

*Taken from Departmental Handbook issued to faculty and graduate students each fall.*

## Graduate Degree Programs

### Master's Degree in Mathematics

The Department of Mathematics and Statistics offers the Master's Degree in Mathematics in each of the four options discussed below. The requirements listed below are in addition to the University and Graduate School requirements found in the Catalog of the Graduate School. Each student's program of study and committee must be approved by the Director of Graduate Studies in the Department of Mathematics and Statistics. The requirements listed here are to be regarded as *minimal* and the student should give very careful consideration to the selection of the particular option which is best suited to his or her needs. Work completed with a grade less than 'B' will not be accepted on a degree plan for any graduate degree in mathematics or statistics.

- A. This option is a 30 hour plan leading to the degree of Master of Science. This plan calls for 24 hours of course work and at least 6 hours of the *thesis* course (MATH 6000). Of the 24 hours of course work, 18 must be in mathematics and must include *one* sequence in a core area. The core areas are: 1) Algebra, 2) Ordinary Differential Equations / Partial Differential Equations 3) Complex Analysis, 4) Probability and Statistics, 5) Real Analysis, 6) Topology, 7) Numerical Analysis. In the case of Real Analysis, 5318-5319 is not considered to be a core sequence for purposes of Option A; likewise with 5310-5311 in the area of Applied Mathematics. A minor of 6 hours in an approved area outside of mathematics is permitted. A thesis defense is required.
- B. This option is a 36 hour plan leading to the degree of Master of Science. This program calls for 33 hours of course work and 3 hours of work on a departmental report (MATH 6310). Of the 33 hours of course work, 24 must be in mathematics and must include two sequences from the core areas listed in part A. A third core area is strongly recommended. A minor of 9 hours in an approved area outside mathematics is permitted. A presentation of the report and a final comprehensive examination is required.
- C. This option is a 36 hour plan leading to the degree of Master of Science in Mathematics, and is designed for students desiring an emphasis in computer science. The plan calls for 18-21 hours of course work in mathematics, 12-15 hours of course work in computer science, and 3 hours of departmental report (MATH 6310). Of the 18-21 hours of mathematics course work, at least two sequences from the following list must be included:

MATH 5318-5319	(Intermediate Analysis)
MATH 5320-5321	(Complex Analysis)
MATH 5322-5323	(Real Analysis)
MATH 5324-5325	(Topology)
MATH 5326-5327	(Modern Algebra)
STAT 5328-5329	(Intermediate Mathematical Statistics)
MATH 5330, 5332	(Ordinary Differential Equations, Partial Differential Equation)
MATH 5334-5335	(Numerical Analysis)

The 12 to 15 hours of computer science course work constitute adjunct requirements (i.e., special requirements by the Department of Mathematics). These required courses must be taken from the graduate offerings in Computer Science and/or Information Systems and Quantitative Science (ISQS) in the Business College with the approval of the Director of Graduate Studies in the Department of Mathematics. The approved ISQS courses are 5338, 5341, 6337, 6339, 7338, and 7340. CS 5301 is not approved for Program credit. All other 5000-level CS courses may be used, with permission from the Graduate Advisor. A presentation of the report and a final comprehensive examination is required for completion of the program.

It is anticipated that a substantial number of students having mathematics degrees but *no background in computer science* will wish to enter this program. Such students will be required to take undergraduate leveling courses, as deemed appropriate in their individual cases, for

*undergraduate credit only.* No such leveling work will be credited toward satisfying the above requirements.

- D. This option is a 36 hour plan leading to the Master of Arts degree. This option is designed for persons who are teaching (or plan to teach) mathematics at the pre-university level. This plan calls for 33 hours of course work and 3 hours of work on a departmental report (MATH 6310). Of the 33 hours of course work at least 24 hours must be in mathematics. Of the seven sequences listed below, the student must complete at least three where the courses within each sequence are taken consecutively: 1) Algebra, 2) Geometry, 3) Analysis, 4) Number Theory, 5) Probability and Statistics, 6) Applied Mathematics, 7) Computer Literacy and Programming. Much of this work will be taken under the course numbers MATH 5360-5361 and 5364-5365. A minor of 9 hours in an approved area outside mathematics is permitted. Normally, work in the student's second field of certification or work towards the Professional Teacher's Certificate will be an acceptable minor area. A presentation of the report and final comprehensive examination is required.

