev	Z-score
Jbb	60	27.125	8.667	3.7930
Jgg	21	27.375	8.704	-0.7325
Jbg	28	54.500	5.220	-5.0763
Total	109	109.000		

- The expected number of joins is calculated based on the proportion of votes each received in the election (for Bush = 109*.499*.499=27.125)
- There are far <u>more</u> Bush/Bush joins (actual = 60) than would be expected (27)
  - Positive autocorrelation
- There are far <u>fewer</u> Bush/Gore joins (actual = 28) than would be expected (54)
  - Positive autocorrelation
- No strong clustering evidence for Gore (actual = 21 slightly less than 27.375)

Moran's I

- The most common measure of Spatial Autocorrelation
- Use for points or polygons
  - Join Count statistic only for polygons
- Use for a continuous variable (any value)
  - Join Count statistic only for binary variable (1,0)

Patrick Alfred Pierce Moran (1917-1988)

Formula for Moran's I

$$I = \frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\right)\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- Where:

N is the number of observations (points or polygons)

$\bar{X}$ is the mean of the variable

X_i is the variable value at a particular location

X_j is the variable value at another location

W_{ij} is a weight indexing location of *i* relative to *j*

Moran's I

- Expectation of Moran's I under no spatial autocorrelation

$$E(\mathrm{I}) = -1/(\mathrm{N}-1)$$

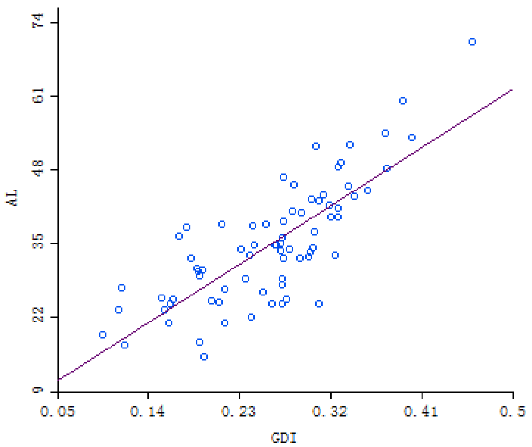
- Variance of Moran's is complex and exact equation is given at textbook d&G&L
- [-1, 1]

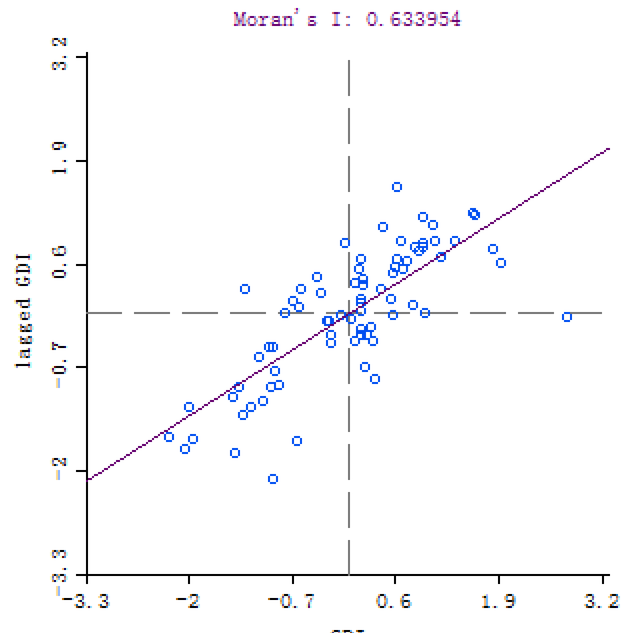
Moran's *I* and Correlation Coefficient

- **Correlation Coefficient [-1, 1]**
  - Relationship between <u>two</u> different variables
- **Moran's I [-1, 1]**

– Spatial autocorrelation and often involves <u>one</u> (spatially indexed) variable only

– Correlation between observations of a spatial variable at location X and "spatial lag" of X formed by averaging all the observation at neighbors of X





Correlation Coefficient

$$\frac{\sum_{i=1}^{n} 1(y_i - \bar{y})(x_i - \bar{x})/n}{\sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Note the similarity of the numerator (top) to the measures of spatial association discussed earlier if we view Yi as being the Xi for the neighboring polygon

(see next slide)

$$\frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij})\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})/\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Spatial auto-correlation

Source: Ron Briggs of UT Dallas

Correlation Coefficient

$$\frac{\sum_{i=1}^{n} 1(y_i-\bar{y})(x_i-\bar{x})/n}{\sqrt{\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}}$$

Spatial weights

Yi is the Xi for the neighboring polygon

$$\frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\right)\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i-\bar{x})(x_j-\bar{x})/\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}}$$

