

**To the Student:**

After your registration is complete and your proctor has been approved, you may take the Credit by Examination for GEOM 1B.

**WHAT TO BRING**

- several sharpened no. 2 pencils
- lined notebook paper
- straightedge ruler
- graphing calculator

**ABOUT THE EXAM**

The examination for the second semester of Geometry consists of 29 items or questions. The exam is based on the Texas Essential Knowledge and Skills (TEKS) for this subject. The full list of TEKS is included in this document (it is also available online at the Texas Education Agency website, <http://www.tea.state.tx.us/>). The TEKS outline specific topics covered in the exam, as well as more general areas of knowledge and levels of critical thinking. Use the TEKS to focus your study in preparation for the exam.

The examination will take place under supervision, and the recommended time limit is three hours. You may not use any notes or books. A percentage score from the examination will be reported to the official at your school.

In preparation for the examination, review the TEKS for this subject. All TEKS are assessed. A list of review topics is included in this document to focus your studies. It is important to prepare adequately. Since questions are not taken from any one source, you can prepare by reviewing any of the state-adopted textbooks that are used at your school. The textbook used with our GEOM 1B course is:

Larson, Ron; Boswell, Laurie, et al (2016). *Geometry, Big Ideas Math*. Erie, PA. Big Ideas Learning. ISBN 13: 978-1-60840-815-3.

The practice exam included in this document will give you a model of the types of questions that will be asked on your examination. It is **not** a duplicate of the actual examination. It is provided to illustrate the format of the exam, not to serve as a complete review sheet.

Good luck on your examination!

## EXAM OBJECTIVES

Be sure you are able to perform each of the tasks in the following skill areas to prepare yourself for the Geometry 1B CBE. The actual exam will contain the formula chart included with this practice exam. When you take your examination, be sure to show *all* of your work and do not leave any questions blank.

### Similarity

- Decide whether polygons are similar and to use similarity statements
- Find corresponding lengths, areas and perimeters of similar polygons
- Use the AA, SSS, and SAS Similarity Theorems
- Prove slope criteria using similar triangles
- Use Triangle Proportionality Theorem and Converse along with other similarity theorems

### Right Triangles and Trigonometry

- Use the Pythagorean Theorem and Converse
- Classify triangles
- Find side lengths of special right triangles and apply special triangle relationships
- Use geometric means
- Solve problems using similar right triangles
- Use the tangent ratio
- Use the sine and cosine ratios
- Use the inverse trigonometric ratios
- Solve right triangles
- Find the areas of triangles
- Use the Law of Sines to solve triangles
- Use the Law of Cosines to solve triangles

### Circles

- Identify special lines and segments of circles
- Draw, identify, and use properties of tangents and common tangents
- Find arc measures
- Identify congruent arcs
- Prove circles are similar

- Use chords to find arc measures and arc lengths
- Use inscribed angles and polygons
- Find angle and arc measures
- Use circumscribed angles
- Use segments of chords, tangents, and secants
- Write and graph equations of circles
- Write coordinate proofs involving circles

### **Circumference and Area**

- Use the Formulas for circumference, area of a circle and population density
- Use arc lengths to find measures
- Measure angles in radians
- Find and use areas of sectors
- Find areas of regular polygons, kites, and rhombuses
- Find angle measures of regular polygons
- Find areas of composite figures
- Find the effects of proportional and non-proportional dimension changes

### **Surface Area and Volume**

- Classify solids
- Describe cross sections of, and sketch solids of revolution
- Find and use lateral and total surface areas of right prisms and cylinders
- Find and use lateral and total surface areas of regular pyramids and right cones
- Find and use volumes of prisms and cylinders
- Find and use volumes of pyramids and cones
- Find surface areas and volumes of spheres
- Compare Euclidean and spherical geometries
- Find distances on a sphere
- Find areas of spherical triangles

### **Probability**

- Find sample spaces
- Find theoretical and experimental probabilities

- Determine the dependence or independence of events
- Find independent and dependent probabilities
- Find conditional probabilities
- Create two-way tables
- Use relative and conditional relative frequencies to find conditional probabilities
- Find probabilities of compound events
- Find permutations and combinations
- Construct and interpret probability distributions and binomial distributions