With regard to all of the above programs, it is expected that the student's final oral defense of his/her thesis or report will be open to all who wish to attend, with scheduling to reflect this.

### Master's Degree in Statistics

In order to enter the program of study leading to a Master of Science Degree in Statistics the applicant must meet the requirements of the Graduate School and of the Department of Mathematics. In addition, the applicant must satisfy the following undergraduate requirements:

- a) differential and integral calculus (Calculus I, II, III),
- b) linear algebra,
- c) FORTRAN programming or some other high-level computer language.

The requirements b) and c) may be satisfied after admission into the program, but must be satisfied before the second year of study begins.

The degree requirements are:

1. Completion of STAT 5328-5329, Intermediate Mathematical Statistics; STAT 5373, Design of Experiments; STAT 5374, Theory of Linear Statistical Models; and STAT 5375, Statistical Multivariate Analysis. In addition, the completion of any two courses from the following list:

- STAT 5372 Nonparametric Statistical Inference
- STAT 5377 Statistical Sampling Theory
- STAT 5378 Stochastic Processes
- STAT 5379 Time Series Analysis
- STAT 5386 Statistical Computing and Simulation I

2. Six hours of mathematics to be selected with the approval of the Director of Graduate Studies and the Statistics coordinator. Those students aspiring toward a Ph.D. (statistics emphasis) should take the Math 5318-5319 sequence during their first year of graduate studies.

3. One of the following two options (to be selected with the approval of the Director of Graduate Studies).

- a) Three hours in an area other than statistics, e.g., mathematics, animal science, computer science, biology, economics, engineering, psychology, or sociology. This option requires approval of the appropriate graduate advisor from the selected area.
- b) Three additional hours in Statistics (**to be selected from the Mathematics Department offerings**).

4. Either a six-hour Master's Thesis or a three-hour Master's Report with an additional three hour course to be selected from requirement 1 or 3 above. A thesis defense or a final comprehensive examination for the report is required.

**NOTE: All statistics courses for the M.S. degree must be taken from the statistics offerings in the Department of Mathematics.**

### Doctoral Program

The following represents an outline of the Department of Mathematics policies concerning the doctoral program. These policies are supplemental to the general Texas Tech University policies as outlined in the official catalogs of the university. Specific questions concerning interpretation of these policies should be directed to the Graduate Advisor.

#### **AREAS OF SPECIALIZATION**

Each doctorate in mathematics at Texas Tech University will be based on the doctoral candidate's choice of an area of specialization from the following three broad specialty areas:

1. Applied Mathematics
2. Pure Mathematics
3. Statistics

Overall policy guidelines have been established by the Department of Mathematics with respect to work within these specialty areas.

#### **PRELIMINARY EXAMINATIONS**

The Doctoral Preliminary Examinations will be administered twice each year (in May and in August) and are offered in the seven areas corresponding to the following graduate core courses:

1. Algebra (MATH 5326-5327)
2. Complex Analysis (MATH 5320-5321)
3. Ordinary Differential Equations (MATH 5330) and Partial Differential Equations (MATH 5332)
4. Numerical Analysis (MATH 5334-5335)
5. Real Analysis (MATH 5322-5323)
6. Probability and Statistics (STAT 5328-5329)
7. Topology (MATH 5324-5325)

Each examination is four hours long with content based on important fundamental concepts in the area. Students should NOT infer that the Preliminary Examination is equivalent to a final examination over the respective core area. Rather, each examination is developed by a committee of faculty in the respective core area in consultation with the Graduate Committee. The topics over which a student can be tested are listed in the Preliminary Examination Topics List which is available from the Graduate Advisor.