Moran's I

Source: Ron Briggs of UT Dallas

Statistical Significance Tests for Moran's I

- Based on the normal frequency distribution with

$$Z = \frac{I - E(I)}{S_{error(I)}}$$

Where: I is the calculated value for Moran's I from the sample

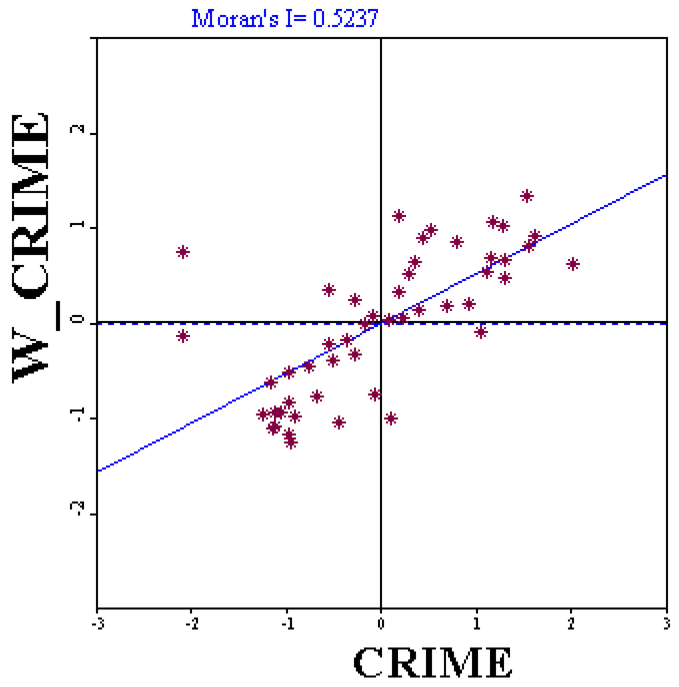
E(I) is the expected value if random

S is the standard error

- Statistical significance test
  - Monte Carlo test, as we did for spatial pattern analysis
  - Permutation test
    - Non-parametric
    - Data-driven, no assumption of the data
    - Implemented in GeoDa

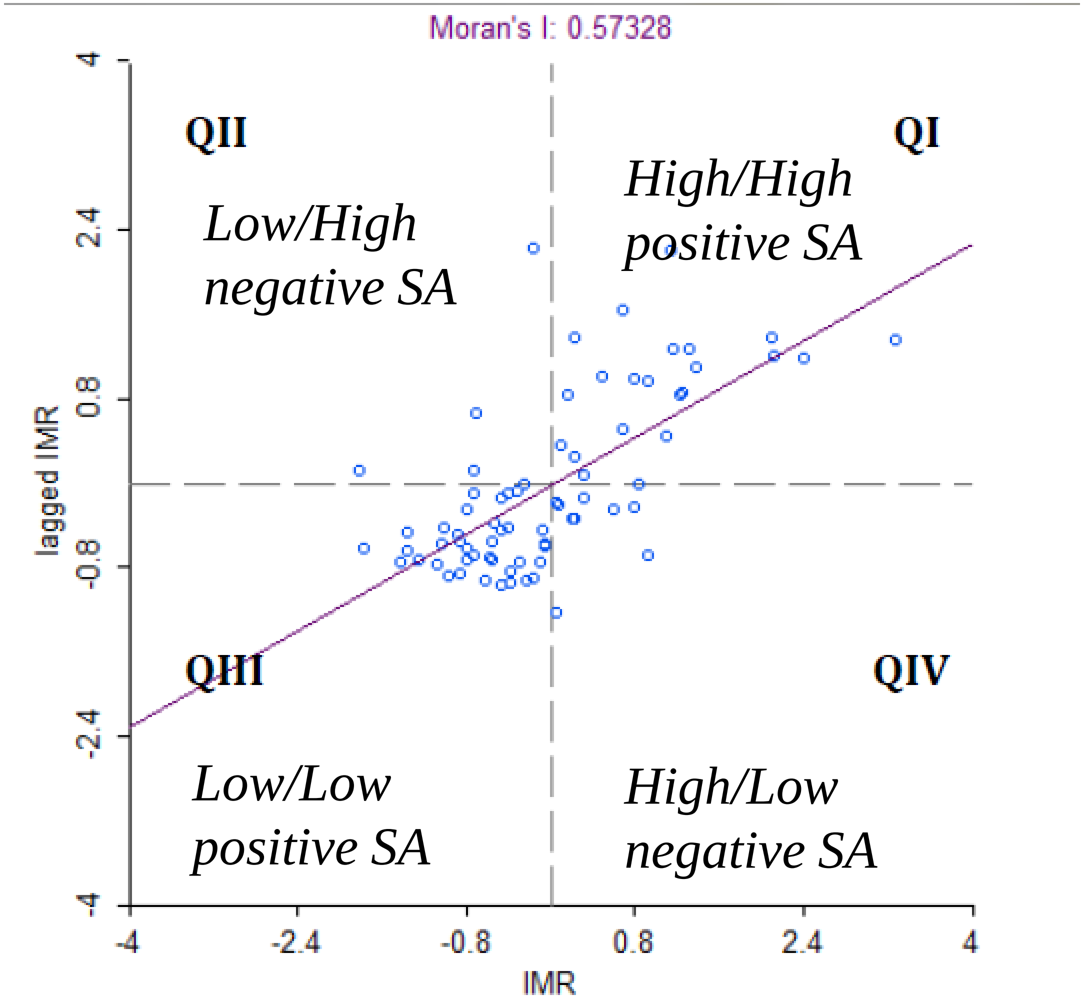
Moran Scatter Plots

We can draw a scatter diagram between these two variables (in standardized form): **X** and **lag-X** (or W_X)

