

# PRELIMINARY EXAMINATION TOPICS LIST FOR COMPLEX ANALYSIS

- 1) Complex Number System and Extended Complex Plane
- 2) Properties of Analytic Functions and Harmonic Functions
  - a. Cauchy-Riemann Equations
  - b. Conformality
- 3) Power Series and Analyticity
- 4) Properties of Elementary Functions
  - a. Mapping Properties
  - b. Branch-of-Log
  - c. Power Functions
  - d. Möbius Transformations
- 5) Complex Integration
  - a. Complex Line Integral
  - b. Index of a Closed Curve
  - c. Cauchy's Theorems
  - d. Cauchy Integral Formulae
  - e. Morera's Theorem
  - f. Cauchy-Goursat Theorem
- 6) Properties of Analytic Functions
  - a. Zeros of an Analytic Function
  - b. Identity Theorem
  - c. Open Mapping Theorem
  - d. Maximum Modulus Theorem
  - e. Schwarz's Lemma
  - f. Liouville's Theorem
  - g. Fundamental Theorem of Algebra
- 7) Singularities
  - a. Laurent Expansions
  - b. Residue Theorem
  - c. Argument Principle
  - d. Rouché's Theorem
  - e. Jensen's Theorem
- 8) Spaces of Analytic Functions
  - a. Normality and Montel's Theorem
  - b. Hurwitz's Theorem
  - c. Riemann Mapping Theorem
  - d. Weierstrass Factorization

9) Spaces of Meromorphic Functions

- a. Runge's Theorem
- b. Mittag-Leffler Theorem

10) Special Functions

- a. Gamma Function
- b. Beta Function
- c. Zeta Function

11) Harmonic Functions

- a. Dirichlet Problem
- b. Mean-Value Theorem
- c. Maximum Principle
- d. Poisson Formula
- e. Harnack's Theorem
- f. Green's Function

12) Analytic Continuation

- a. Schwarz Reflection Principle
- b. Schwarz-Christoffel Formula