## TERMS AND VOCABULARY

The following terms and vocabulary words can and may be used in the exam. You should be able to identify and define each:

30-60-90 triangle

45-45-90 triangle

### A

Adjacent arcs

Angle of depression

Angle of elevation

Antipodal points

Apothem of a regular polygon

Approximation

Arc length

Axis of revolution

### B

Binomial distribution

### C

Cavalieri's Principle

Center of a circle

Central angle of a regular polygon

Central angle of a circle

Chord of a circle

Chord of a sphere

Circle

Circumference

Circumscribed angle

Circumscribed circle

Combination

Common tangent

Compass

Compound event

Concentric circles

Conditional probability

Congruent circles

Congruent arcs

Construction

Corresponding parts

Cosine

Cross section

### D

Dependent events

Diameter

Dimension

Dimensional analysis

Disjoint

### E

Edge

Event

Experimental probability

External segment

### F

Face

### G

Geometric mean

Geometric probability

Great circle

### I

Independent events

Inscribed angle

Inscribed polygon

Intercepted arc

Inverse cosine

Inverse sine

Inverse tangent

### L

Lateral area

Lateral edges

Lateral faces

Lateral surface of a cone

Law of Cosines

Law of Sines

### M

Major arc

Measure of a major arc

Measure of a minor arc

Minor arc

### N

N factorial

Net

### O

Oblique cone

Oblique cylinder

Oblique prism

Outcome

Overlapping events

**P**

Permutation  
 Point of tangency  
 Polyhedron  
 Population density  
 Probability distribution  
 Probability experiment  
 Probability of an event  
 Proof  
 Proportional statement  
 Proportionality  
 Pythagorean Theorem

**R**

Radian  
 Radius of a circle  
 Radius of a regular polygon  
 Random variable  
 Ratio  
 Regular pyramid  
 Right cones  
 Right cylinder  
 Right prism

**S**

Sample space  
 Scale factor  
 Secant  
 Secant segment  
 Sector of a circle  
 Segments of a chord  
 Semicircle  
 Similar arcs  
 Similar figures  
 Similarity  
 Similarity statement  
 Similarity transformation  
 Simplified radical form  
 Sine  
 Slant height of a regular pyramid  
 Slant height of a right cone  
 Solid of revolution  
 Solving a right triangle  
 Special right triangle  
 Standard equation of a circle

Standard position

Subtend

Surface area

**T**

Tangent  
 Tangent circles  
 Tangent of a circle  
 Tangent segment  
 Theoretical probability  
 Trigonometric ratio  
 Two-way table

**V**

Vertex  
 Vertex of a cone  
 Vertex of a polyhedron  
 Vertex of a pyramid  
 Volume

## GEOM 1B Formula Chart

Segment and Slope Formulas	
<b>Partitioning a Segment:</b>	$\frac{ax_1 + bx_2}{a + b}$ ; with endpoints at coordinates $x_1$ and $x_2$ , that partitions the segment in the ratio of $b : a$ .
<b>Point–Slope Form:</b>	$y - y_1 = m(x - x_1)$
<b>Slope:</b>	$m = \frac{y_2 - y_1}{x_2 - x_1}$
<b>Slope–Intercept Form:</b>	$y = mx + b$
<b>Sum of Interior Angles of a Polygon:</b> $(n - 2)180^\circ$	
Congruence Transformations	
<b>Translation Coordinate Notation:</b>	$(x, y) \rightarrow (x + a, y + b)$
<b>Reflection Coordinate Notation:</b>	<p>About the x-axis: <math>(x, y) \rightarrow (x, -y)</math></p> <p>About the y-axis: <math>(x, y) \rightarrow (-x, y)</math></p> <p>About <math>y = x</math>: <math>(x, y) \rightarrow (y, x)</math></p> <p>About <math>y = -x</math>: <math>(x, y) \rightarrow (-y, -x)</math></p>
<b>Rotation Coordinate Notation About the Origin:</b>	<p>90-degree counterclockwise: <math>(x, y) \rightarrow (-y, x)</math></p> <p>180-degree counterclockwise: <math>(x, y) \rightarrow (-x, -y)</math></p> <p>270-degree counterclockwise: <math>(x, y) \rightarrow (y, -x)</math></p>
<b>Similarity Transformation:</b>	Dilation coordinate notation about the origin: $(x, y) \rightarrow (kx, ky)$ , where $k$ is the scale factor