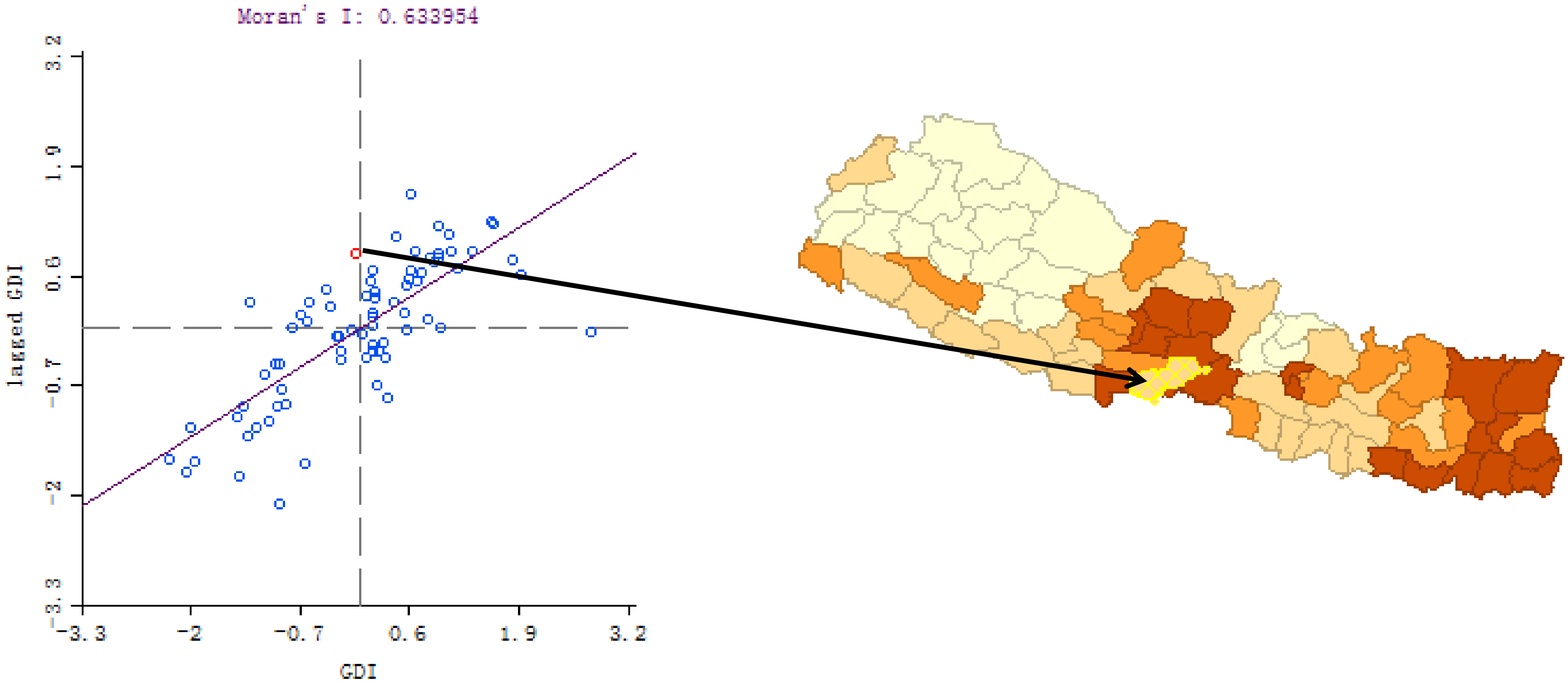


The <u>slope</u> of this *regression line* is Moran's I

Moran Scatter Plots



Moran Scatterplot: Example



Moran's I for rate-based data

- Moran's I is often calculated for rates, such as crime rates (e.g. number of crimes per 1,000 population) or infant mortality rates (e.g. number of deaths per 1,000 births)
- An adjustment should be made, especially if the denominator in the rate (population or number of births) varies greatly (as it usually does)
- Adjustment is know as the *EB adjust*ment:
  - see Assuncao-Reis *Empirical Bayes Standardization* Statistics in Medicine, 1999
- *GeoDA* software includes an option for this adjustment

Geary's C

- Calculation is similar to Moran's I,
  - For Moran, the cross-product is based on the deviations from the mean for the two location values
  - For Geary, the cross-product uses the actual values themselves at each location
  - Covariance vs. variogram

$$C = \frac{N \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} (x_i - x_j)^2}{2\left(\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}\right) \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Geary's C vs. Moran's I

- Interpretation is very different, essentially the opposite!
  Geary's C varies on a scale from 0 to 2
  - 0 indicates perfect positive autocorrelation/clustered
  - 1 indicates no autocorrelation/random
  - 2 indicates perfect negative autocorrelation/dispersed
- Can convert to a -/+1 scale by: calculating $C^* = 1 – C$
- Morain's I is more often used

Statistical Significance Tests for Geary's C

- Similar to Moran
- Again, based on the normal frequency distribution with

$$Z = \frac{C - E(C)}{S_{error(I)}}$$

Where: C is the calculated value for Geary's C from the sample

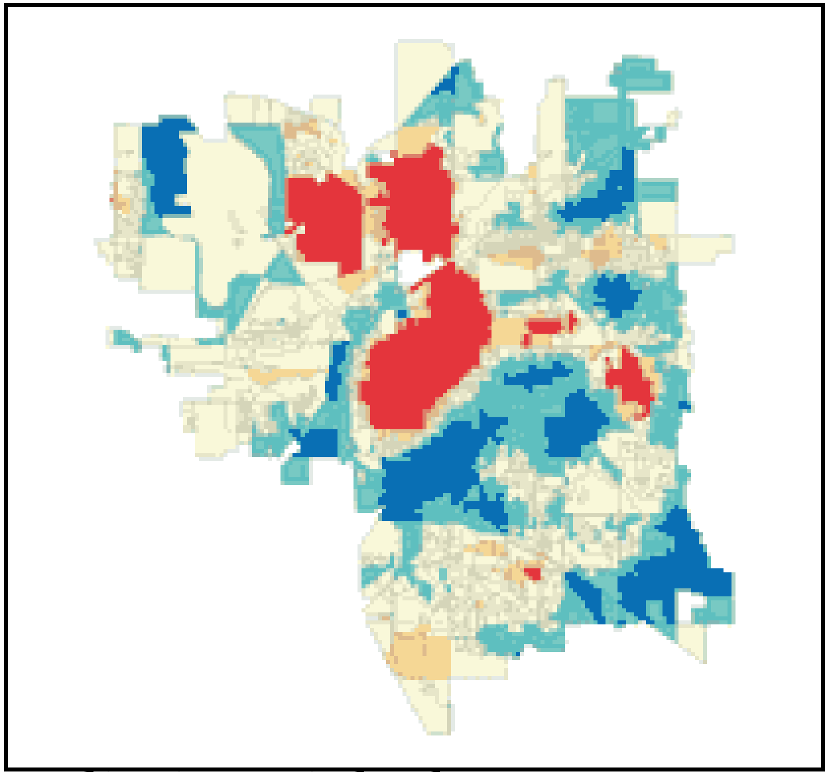
E(C) is the expected value if no autocorrelation

