

GEOM 1B Geometry I, Second Semester CBE Review (ONLINE)

(v.3.1)

To the Student:

After your registration is complete, you may take the online Credit by Examination for GEOM 1B.

WHAT TO BRING

- several sharpened no. 2 pencils
- lined notebook paper
- straightedge ruler
- graphing calculator

ABOUT THE EXAM

The examination for the second semester of Geometry consists of 30 problems or questions. The exam is based on the Texas Essential Knowledge and Skills (TEKS) for this subject. The full list of TEKS is included in this document (it is also available online at the <u>Texas Education Agency website</u>). The TEKS outline specific topics covered in the exam, as well as more general areas of knowledge and levels of critical thinking. Use the TEKS to focus your study in preparation for the exam.

A formula chart is provided with the exam. A copy of the formula chart is included in this document.

The examination will take place under supervision, and the recommended time limit is three hours. You may not use any notes or books. A percentage score from the examination will be reported to the official at your school.

In preparation for the examination, review the TEKS for this subject. All TEKS are assessed. A list of key concepts is included in this document to focus your studies. It is important to prepare adequately. Since questions are not taken from any one source, you can prepare by reviewing any of the state-adopted textbooks that are used at your school. The textbook used with our GEOM 1B course is:

Larson, Ron; Boswell, Laurie, et al (2016). *Geometry, Big Ideas Math.* Erie, PA. Big Ideas Learning ISBN 13: 978–1–60840–815–3.

Good luck on your examination!

GEOM 1B Key Concepts

Be sure you are able to perform each of the tasks in the following skill areas to prepare yourself for the Geometry 1B CBE. The actual exam will contain the formula chart included in this document. When you take your examination, be sure to show *all* of your work and do not leave any questions blank.

Similarity

- Decide whether polygons are similar and to use similarity statements
- Find corresponding lengths, areas and perimeters of similar polygons
- Use the AA, SSS, and SAS Similarity Theorems
- Prove slope criteria using similar triangles
- Use Triangle Proportionality Theorem and Converse along with other similarity theorems

Right Triangles and Trigonometry

- Use the Pythagorean Theorem and Converse
- Classify triangles
- Find side lengths of special right triangles and apply special triangle relationships
- Use geometric means
- Solve problems using similar right triangles
- Use the tangent ratio
- Use the sine and cosine ratios
- Use the inverse trigonometric ratios
- Solve right triangles
- Find the areas of triangles
- Use the Law of Sines to solve triangles
- Use the Law of Cosines to solve triangles

Circles

- Identify special lines and segments of circles
- Draw, identify, and use properties of tangents and common tangents
- Find arc measures
- Identify congruent arcs
- Prove circles are similar

- Use chords to find arc measures and arc lengths
- Use inscribed angles and polygons
- Find angle and arc measures
- Use circumscribed angles
- Use segments of chords, tangents, and secants
- Write and graph equations of circles
- Write coordinate proofs involving circles

Circumference and Area

- Use the Formulas for circumference, area of a circle and population density
- Use arc lengths to find measures
- Measure angles in radians
- Find and use areas of sectors
- Find areas of regular polygons, kites, and rhombuses
- Find angle measures of regular polygons
- Find areas of composite figures
- Find the effects of proportional and non-proportional dimension changes

Surface Area and Volume

- Classify solids
- Describe cross sections of, and sketch solids of revolution
- Find and use lateral and total surface areas of right prisms and cylinders
- Find and use lateral and total surface areas of regular pyramids and right cones
- Find and use volumes of prisms and cylinders
- Find and use volumes of pyramids and cones
- Find surface areas and volumes of spheres
- Compare Euclidean and spherical geometries
- Find distances on a sphere
- Find areas of spherical triangles

Probability

- Find sample spaces
- Find theoretical and experimental probabilities

- Determine the dependence or independence of events
- Find independent and dependent probabilities
- Find conditional probabilities
- Create two-way tables
- Use relative and conditional relative frequencies to find conditional probabilities
- Find probabilities of compound events
- Find permutations and combinations
- Construct and interpret probability distributions and binomial distributions