Perimeter and Area Formulas		
Perimeter:	sum of all sides	
Circumference of a Circle:	$C = 2\pi r$	
Area		
Square:	$A = s^2$	
Rectangle:	$A = lw$	
Parallelogram:	$A = bh$	
Triangle:	$A = \frac{1}{2}bh$	
Trapezoid:	$A = \frac{1}{2}(b_1 + b_2)h$	
Regular Polygon:	$A = \frac{1}{2}ap$	$a$ = apothem, $p$ = perimeter
Circle	$A = \pi r^2$	
Surface Area and Volume Formulas for Solids		
Prisms		
Lateral Area:	$LA = ph$	$p$ = perimeter of base, $h$ = height of figure
Surface Area:	$SA = ph + 2B$	$B$ = area of base
Volume:	$V = Bh$	
Cylinder		
Lateral Area:	$LA = 2\pi rh$	
Surface Area:	$SA = 2\pi rh + 2\pi r^2$	
Volume:	$V = \pi r^2 h$	
Pyramid		
Lateral Area:	$LA = \frac{1}{2}ps$	$s$ = slant height
Surface Area:	$SA = \frac{1}{2}ps + B$	
Volume:	$V = \frac{1}{3}Bh$	



**Cone**

Lateral Area:  $LA = \pi rs$

Surface Area:  $SA = \pi rs + \pi r^2$

Volume:  $V = \frac{1}{3} \pi r^2 h$

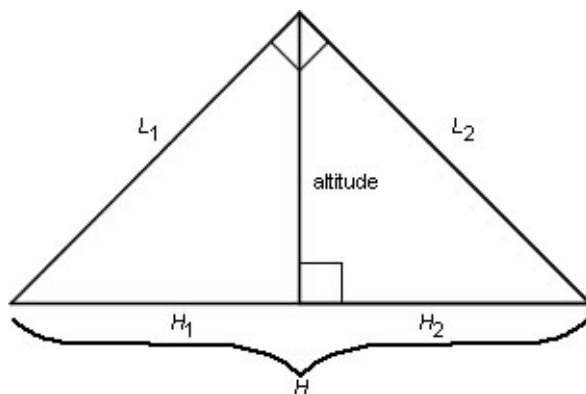
**Sphere**

Surface Area:  $SA = 4\pi r^2$

Volume:  $V = \frac{4}{3} \pi r^3$

**Right Triangle formulas****Geometric Mean**

$x$  is the geometric mean of two values  $a$  and  $b$  if  $\frac{x}{a} = \frac{b}{x}$  or  $x = \sqrt{ab}$

**Similar Triangle Theorems**

altitude is the geometric mean between  $H_1$  and  $H_2$

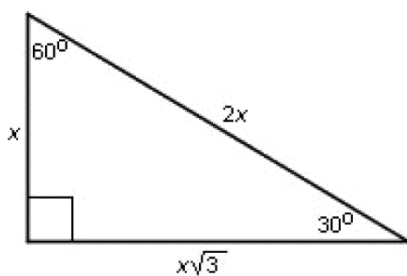
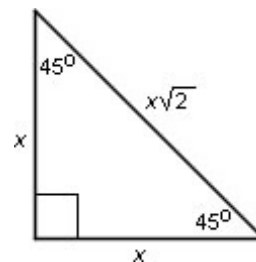
$$\text{altitude} = \sqrt{H_1 \cdot H_2}$$

$L_1$  is the geometric mean between  $H_1$  and  $H$

$$L_1 = \sqrt{H_1 \cdot H}$$

$L_2$  is the geometric mean between  $H_2$  and  $H$

$$L_2 = \sqrt{H_2 \cdot H}$$

**Special Right Triangles****30–60–90 Right Triangle****45–45–90 Right Triangle**

## Midpoint Formula

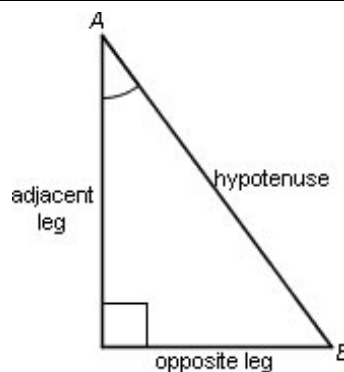
$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