At least three weeks prior to taking a Preliminary Examination the student must inform the Graduate Advisor which examinations he/she wishes to take. Up to three different examinations can be taken in each administration of the Preliminary examinations. The student must pass a total of three different examinations in four consecutive administrations of the Preliminary Examinations. This includes a requirement that at least one of the examinations must be in either Algebra, Complex Analysis, Real Analysis, or Topology. A grade of *P* (pass) or *F* (fail) will be given in each examination.

Any student who in four consecutive administrations of the Preliminary Examinations does not successfully complete the Doctoral Preliminary Examinations may not continue in the Doctoral Program in the Department of Mathematics at Texas Tech University.

#### **QUALIFYING EXAMINATION**

Each doctoral student will be required to pass a Qualifying Examination on advanced topics beyond those covered in the Preliminary Examinations. In general, the Qualifying Examination will follow the format established by the Texas Tech University Graduate Catalog. Any exceptions to this format must be agreed upon by both the student's Doctoral Advisory Committee and the Graduate Advisor.

#### **FOREIGN LANGUAGE REQUIREMENT**

Any foreign language requirement will be at the discretion of the student's dissertation advisor

### AREA PROGRAMS

The following list of area programs is meant to be an outline of minimal requirements. The individual student is encouraged to supplement specified requirements with as much advanced work as possible.

Specific course requirement may be waived at the request of the student's Doctoral Advisory Committee with the consent of the Graduate Advisor.

Advanced topics seminars which contribute to the student's overall mathematical background will be offered each semester. It is expected that each student will participate in seminar work in his/her area of specialty.

Note that Math 5316, and Math 5318-5319 will not be counted toward a doctoral degree.

### APPLIED MATHEMATICS

1. All of the following three sequences:

MATH 5322-5323 Functions of a Real Variable I and II

MATH 5330 Ordinary Differential Equations I and

MATH 5332 Partial Differential Equations I

MATH 5334-5335 Numerical Analysis I and II

2. At least one of the following sequences:

MATH 5320-5321 Functions of a Complex Variable I and II

STAT 5328-5329 Intermediate Mathematical Statistics I and II

3. At least six courses, different from the courses used in part 2, chosen from:

MATH 5312-5313 Control Theory I and II

MATH 5320-5321 Functions of a Complex Variable I and II

MATH 5324-5325 Topology I and II

MATH 5326-5327 Modern Algebra I and II

MATH 5331 Ordinary Differential Equations II

MATH 5333 Partial Differential Equations II

MATH 5340-5341 Functional Analysis I and II

MATH 5354-5355 Biomathematics I and II

MATH 5382-5383 Advanced Probability I and II

STAT 5328-5329 Intermediate Mathematical Statistics I and II

STAT 5378 Stochastic Processes

STAT 5379 Time Series Analysis

4. At least six additional courses which are usually related to the student's dissertation area. These courses may be listed under MATH 5342-5343 or MATH 5344-5345.

5. Twelve hours of MATH 8000

### PURE MATHEMATICS

1. All of the following four sequences:

MATH 5320-5321 Functions of a Complex Variable I and II

MATH 5322-5323 Functions of a Real Variable I and II

MATH 5324-5325 Topology I and II

MATH 5326-5327 Modern Algebra I and II

2. An additional six hours in mathematics to serve a unifying role to the courses in Section 1. This requirement will usually be met by MATH 5340-5341 Functional Analysis I and II. With approval of the student's Doctoral Advisory Committee and the Graduate Advisor, other courses serving the same purpose may be acceptable.

3. Twelve hours of course work providing breadth in training. These will normally come from the following courses:

STAT 5328-5329 Intermediate Mathematical Statistics I and II  
MATH 5330 Ordinary Differential Equations I  
MATH 5332 Partial Differential Equations I  
MATH 5334-5335 Numerical Analysis I and II

With the approval of the student's Doctoral Advisory Committee and the Graduate Advisor, other courses serving the same purpose may be acceptable.

4. At least eighteen additional hours of advanced course work to be approved by the student's Doctoral Advisory Committee and the Graduate Advisor. These courses will normally include work related to the student's area of specialization but need not be confined to that area.