S is the standard error

however, E(C) = 1

Hot Spots and Cold Spots

- What is a *hot spot*?
  – A place where high values cluster together
- What is a *cold spot*?
  – A place where low values cluster together



- Moran's I and Geary's C cannot distinguish them
  - They only indicate clustering
  - Cannot tell if these are hot spots, cold spots, or both

Getis-Ord General/Global G-Statistic

- The G statistic distinguishes between hot spots and cold spots. It identifies *spatial concentrations*.
  - G is relatively large if high values cluster together
  - G is relatively low if low values cluster together
- The General G statistic is interpreted relative to its *expected value*
  - The value for which there is no spatial association
  - G > (larger than) *expected value* ➔ potential "hot spots"
  - G < (smaller than) *expected value* ➔ potential "cold spots"
- A Z *test statistic* is used to test if the difference is statistically significant
- Calculation of G based on a *neighborhood distance* within which cluster is expected to occur

Getis, A. and Ord, J.K. (1992) *The analysis of spatial association by use of distance statistics* Geographical Analysis, *24(3) 189-206*

Formulae of General G

The General *G* statistic of overall spatial association is given as:

$$G = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{i,j} x_i x_j}{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j}, \ \forall j \neq i \qquad (1)$$

where x_i and x_j are attribute values for features i and j, and $w_{i,j}$ is the spatial weight between feature i and j. n is the number of features in the dataset and $\forall j \neq i$ indicates that features i and j cannot be the same feature.

The z_G-score for the statistic is computed as:

$$z_G = \frac{G - \mathrm{E}[G]}{\sqrt{\mathrm{V}[G]}} \qquad (2)$$

where:

$$\mathrm{E}[G] = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{i,j}}{n(n-1)}, \ \forall j \neq i \qquad (3)$$

$$\mathrm{V}[G] = \mathrm{E}[G^2] - \mathrm{E}[G]^2 \qquad (4)$$

Comments on General G

- General G will <u>not</u> show <u>negative</u> spatial autocorrelation
- Should <u>only</u> be calculated for <u>ratio scale</u> data
  - data with a "natural" zero such as crime rates, birth rates
- Although it was defined using a contiguity (0,1) weights matrix, <u>any</u> type of spatial weights matrix can be used
  - ArcGIS gives multiple options
- There are two global versions: G and G*
  - G does <u>not</u> include the value of X_i itself, only "neighborhood" values
  - G* includes X_i as well as "neighborhood" values
- Significance test on General G and G* follows the similar procedure as used for Moran's I

Local Measures of Spatial Autocorrelation

Local Indicators of Spatial Association (LISA)

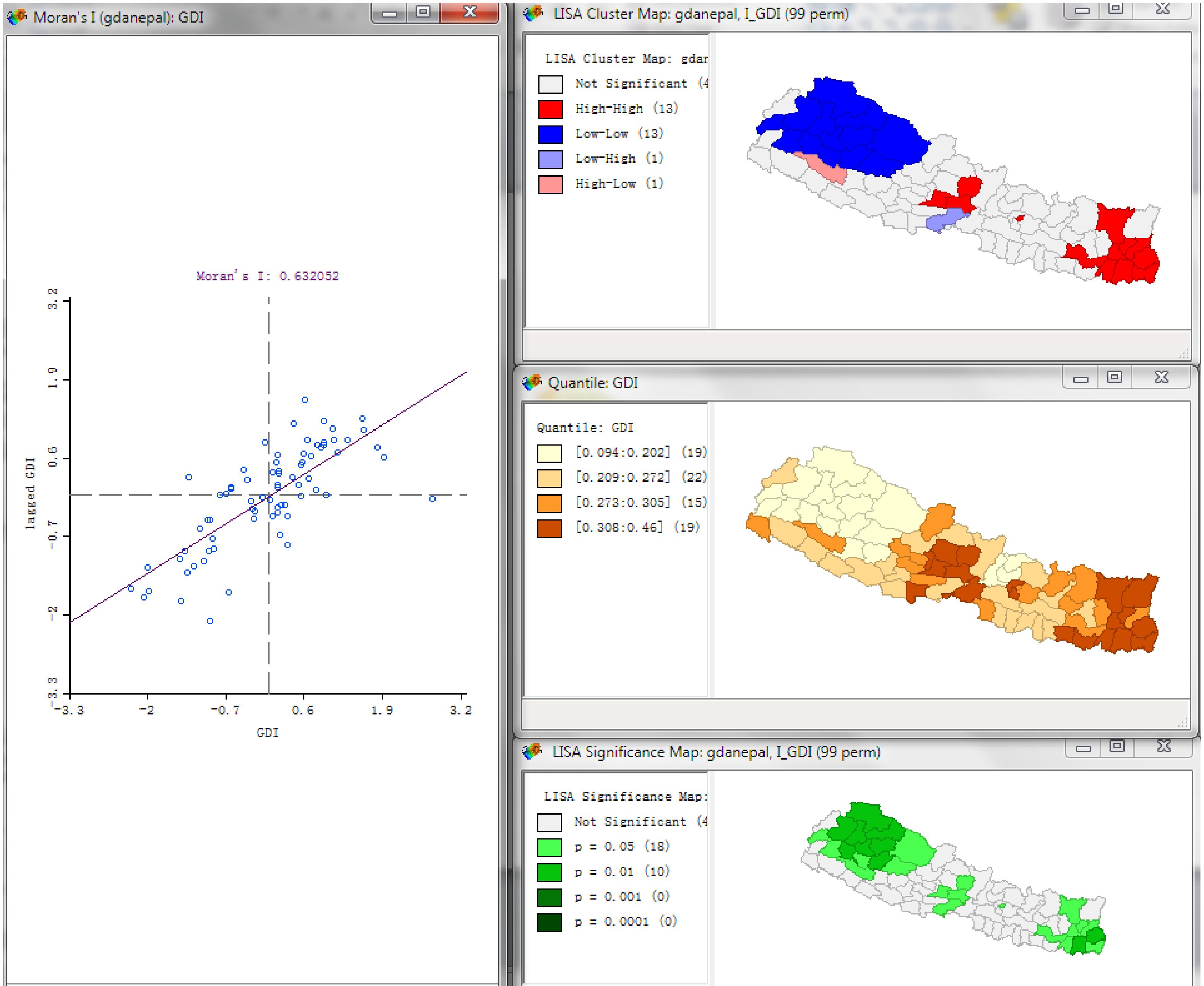
- Local versions of *Moran's I, Geary's C, and the Getis-Ord G statistic*
- Moran's I is most commonly used, and the local version is often called Anselin's LISA, or just LISA