## Trig Formulas: SOH CAH TOA

Sine:  $\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$

Cosine:  $\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$

Tangent:  $\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

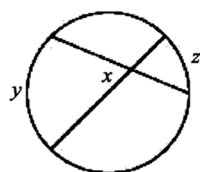


## Distance formula

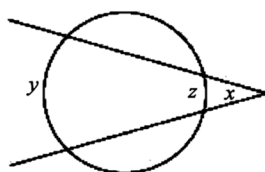
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

## Circle Formulas

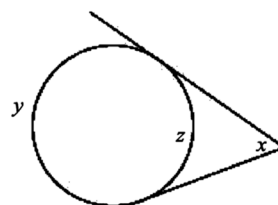
Circle in Standard Form:  $(x - h)^2 + (y - k)^2 = r^2$  at center  $(h, k)$  and radius  $r$



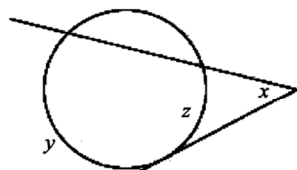
$$x = \frac{1}{2}(y + z)$$



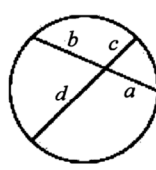
$$x = \frac{1}{2}(y - z)$$



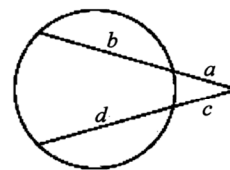
$$x = \frac{1}{2}(y - z)$$



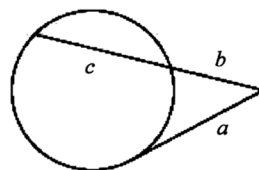
$$x = \frac{1}{2}(y - z)$$



$$a(b) = c(d)$$



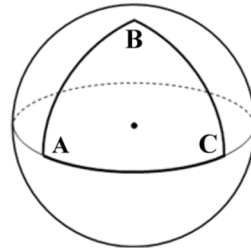
$$a(a + b) = c(c + d)$$



$$a^2 = b(b + c)$$

### Area of Triangle Formula in Spherical Geometry

$$A = \frac{\pi r^2}{180} (m\angle A + m\angle B + m\angle C - 180^\circ)$$



### Area of the Sector of a Circle

$$A = \frac{\text{arc measure}}{360^\circ} \cdot \pi r^2$$

### Conditional Probability

$$P(B / A) = \frac{P(A \text{ and } B)}{P(A)}$$

### Permutations

For  $n$  objects:  ${}_nP_n = n!$

For  $n$  objects taken  $r$  at a time:  ${}_nP_r = \frac{n!}{(n-r)!}$

### Combinations Formula

The number of combinations of  $n$  objects taken  $r$  at a time, where  $r \leq n$ , is given by:

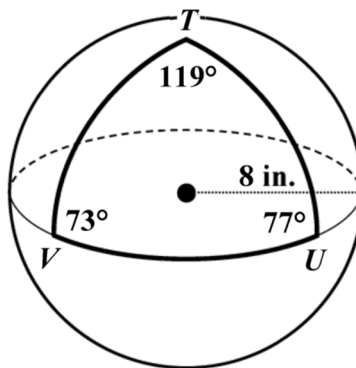
$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

## GEOM 1B Practice Exam

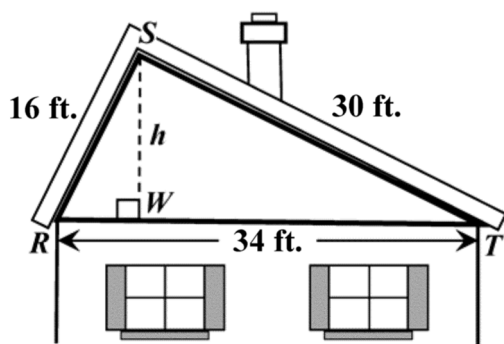
The following practice exam represents the form and types of questions on the Credit-by-Examination that you will take. Approximately one third of the problems on the GEOM 1B CBE will be in multiple choice format. The other two thirds of the exam will be in short answer or free response format.