TERMS AND VOCABULARY

The following terms and vocabulary words can and may be used in the exam. You should be able to identify and define each:

# 30-60-90 triangle 45-45-90 triangle A Adjacent arcs Angle of depression Angle of elevation Antipodal points Apothem of a regular polygon Approximation Arc length Axis of revolution B Binomial distribution C Cavalieri's Principle Center of a circle	Central angle of a regular polygon Central angle of a circle Chord of a circle Chord of a sphere Circle Circumference Circumscribed angle Circumscribed circle Combination Common tangent Compass Compound event Concentric circles Conditional probability Congruent circles Construction Corresponding parts	Cross section D Dependent events Diameter Dimension Dimensional analysis Disjoint E Edge Event Experimental probability External segment F Face G Geometric mean Geometric probability
Center of a circle	Corresponding parts Cosine	Great circle

Ρ ı Similar figures Permutation Independent events Similarity Inscribed angle Point of tangency Similarity statement Inscribed polygon Polyhedron Similarity transformation Population density Intercepted arc Simplified radical form Inverse cosine Sine Probability distribution Inverse sine Probability experiment Slant height of a regular pyramid Probability of an event Inverse tangent Slant height of a right cone Proof Solid of revolution Proportional statement Lateral area Solving a right triangle Proportionality Lateral edges Special right triangle Pythagorean Theorem Lateral faces Standard equation of a Lateral surface of a cone R circle Law of Cosines Radian Standard position Law of Sines Radius of a circle Subtend Radius of a regular Surface area M polygon Major arc Т Random variable Measure of a major arc **Tangent** Ratio Measure of a minor arc Tangent circles Regular pyramid Minor arc Tangent of a circle Right cones Tangent segment Ν Right cylinder Theoretical probability N factorial Right prism Trigonometric ratio Net S Two-way table 0 Sample space V Oblique cone Scale factor Vertex Oblique cylinder Secant Vertex of a cone Oblique prism Secant segment Vertex of a polyhedron Outcome Sector of a circle

Segments of a chord

Semicircle

Similar arcs

Overlapping events

Vertex of a pyramid

Volume

GEOM 1B Formula Chart

Segment and Slope Formulas

Partitioning a Segment: $\frac{ax_1 + bx_2}{a + b}$; with endpoints at coordinates x_1 and x_2 , that

partitions the segment in the ratio of b: a.

Point–Slope Form: $y - y_1 = m(x - x_1)$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope–Intercept Form: y = mx + b

Sum of Interior Angles of a Polygon: $(n-2)180^{\circ}$

Congruence Transformations

Translation Coordinate Notation: $(x, y) \rightarrow (x + a, y + b)$

Reflection Coordinate Notation:

About the *x*-axis: $(x, y) \rightarrow (x, -y)$

About the *y*-axis: $(x, y) \rightarrow (-x, y)$

About y = x: $(x, y) \rightarrow (y, x)$

About y = -x: $(x, y) \rightarrow (-y, -x)$

Rotation Coordinate Notation About the Origin:

90-degree counterclockwise: $(x, y) \rightarrow (-y, x)$

180-degree counterclockwise: $(x, y) \rightarrow (-x, -y)$

270-degree counterclockwise: $(x, y) \rightarrow (y, -x)$

Similarity Transformation:

Dilation coordinate notation about the origin: $(x, y) \rightarrow (kx, ky)$, where k is the scale factor

Perimeter and Area Formulas

Perimeter: sum of all sides

Circumference of a Circle: $C = 2\pi r$

Area

Square: $A = s^2$

Rectangle: A = lw

Parallelogram: A = bh

Triangle: $A = \frac{1}{2}bh$

Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Regular Polygon: $A = \frac{1}{2}ap$ a = apothem, p = perimeter