See:
Luc Anselin 1995 *Local Indicators of Spatial Association-LISA* Geographical Analysis 27: 93-115

Local Indicators of Spatial Association (LISA)

- The statistic is calculated for **each** areal unit in the data
- For each polygon, the index is calculated based on neighboring polygons with which it shares a border
- A measure is available for each polygon, these can be mapped to indicate how spatial autocorrelation varies over the study region
- Each index has an associated test statistic, we can also map which of the polygons has a statistically significant relationship with its neighbors, and show type of relationship

Example:


Moran's I (gdanepal): GDI
Moran's I: 0.632052
lagged GDI
GDI
-3.3
-2
-0.7
0.6
1.9
3.2
LISA Cluster Map: gdanepal, I_GDI (99 perm)
LISA Cluster Map: gdar
Not Significant (4
High-High (13)
Low-Low (13)
Low-High (1)
High-Low (1)
Quantile: GDI
Quantile: GDI
[0.094:0.202] (19)
[0.209:0.272] (22)
[0.273:0.305] (15)
[0.308:0.46] (19)
LISA Significance Map: gdanepal, I_GDI (99 perm)
LISA Significance Map:
Not Significant (4
p = 0.05 (18)
p = 0.01 (10)
p = 0.001 (0)
p = 0.0001 (0)

Calculating Anselin's LISA

- The local Moran statistic for areal unit *i* is:

$$I_i = z_i \sum_j w_{ij} z_j$$

where z_i is the original variable x_i in "standardized form"

$$z_i = \frac{x_i - \bar{x}}{SD_x}$$

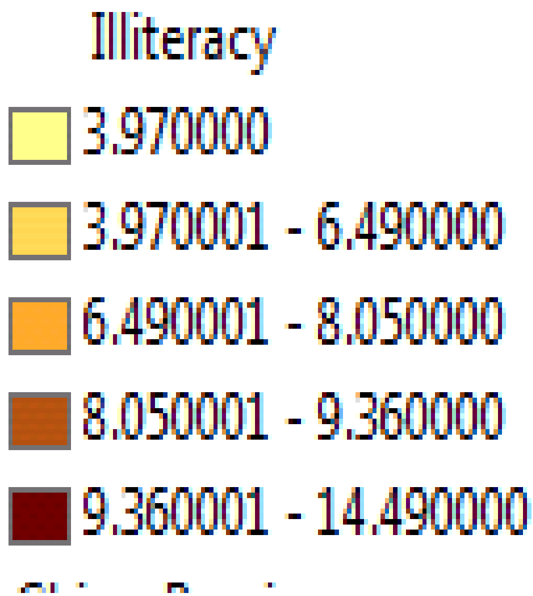
or it can be in "deviation form" $x_i - \bar{x}$

and w_{ij} is the spatial weight

The summation $\sum_j$ is across each row *i* of the spatial weights matrix.

An example follows

Contiguity Matrix	Code	1 Anhui	2 Zhejiang	3 Jiangxi	4 Jiangsu	5 Henan	6 Hubei	7 Shanghai	Sum	Neighbors	**Illiteracy**
Anhui	1	0	1	1	1	1	1	0	5	6 5 4 3 2	**14.49**
Zhejiang	2	1	0	1	1	0	0	1	4	7 4 3 1	**9.36**
Jiangxi	3	1	1	0	0	0	1	0	3	6 2 1	**6.49**
Jiangsu	4	1	1	0	0	0	0	1	3	7 2 1	**8.05**
Henan	5	1	0	0	0	0	1	0	2	6 1	**7.36**
Hubei	6	1	0	1	0	1	0	0	3	1 3 5	**7.69**
Shanghai	7	0	1	0	1	0	0	0	2	2 4	**3.97**



Source: Ron Briggs of UT Dallas

Contiguity Matrix and Row Standardized Spatial Weights Matrix

Contiguity Matrix

	Code	1 Anhui	2 Zhejiang	3 Jiangxi	4 Jiangsu	5 Henan	6 Hubei	7 Shanghai	Sum
Anhui	1	0	1	1	1	1	1	0	5
Zhejiang	2	1	0	1	1	0	0	1	4
Jiangxi	3	1	1	0	0	0	1	0	3
Jiangsu	4	1	1	0	0	0	0	1	3
Henan	5	1	0	0	0	0	1	0	2
Hubei	6	1	0	1	0	1	0	0	3
Shanghai	7	0	1	0	1	0	0	0	2

1/3

Row Standardized Spatial Weights Matrix

	Code	Anhui	Zhejiang	Jiangxi	Jiangsu	Henan	Hubei	Shanghai	Sum
Anhui	1	0.00	0.20	0.20	0.20	0.20	0.20	0.00	1
Zhejiang	2	0.25	0.00	0.25	0.25	0.00	0.00	0.25	1
Jiangxi	3	0.33	0.33	0.00	0.00	0.00	0.33	0.00	1
Jiangsu	4	0.33	0.33	0.00	0.00	0.00	0.00	0.33	1
Henan	5	0.50	0.00	0.00	0.00	0.00	0.50	0.00	1
Hubei	6	0.33	0.00	0.33	0.00	0.33	0.00	0.00	1
Shanghai	7	0.00	0.50	0.00	0.50	0.00	0.00	0.00	1

Source: Ron Briggs of UT Dallas

Calculating standardized (z) scores

$$z_i = \frac{x_i - \bar{x}}{SD_x}$$

Deviations from Mean and z scores.