**Answer the questions below on your own paper.**

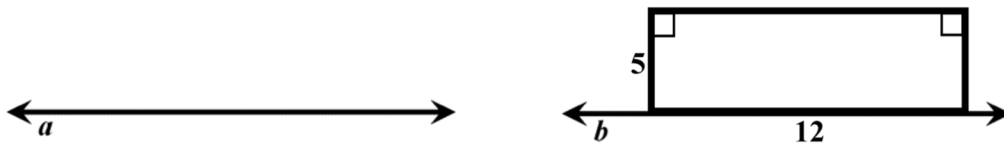
1. Find the area of the spherical triangle  $\triangle TUV$ . Find the exact answer in terms of  $\pi$ , and the approximation rounded to the nearest tenths' place.



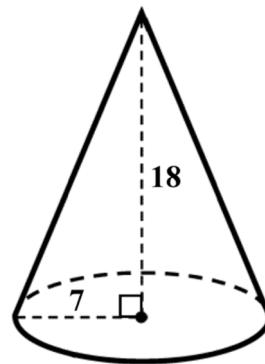
2. Jennie rides her bike 6 kilometers south and then 1 kilometer east. Use the Pythagorean Theorem to find how far she is from her starting point at the end of her ride. Draw and label a diagram of the situation. Round your answer to the nearest hundredth of a kilometer.
3. A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height  $h$  of the roof as a mixed number.



4. You are measuring the height of a tree. You stand 55 feet from the base of the tree. The angle of elevation from the ground to the top of the tree is  $49^\circ$ . Find the height of the tree to the nearest foot.
- A. 11 ft.  
 B. 70 ft.  
 C. 63 ft.  
 D. 58 ft.  
 E. None of the above
5. On the axis provided named  $a$ , sketch the solid produced by rotating the figure around the given axis  $b$ . Then identify and describe the solid. Find the exact volume of the resulting solid in terms of  $\pi$ .

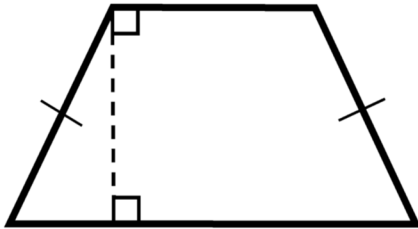


6. Using a straight edge, draw a plane that vertically intersects the following solid. Then state what kind of two-dimensional figure is formed by the cross section of the plane and the solid. What are the dimensions of the two-dimensional cross section?



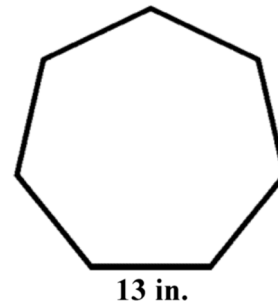
7. By what factor does doubling all the linear dimensions of the figure below, affect the perimeter, and affect the area?

- A. Perimeter factor = 2; area factor = 2
- B. Perimeter factor = 2; area factor = 4
- C. Perimeter factor = 4; area factor = 4
- D. Perimeter factor = 4; area factor = 8
- E. None of the above

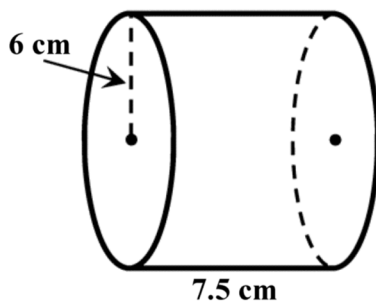


8. A table top is in the shape of a regular heptagon with 13 inch sides. What is the area of the table top rounded to the nearest tenths' place?