Circle $A = \pi r^2$

Surface Area and Volume Formulas for Solids

Prisms

Lateral Area: LA = ph p = perimeter of base,

h = height of figure

Surface Area: SA = ph + 2B B = area of base

Volume: V = Bh

Cylinder

Lateral Area: $LA = 2\pi rh$

Surface Area: $SA = 2\pi rh + 2\pi r^2$

Volume: $V = \pi r^2 h$

Pyramid

Lateral Area: $LA = \frac{1}{2}ps$ s = slant height

Surface Area: $SA = \frac{1}{2}ps + B$

Volume: $V = \frac{1}{3}Bh$

Cone

Lateral Area:
$$LA = \pi rs$$

Surface Area:
$$SA = \pi rs + \pi r^2$$

Volume:
$$V = \frac{1}{3}\pi r^2 h$$

Sphere

Surface Area:
$$SA = 4\pi r^2$$

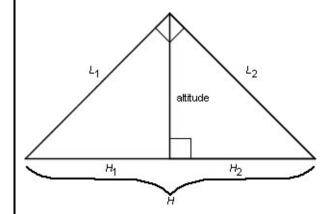
Volume:
$$V = \frac{4}{3}\pi r^3$$

Right Triangle formulas

Geometric Mean

x is the geometric mean of two values a and b if $\frac{x}{a} = \frac{b}{x}$ or $x = \sqrt{ab}$

Similar Triangle Theorems



altitude is the geometric mean between H_1 and H_2

altitude =
$$\sqrt{H_1 \cdot H_2}$$

 L_1 is the geometric mean between H_1 and H

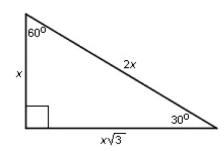
$$L_1 = \sqrt{H_1 \cdot H}$$

 L_2 is the geometric mean between H_2 and H

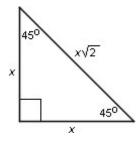
$$L_2 = \sqrt{H_2 \cdot H}$$

Special Right Triangles

30-60-90 Right Triangle



45–45–90 Right Triangle



Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Trig Formulas: SOH CAH TOA

Sine:

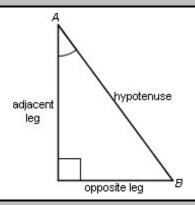
$$\sin A = \frac{opposite \ leg}{hypotenuse}$$

Cosine:

$$\cos A = \frac{adjacent\ leg}{hypotenuse}$$

Tangent:

$$\tan A = \frac{opposite\ leg}{adjacent\ leg}$$

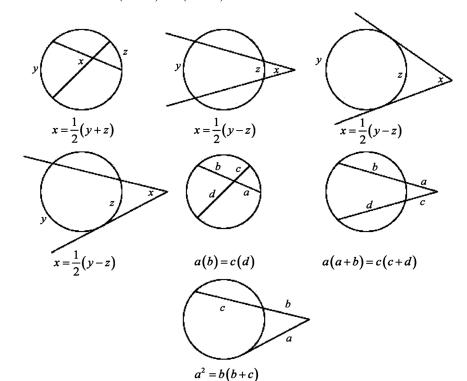


Distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

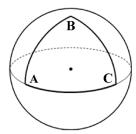
Circle Formulas

Circle in Standard Form: $(x-h)^2 + (y-k)^2 = r^2$ at center (h, k) and radius r



Area of Triangle Formula in Spherical Geometry

$$A = \frac{\pi r^2}{180} \left(m \angle A + m \angle B + m \angle C - 180^{\circ} \right)$$



Area of the Sector of a Circle

$$A = \frac{\text{arc measure}}{360^{\circ}} \bullet \pi r^2$$

Conditional Probability

$$P(B/A) = \frac{P(A \text{ and } B)}{P(A)}$$

Permutations

For *n* objects: ${}_{n}P_{n} = n!$

For *n* objects taken *r* at a time: ${}_{n}P_{r} = \frac{n!}{(n-r)!}$

Combinations Formula

The number of combinations of *n* objects taken *r* at a time, where $r \le n$, is given by:

$$_{n}C_{r}=\frac{n!}{(n-r)! \cdot r!}$$

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Texas Essential Knowledge and Skills GEOM 1 – Geometry

