

PRELIM TOPICS FOR PDE EXAM

- (1) Linear and Quasilinear equations of First Order
 - (a) Single first order equations
 - (b) Complete integrals, envelopes
 - (c) Vector fields and integral curves and surfaces
 - (d) Quasilinear equations in $\mathbb{R}^2$: shocks, conservation laws, etc.
 - (e) Characteristics (Transport equation)
 - (f) Hamilton-Jacobi equation, weak solution
 - (g) Hopf-Lax formula
 - (h) Conservation Laws,
 - (i) Burgers' equation,
 - (ii) Lax-Oleinik formula,
 - (iii) Integral solution
 - (iv) Rankine-Hugoniot condition
 - (i) Similarity solutions
 - (j) Dispersive equations (Airy's equation, Schrodinger's equation, Korteweg-De Vries equation)
 - (k) Smoothness of solutions, Cauchy-Kovalevski theorem
- (2) Elliptic Equations
 - (a) Laplace and Poisson's equations
 - (b) Harmonic functions
 - (c) Boundary value problems
 - (d) Green's identities and uniqueness
 - (e) Fundamental solutions and Green's functions
 - (f) Maximum principle and its consequences
 - (g) Mean value theorem
 - (h) Separation of variables and eigenfunction expansions
- (3) Parabolic Equations
 - (a) Some results from real and functional analysis
 - (i) Lebesgue dominated convergence theorem, monotone convergence theorem
 - (ii) L^p spaces, H^s spaces, $W^{m,p}$ spaces, $C^{k,\alpha}$, etc.
 - (iii) Weak derivatives
 - (iv) Hölder's inequality
 - (v) Young's inequality (Cauchy's inequality)
 - (vi) Convolutions, Young's inequality for convolutions
 - (vii) Jensen's inequality
 - (viii) Rademacher's theorem
 - (ix) Mollifiers and Approximate Identity

- (x) Schwartz class $\mathcal{S}$, space of smooth functions with compact support
- (xi) Fourier transform and Laplace transform (to solve equations)
- (xii) Riemann-Lebesgue Lemma
- (xiii) Plancherel Formula
- (xiv) Sobolev inequalities, Poincaré inequalities
- (b) Heat equation in $\mathbb{R}^n$
- (c) Heat equation in bounded domains
- (d) Maximum principle and uniqueness theorems
- (e) Distributions
- (f) Fundamental solutions
- (g) Separation of variables and eigenfunction expansions
- (4) Hyperbolic Equations
 - (a) One-dimensional wave equation
 - (b) D'Alembert's formula
 - (c) Energy methods
 - (d) Domain of dependance, range of influence
 - (e) Forward and backward characteristic cones
 - (f) Cauchy problem in $\mathbb{R}^n$
 - (g) Spherical means and the solution in $\mathbb{R}^3$
 - (h) The method of descent and the solution in $\mathbb{R}^2$
 - (i) Duhamel's principle
 - (j) Wave equation in bounded domains
 - (k) Conservation of energy
 - (l) Examples of boundary value problems
 - (m) Separation of variables and eigenfunction expansions