	X	X-Xmean	X-Mean2	z
Anhui	14.49	6.29	39.55	2.101
Zhejiang	9.36	1.16	1.34	0.387
Jiangxi	6.49	(1.71)	2.93	(0.572)
Jiangsu	8.05	(0.15)	0.02	(0.051)
Henan	7.36	(0.84)	0.71	(0.281)
Hubei	7.69	(0.51)	0.26	(0.171)
Shanghai	3.97	(4.23)	17.90	(1.414)

Mean and Standard Deviation

Sum	57.41			0.00	62.71
Mean	57.41	/	7	=	8.20
Variance	62.71	/	7	=	8.96
SD	√ 8.96			=	2.99

Source: Ron Briggs of UT Dallas

Calculating LISA

Row Standardized Spatial Weights Matrix

	Code	Anhui	Zhejiang	Jiangxi	Jiangsu	Henan	Hubei	Shanghai
Anhui	1	0.00	0.20	0.20	0.20	0.20	0.20	0.00
Zhejiang	2	0.25	0.00	0.25	0.25	0.00	0.00	0.25
Jiangxi	3	0.33	0.33	0.00	0.00	0.00	0.33	0.00
Jiangsu	4	0.33	0.33	0.00	0.00	0.00	0.00	0.33
Henan	5	0.50	0.00	0.00	0.00	0.00	0.50	0.00
Hubei	6	0.33	0.00	0.33	0.00	0.33	0.00	0.00
Shanghai	7	0.00	0.50	0.00	0.50	0.00	0.00	0.00

w_{ij}

$$I_i = z_i \sum_j w_{ij} z_j$$

Z-Scores for row Province and its potential neighbors

	Zi	Anhui	Zhejiang	Jiangxi	Jiangsu	Henan	Hubei	Shanghai
Anhui	2.101	***2.101***	0.387	(0.572)	(0.051)	(0.281)	(0.171)	(1.414)
Zhejiang	0.387	2.101	***0.387***	(0.572)	(0.051)	(0.281)	(0.171)	(1.414)
Jiangxi	(0.572)	2.101	0.387	***(0.572)***	(0.051)	(0.281)	(0.171)	(1.414)
Jiangsu	(0.051)	2.101	0.387	(0.572)	***(0.051)***	(0.281)	(0.171)	(1.414)
Henan	(0.281)	2.101	0.387	(0.572)	(0.051)	*(0.281)*	(0.171)	(1.414)
Hubei	(0.171)	2.101	0.387	(0.572)	(0.051)	(0.281)	***(0.171)***	(1.414)
Shanghai	(1.414)	2.101	0.387	(0.572)	(0.051)	(0.281)	(0.171)	***(1.414)***

z_j

$w_{ij} z_j$

Spatial Weight Matrix multiplied by Z-Score Matrix (cell by cell multiplication)

	Zi	**Anhui**	**Zhejiang**	**Jiangxi**	**Jiangsu**	**Henan**	**Hubei**	**Shanghai**	**SumWijZj**	**LISA**	**Lisa from GeoDA**
									0.000		
Anhui	**2.101**	-	0.077	(0.114)	(0.010)	(0.056)	(0.034)	-	**(0.137)**	**-0.289**	-0.248
Zhejiang	**0.387**	0.525	-	(0.143)	(0.013)	-	-	(0.353)	**0.016**	**0.006**	0.005
Jiangxi	**(0.572)**	0.700	0.129	-	-	-	(0.057)	-	**0.772**	**-0.442**	-0.379
Jiangsu	**(0.051)**	0.700	0.129	-	-	-	-	(0.471)	**0.358**	**-0.018**	-0.016
Henan	**(0.281)**	1.050	-	-	-	-	(0.085)	-	**0.965**	**-0.271**	-0.233
Hubei	**(0.171)**	0.700	-	(0.191)	-	(0.094)	-	-	**0.416**	**-0.071**	-0.061
Shanghai	**(1.414)**	-	0.194	-	(0.025)	-	-	-	**0.168**	**-0.238**	-0.204

Source: Ron Briggs of UT Dallas

Local Getis-Ord G and G* Statistics

- ## Local Getis-Ord G
  - It is the proportion of all x values in the study area accounted for by the neighbors of location *I*
  - G* will include the self value

$$G_i(d) = \frac{\sum_j w_{ij} x_j}{\sum_j x_j}$$

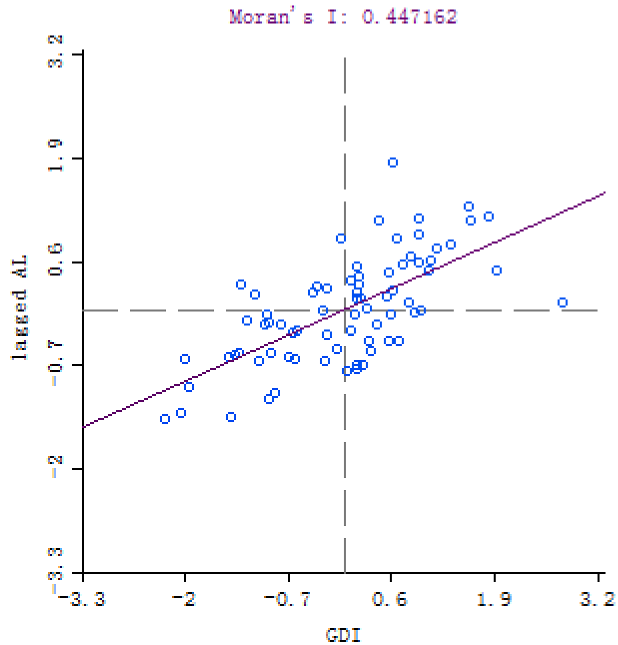
G will be high where high values cluster
G will be low where low values cluster
Interpreted relative to expected value
if randomly distributed.