§111.41. Geometry, Adopted 2012. (One-Half Credit)

- (a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.
- (b) Introduction.
- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) In Geometry, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I to strengthen their mathematical reasoning skills in geometric contexts. Within the course, students will begin to focus on more precise terminology, symbolic representations, and the development of proofs. Students will explore concepts covering coordinate and transformational geometry; logical argument and constructions; proof and congruence; similarity, proof, and trigonometry; two- and three-dimensional figures; circles; and probability. Students will connect previous knowledge from Algebra I to Geometry through the coordinate and transformational geometry strand. In the logical arguments and constructions strand, students are expected to create formal constructions using a straight edge and compass. Though this course is primarily Euclidean geometry, students should complete the course with an understanding that non-Euclidean geometries exist. In proof and congruence, students will use deductive reasoning to justify, prove and apply theorems about geometric figures. Throughout the standards, the term "prove" means a formal proof to be shown in a paragraph, a flow chart, or two-column formats. Proportionality is the unifying component of the similarity, proof, and trigonometry strand. Students will use their proportional reasoning skills to prove and apply theorems and solve problems in this strand. The two- and three-dimensional figure strand focuses on the application of formulas in multi-step situations since students have developed background knowledge in two- and three-dimensional figures. Using patterns to identify geometric properties, students will apply theorems about circles to determine relationships between special segments and angles in circles. Due to the emphasis of probability and statistics in the college and career readiness standards, standards dealing with probability have been added to the geometry curriculum to ensure students have proper exposure to these topics befor
- (4) These standards are meant to provide clarity and specificity in regards to the content covered in the high school geometry course. These standards are not meant to limit the methodologies used to convey this knowledge to students. Though the standards are written in a particular order, they are not necessarily meant to be taught in the given order. In the standards, the phrase "to solve problems" includes both contextual and non-contextual problems unless specifically stated.
- (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (c) Knowledge and skills.
- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (E) create and use representations to organize, record, and communicate mathematical ideas;
- (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
- (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:
- (A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint;
- (B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines; and
- (C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point.
- (3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:
- (A) describe and perform transformations of figures in a plane using coordinate notation;
- (B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;
- (C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and
- (D) identify and distinguish between reflectional and rotational symmetry in a plane figure.
- (4) Logical argument and constructions. The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to:
- (A) distinguish between undefined terms, definitions, postulates, conjectures, and theorems;
- (B) identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse;
- (C) verify that a conjecture is false using a counterexample; and
- (D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle.
- (5) Logical argument and constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to:
- (A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools;
- (B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge;
- (C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships; and
- (D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems.
- (6) Proof and congruence. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:
- (A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems;
- (B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions;
- (C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles;
- (D) verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and apply these relationships to solve problems; and
- (E) prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships-to solve problems.
- (7) Similarity, proof, and trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:
- (A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and
- (B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.
- (8) Similarity, proof, and trigonometry. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

- (A) prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems; and
- (B) identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems.
- (9) Similarity, proof, and trigonometry. The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:
- (A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems; and
- (B) apply the relationships in special right triangles 30° - 60° - 90° and 45° - 45° - 90° and the Pythagorean theorem, including Pythagorean triples, to solve problems.
- (10) Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:
- (A) identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two-dimensional shapes; and
- (B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.
- (11) Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:
- (A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure;
- (B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;
- (C) apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and
- (D) apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.
- (12) Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:
- (A) apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems;
- (B) apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems;
- (C) apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems;
- (D) describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle; and
- (E) show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k), $(x h)^2 + (y k)^2 = r^2$.
- (13) Probability. The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to:
- (A) develop strategies to use permutations and combinations to solve contextual problems;
- (B) determine probabilities based on area to solve contextual problems;
- (C) identify whether two events are independent and compute the probability of the two events occurring together with or without replacement;
- (D) apply conditional probability in contextual problems; and
- (E) apply independence in contextual problems.

Source: The provisions of this §111.41 adopted to be effective September 10, 2012, 37 TexReg 7109.