5. Twelve hours of MATH 8000

### **STATISTICS**

1. All of the following sixteen courses:

MATH 5322-5323 Functions of a Real Variable I and II  
MATH 5382-5383 Advanced Probability I and II  
STAT 5328-5329 Intermediate Mathematical Statistics I and II  
STAT 5370 Decision Theory  
STAT 5372 Nonparametric Statistical Inference  
STAT 5373 Design of Experiments

STAT 5374 Theory of Linear Statistical Models  
STAT 5375 Statistical Multivariate Analysis  
STAT 5376 Advanced Statistical Methods  
STAT 5377 Statistical Sampling Theory  
STAT 5378 Stochastic Processes  
STAT 5379 Time Series Analysis  
STAT 5380 Advanced Mathematical Statistics I

2. At least two of the following sequences:

MATH 5320-5321 Functions of a Complex Variable I and II  
MATH 5330 Ordinary Differential Equations I and II  
MATH 5332 Partial Differential Equations I  
MATH 5334-5335 Numerical Analysis I and II

3. Twelve hours of MATH 8000

**A DISSERTATION IS REQUIRED OF EVERY CANDIDATE FOR THE DOCTORAL DEGREE. THIS REQUIREMENT IS SEPARATE AND APART FROM OTHER REQUIREMENTS IN THE DOCTORAL PROGRAM. CONSEQUENTLY, SUCCESSFUL PERFORMANCE IN OTHER AREAS DOES NOT NECESSARILY GUARANTEE THE ACCEPTANCE OF A DISSERTATION. THE DISSERTATION SHOULD EMBODY A SIGNIFICANT CONTRIBUTION TO NEW INFORMATION TO THE SUBJECT.**

### **FINAL EXAMINATION**

A final public oral examination over the student's dissertation topics is required of every candidate for the doctorate.

**EACH DOCTORAL STUDENT SHOULD BECOME FAMILIAR WITH THE UNIVERSITY AND DEPARTMENTAL REQUIREMENTS AND DEADLINES FOR THE DOCTORAL DEGREE.**

## Graduate Course Outlines

The pages that follow contain information about certain courses in the main body of the graduate program. Recent textbooks and course content are included for some of the courses.

**Course Number:** Mathematics 5310-5311  
**Descriptive Title:** Principles of Classical Applied Analysis  
**Text:** *Applied Linear Algebra & notes from Applied Mathematics*, both by P. Oliver, published by Person

Partial differential equations, separation of variables, Fourier series, Sturm-Liouville theory, Green's functions, Laplace and Fourier transforms, calculus of variations

**Course Number:** Mathematics 5316  
**Descriptive Title:** Applied Linear Algebra  
**Text:** *Matrix Theory: A Second Course*, by Ortega, Plenum Press.

Solution of linear systems, matrix inversion, vector spaces, projections, determinants, eigenvalues and eigenvectors, Jordan forms, computational methods, and applications.

**Course Number:** Mathematics 5318-5319  
**Descriptive Title:** Intermediate Analysis  
**Text:** *Principles of mathematical analysis*, by W. Rudin, 3<sup>rd</sup> Ed.

This sequence covers the topics of single and multivariable advanced calculus in greater depth and with more rigor than in an ordinary advanced calculus course. It forms a bridge between senior-level analysis and the more advanced graduate studies in real analysis and functional analysis. Students with a good background in basic analysis can go to 5322-5323 directly.

*Learning Outcomes:* Upon completion of this two-semester series, students should master concepts and theories of single and multi variable calculus, including: sets, real number system, formal definition of limits of sequences, Cauchy sequences, epsilon-delta definition of limits of functions, continuous functions, differentiation, mean value theorems, Taylor's theorem, Riemann integrals, fundamental theorems of calculus, infinite series, sequences and series of functions, linear transformation and differentiation of multi-variable functions, inverse and implicit function theorems, vector calculus, and Green's, Stokes', and divergence theorems.

**Course Number:** Mathematics 5320-5321  
**Descriptive Title:** Functions of a Complex Variable  
**Text:** *Functions of One Complex Variable*, by Conway, Springer-Verlag, 2nd Edition.