$$E(G_i(d)) = \frac{\sum_j w_{ij}(d)}{n-1}$$

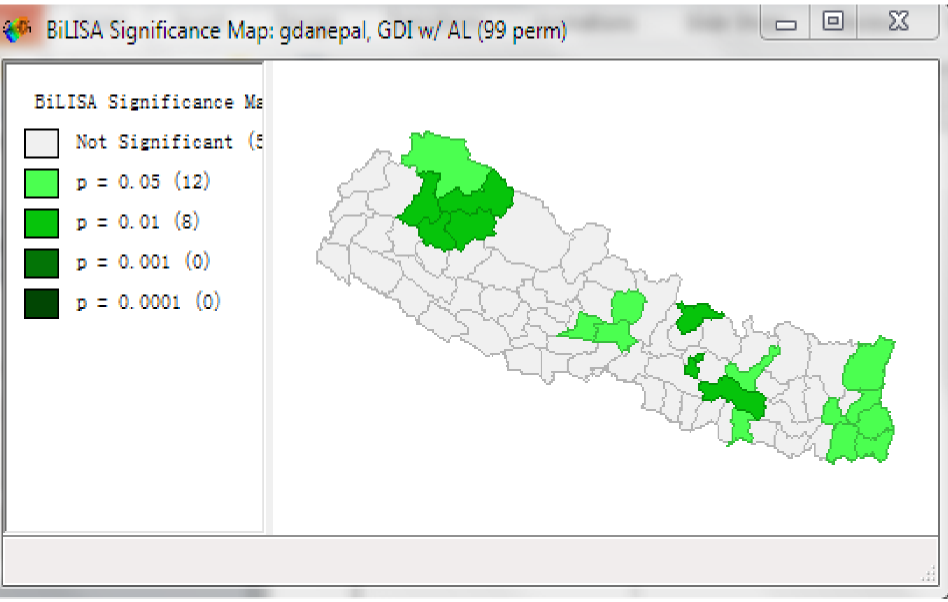
Bivariate LISA

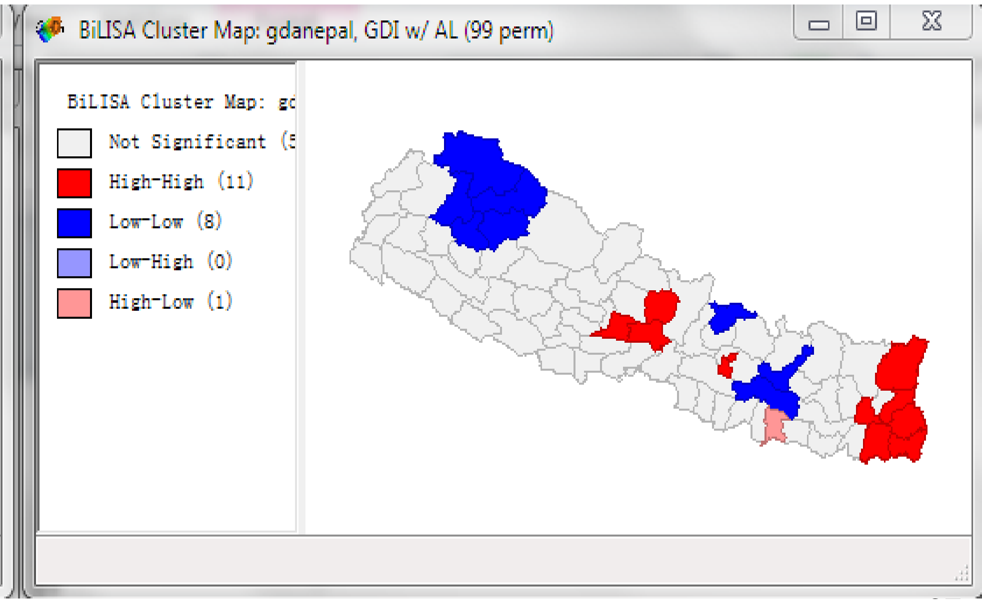
- Moran's I is the correlation between X and Lag-X--the same variable but in nearby areas
  - Univariate Moran's I
- Bivariate Moran's I is a correlation between X and a different variable in nearby areas.