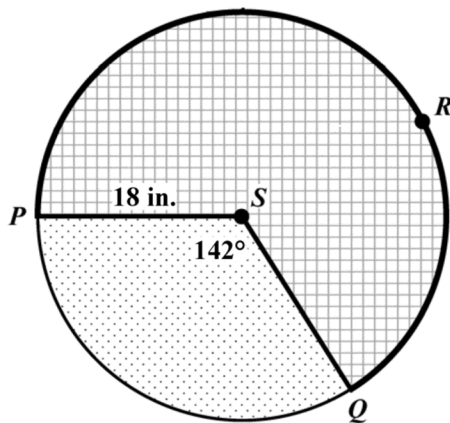
- A. 501.2 in.<sup>2</sup>
- B. 586.8 in.<sup>2</sup>
- C. 614.3 in.<sup>2</sup>
- D. 679.4 in.<sup>2</sup>
- E. None of the above



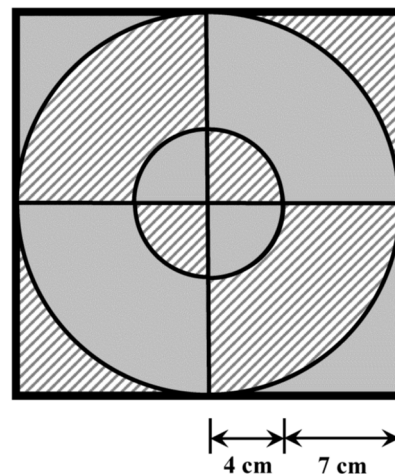
9. Find the volume, lateral surface area, and total surface area of the cylinder, all in terms of  $\pi$ .



10. A super-large pizza was purchased for a party. Pictured is the pizza with a couple of slices taken out. How much pizza in square inches, remains to serve the guests at the party. How much was taken? Round to the nearest hundredths' place.

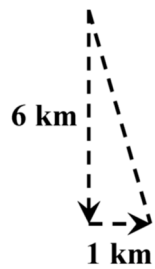


11. The point  $(-2, 8)$  is on a circle with the center at  $(-4, 7)$ . Write the standard equation of the circle.
12. You throw a dart at the board shown. Your dart is equally likely to hit any point inside the square board. What is the probability that your dart lands on one of the two solid sectors in the larger sector? Write the probability as a percent rounded to the tenths' place.

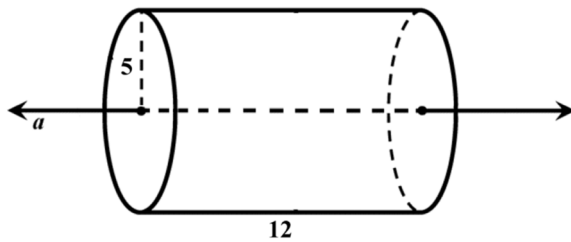


## GEOM 1B Practice Exam Answer Key:

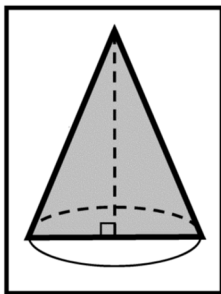
1.  $99.4 \text{ in}^2$
2.  $\approx 6.08 \text{ km}$



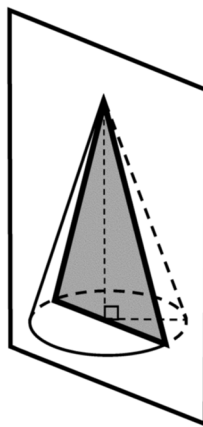
3.  $14\frac{2}{17} \text{ ft.}$
4. C. 63 ft.
5. Cylinder  
Radius of 5 and height of 12;  
 $V = 300\pi \text{ units}^3$



6. Triangle  
Height of 18 and base of 14



or





- 7. B. Perimeter factor = 2; area factor = 4
- 8. C.  $614.3 \text{ in.}^2$
- 9. L.A. =  $90 \pi \text{ cm}^2$ ; S.A. =  $162 \pi \text{ cm}^2$ ; V. =  $270 \pi \text{ cm}^3$
- 10. Pizza remaining =  $616.38 \text{ in}^2$ ; pizza remaining =  $410.50 \text{ in}^2$
- 11.  $(x + 4)^2 + (y - 7)^2 = 5$
- 12.  $\frac{52.5\pi}{484} \approx 34.1\%$