Geometry and analysis of the complex plane. Topology of the plane. Analytic functions and conformal mapping. Complex integration. Residue theory. Maximum Modulus Theorem and its applications. Normal families and meromorphic functions. Analytic continuations and Riemann surfaces. Harmonic function theory.

*Learning Outcomes:* Upon completion of this two-semester series, students should master concepts and theories of geometry and analysis of complex plane, topology of the plane, analytic functions, conformal mapping, complex integration, residue theory, maximum modulus theorem and its applications, normal families, meromorphic functions, analytic continuations, and harmonic function theory.

**Course Number:** Mathematics 5322-5323

**Descriptive Title:** Real Analysis  
**Text:** *Real Analysis and Modern Techniques*, by G. Folland, 2nd Edition

Topics covered primarily during the first semester of this course include a general development of measure, integration, convergence theorems, decomposition of measures and  $L^p$  theory. Chapters 2, 3, 5 and 6 represent the core of material covered in 5322. In 5323 basic elements of functional analysis, as well as topics in Fourier analysis and probability theory, are covered. Other topics that might be covered are selected at the instructor's discretion. The necessary auxiliary topological facts are taught as they are needed for the main developments.

*Learning Outcomes:* Upon completion of this two-semester series, students should master concepts and theories of outer measures, the Caratheodory extension theorem, general measures, Lebesgue integrals with respect to a measure, Lebesgue measures, Lebesgue-Stieltjes measures, product measures, convergence theorems, Fubine-Tonelli theorem, signed measures, functions of bounded variation, absolutely continuous functions, differentiation theory, differentiation of a measure, metric spaces, compactness, Banach spaces,  $L_p$  spaces, Hilbert spaces, basic Fourier analysis, bounded linear functionals, dual spaces, and bounded linear operators.

**Course Number:** **Mathematics 5324-5325**  
**Descriptive Title:** Topology  
**Text:** *General Topology*, by S. Willard.

Mathematics 5324 will deal with the basics of metric and general topology: topologies, closure, bases and refinements, continuity, topological equivalence, compactness, paracompactness, connectedness, subspaces and embeddings, and separation properties. Core theory to be covered will include the metrization theorems of Urysohn and Bing-Nagata-Smirnov, the Tietze Extension Theorem, the Tychonoff Product Theorem, and the Stone-Cech Compactification, this fundamental cluster of material to be supplemented at the instructor's discretion.

Mathematics 5325 will deal with basic topics in algebraic topology: homotopy, fundamental groups, covering spaces, homology and exact sequence analysis, and special topics at the discretion of the instructor.

*Learning Outcomes:* In the two-semester sequence students should develop an understanding of basic concepts and relations of general topology. The first semester covers bases, subbases, subspaces, continuity and homeomorphisms, connectedness in its various forms, compactness, separation axioms, countability properties, products, quotients and metrization theorems. The second semester further covers metric and complete metric spaces, function spaces, as well as basics of homotopy theory, including covering spaces, fundamental group, surfaces and applications.

**Course Number:** **Mathematics 5326-5327**  
**Descriptive Title:** Modern Algebra  
**Text:** *Abstract Algebra*, by Dummit and Foote.

This two-semester sequence assumes the student has had undergraduate courses in abstract algebra and linear algebra. Topics covered are groups (solubility, nilpotency, Sylow theorems, groups acting on sets), rings (ideals, factorization in commutative rings), modules (standard functions and constructions, modules over principal ideal domains with applications to linear algebra), fields (extensions, Galois theory, structure of finite fields) and commutative algebra (localization, primary decomposition, polynomial and power series rings). Time permitting, topics in noncommutative ring theory will also be studied.

*Learning Outcomes:* Upon completion of this two-semester series, students should master concepts and theories of groups (solubility, nilpotency, Sylow theorems, groups acting on sets), rings (ideals, factorization in commutative rings), modules (standard functions and constructions, modules over principal ideal domains with applications to linear algebra), fields (extensions, Galois theory, structure of

finite fields), and commutative algebra (localization, primary decomposition, polynomial and power series rings).

**Course Number:**       **Statistics 5328-5329**  
**Descriptive Title:**    Intermediate Mathematical Statistics  
**Text:**                    *Statistical Inference*, by George Casella, Roger Berger, 2nd Edition.