Moran Scatter Plot for GDI vs AL



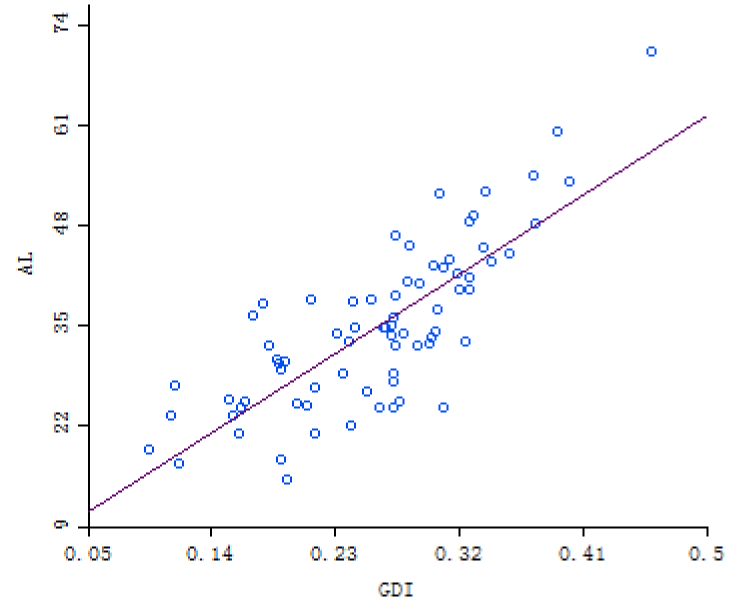
Moran Significance Map for GDI vs. AL

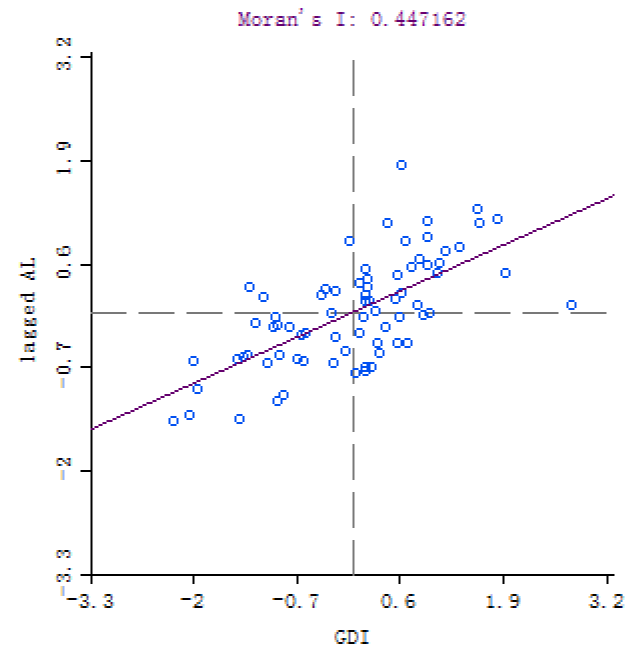




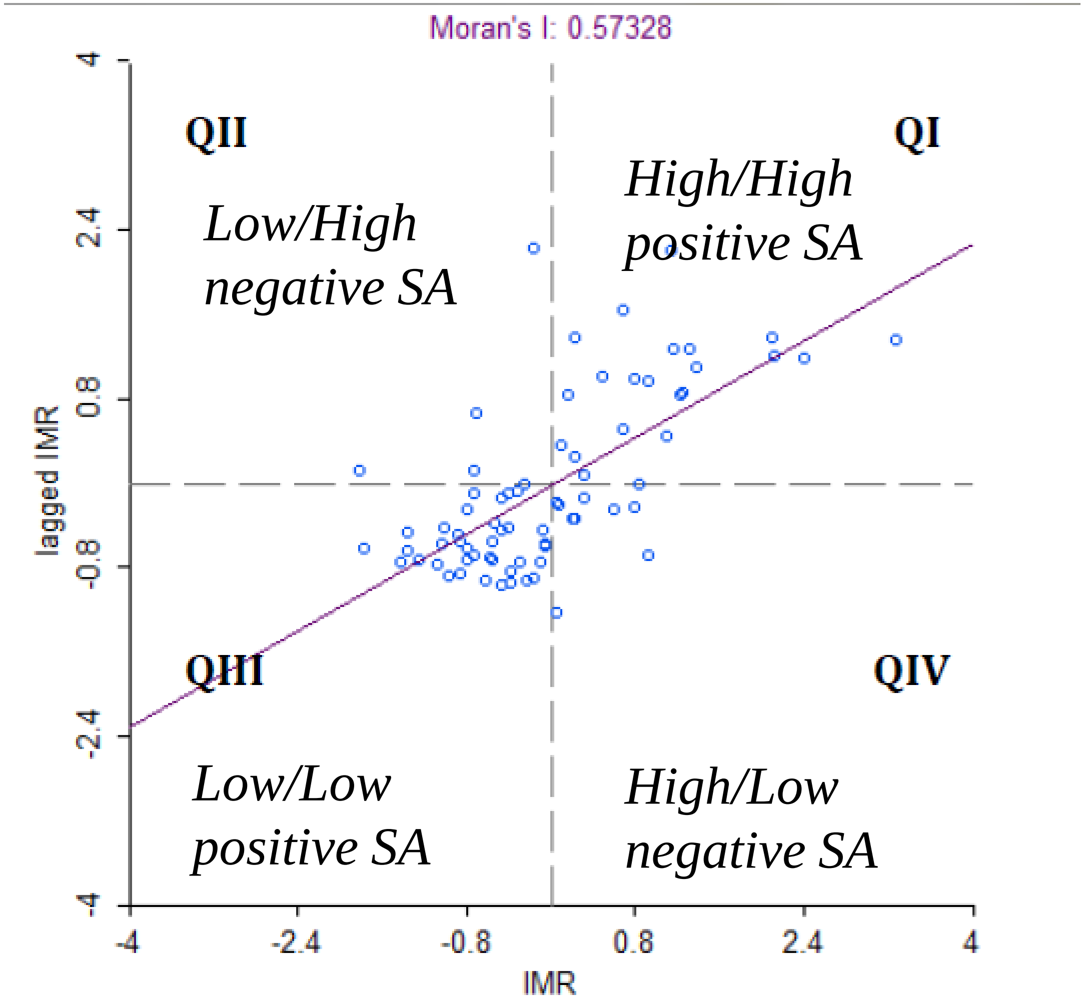
Bivariate LISA
and the Correlation Coefficie

- Correlation Coefficient is the relationship between two different variables in the same area
- Bivariate LISA is a correlation between two different variables in an area and in nearby areas.





Bivariate Moran Scatter Plot



Summary

- Spatial autocorrelation of areal data
- Spatial weight matrix
- Measures of spatial autocorrelation
- Global Measure
  – Moran's I/Geary's C/General G and G*
- Local
  - LISA: Moran's I/Geary's C/General G and G*
  - Bivariate LISA
– Significance test

- End of this topic