## Texas Essential Knowledge and Skills GEOM I – Geometry

### §111.41. Geometry, Adopted 2012. (One-Half Credit)

(a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Geometry, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I to strengthen their mathematical reasoning skills in geometric contexts. Within the course, students will begin to focus on more precise terminology, symbolic representations, and the development of proofs. Students will explore concepts covering coordinate and transformational geometry; logical argument and constructions; proof and congruence; similarity, proof, and trigonometry; two- and three-dimensional figures; circles; and probability. Students will connect previous knowledge from Algebra I to Geometry through the coordinate and transformational geometry strand. In the logical arguments and constructions strand, students are expected to create formal constructions using a straight edge and compass. Though this course is primarily Euclidean geometry, students should complete the course with an understanding that non-Euclidean geometries exist. In proof and congruence, students will use deductive reasoning to justify, prove and apply theorems about geometric figures. Throughout the standards, the term "prove" means a formal proof to be shown in a paragraph, a flow chart, or two-column formats. Proportionality is the unifying component of the similarity, proof, and trigonometry strand. Students will use their proportional reasoning skills to prove and apply theorems and solve problems in this strand. The two- and three-dimensional figure strand focuses on the application of formulas in multi-step situations since students have developed background knowledge in two- and three-dimensional figures. Using patterns to identify geometric properties, students will apply theorems about circles to determine relationships between special segments and angles in circles. Due to the emphasis of probability and statistics in the college and career readiness standards, standards dealing with probability have been added to the geometry curriculum to ensure students have proper exposure to these topics before pursuing their post-secondary education.

(4) These standards are meant to provide clarity and specificity in regards to the content covered in the high school geometry course. These standards are not meant to limit the methodologies used to convey this knowledge to students. Though the standards are written in a particular order, they are not necessarily meant to be taught in the given order. In the standards, the phrase "to solve problems" includes both contextual and non-contextual problems unless specifically stated.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint;

(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines; and

(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point.

(3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(A) describe and perform transformations of figures in a plane using coordinate notation;

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and

(D) identify and distinguish between reflectional and rotational symmetry in a plane figure.

(4) Logical argument and constructions. The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to:

(A) distinguish between undefined terms, definitions, postulates, conjectures, and theorems;

(B) identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse;

(C) verify that a conjecture is false using a counterexample; and

(D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle.

(5) Logical argument and constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to:

(A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools;

(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge;

(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships; and

(D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems.

(6) Proof and congruence. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

(A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems;

(B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions;

(C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles;

(D) verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and apply these relationships to solve problems; and

(E) prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships to solve problems.

(7) Similarity, proof, and trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:

(A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and

(B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.

(8) Similarity, proof, and trigonometry. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

- (A) prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems; and
  - (B) identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems.
- (9) Similarity, proof, and trigonometry. The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:
- (A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems; and
  - (B) apply the relationships in special right triangles  $30^\circ$ - $60^\circ$ - $90^\circ$  and  $45^\circ$ - $45^\circ$ - $90^\circ$  and the Pythagorean theorem, including Pythagorean triples, to solve problems.
- (10) Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:
- (A) identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two-dimensional shapes; and
  - (B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.
- (11) Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:
- (A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure;
  - (B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;
  - (C) apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and
  - (D) apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.
- (12) Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:
- (A) apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems;
  - (B) apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems;
  - (C) apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems;
  - (D) describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle; and
  - (E) show that the equation of a circle with center at the origin and radius  $r$  is  $x^2 + y^2 = r^2$  and determine the equation for the graph of a circle with radius  $r$  and center  $(h, k)$ ,  $(x - h)^2 + (y - k)^2 = r^2$ .
- (13) Probability. The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to:
- (A) develop strategies to use permutations and combinations to solve contextual problems;
  - (B) determine probabilities based on area to solve contextual problems;
  - (C) identify whether two events are independent and compute the probability of the two events occurring together with or without replacement;
  - (D) apply conditional probability in contextual problems; and
  - (E) apply independence in contextual problems.

*Source: The provisions of this §111.41 adopted to be effective September 10, 2012, 37 TexReg 7109.*