Statistics 5328: Random variables, mathematical expectation, probability density functions, cumulative distribution functions, conditional distributions, special distributions, transformation of random variables, order statistics, moment generating functions, limiting distributions, central limit theorem, stochastic convergence.

Statistics 5329: Point estimation, interval estimation, confidence intervals for means, confidence intervals for difference of means, efficiency of estimators, statistical hypotheses, uniformly most powerful tests, likelihood ratio tests, chi-square tests, noncentral chi-square, noncentral F, sufficient statistics.

*Learning Outcomes:* Upon completion of this two-semester series, students should master concepts and theories of random variables, mathematical expectation, probability density functions, cumulative distribution functions, conditional distributions, special distributions, transformation of random variables, order statistics, moment generating functions, limiting distributions, central limit theorem, stochastic convergence, point estimators, statistical hypotheses, uniformly most powerful tests, likelihood ratio tests,  $\chi^2$  tests, noncentral  $\chi^2$ , noncentral F, and sufficient statistics.

**Course Number:**       **Mathematics 5330-5331**  
**Descriptive Title:**    **Ordinary Differential Equations**  
**Text:**                    *Nonlinear Systems*, by H. Khalil, 3<sup>rd</sup> Ed.

M5330 is regarded as the first semester of a two-semester sequence that includes M5332. M5330 is a first course in the theory of ordinary differential equations. Prerequisites are undergraduate differential equations. Mathematical maturity at the level of 5318 and 5319 is desired. The topics to be covered in the first semester include: Existence and uniqueness results for initial value problems, dependence on data, continuation of solutions and maximal intervals of existence, linear equations and systems, oscillation theory, stability, Sturm-Liouville theory and boundary value problems, Green's functions.

M5331 is an advanced topics course in ordinary differential equations. The content is selected at the discretion of the instructor.

*Learning Outcomes:* Upon completion of this course, students should master concepts and theories of existence and uniqueness of initial value problems, dependence on data, continuation of solutions and maximal intervals of existence, linear equations and systems, oscillation theory, stability. Students will acquire an introductory level knowledge on basic concepts of the theory of dynamical systems, such as invariant sets, manifolds and their stability, and chaos.

**Course Number:**       **Mathematics 5332-5333**  
**Descriptive Title:**    Partial Differential Equations  
**Text:**                    *Partial Differential Equations*, Lawrence Evans

M5332 is the second semester of the two-semester sequence that includes M5330 and is designed as a first course in partial differential equations. The content includes quasi-linear and linear first order equations, classification of PDE's, hyperbolic equations, elliptic equations, and parabolic equations.

M5333 is an advanced topics course in partial differential equations. The content is selected at the discretion of the instructor.

*Learning Outcomes:* Upon completion of this course, students should master concepts and theories of quasi-linear and linear first order partial differential equations, classification of partial differential equations, hyperbolic equations, elliptic equations, and parabolic equations.

**Course Number:**       **Mathematics 5334-5335**  
**Descriptive Title:**    Numerical Analysis  
**Text:**                    Recommended: *Concise Numerical Mathematics*, by Robert Plato.

Mathematics 5334 covers computer arithmetic and error analysis; interpolation techniques; numerical differentiation and numerical quadrature; direct and iterative methods for solution of systems of linear equations.

Mathematics 5335 covers numerical solution of ordinary differential equations; solution of nonlinear systems of equations; calculation of eigenvalues and eigenvectors; special topics.

*Learning Outcomes:* Upon completion of this two-semester series, students should become proficient in the theoretical, analytical, and computational study of numerical analysis. Students should master concepts in computer arithmetic, rounding error analysis, numerical solution of nonlinear equations in one variable, interpolation theory, numerical differentiation, numerical quadrature, numerical linear algebra, approximation theory, direct and iterative methods for solution of linear systems, computational solution of eigenvalues-eigenvectors problems, numerical solution of initial-value differential equation systems, computational solution of systems of nonlinear equations, numerical optimization, and computational solution of boundary-value problems.