

PRELIMINARY EXAMINATION TOPICS LIST FOR COMPLEX ANALYSIS

- 1) Complex Number System and Extended Complex Plane
- 2) Properties of Analytic Functions and Harmonic Functions
 - a. Cauchy-Riemann Equations
 - b. Conformality
- 3) Power Series and Analyticity
- 4) Properties of Elementary Functions
 - a. Mapping Properties
 - b. Branch-of-Log
 - c. Power Functions
 - d. Möbius Transformations
- 5) Complex Integration
 - a. Complex Line Integral
 - b. Index of a Closed Curve
 - c. Cauchy's Theorems
 - d. Cauchy Integral Formulae
 - e. Morera's Theorem
 - f. Cauchy-Goursat Theorem
- 6) Properties of Analytic Functions
 - a. Zeros of an Analytic Function
 - b. Identity Theorem
 - c. Open Mapping Theorem
 - d. Maximum Modulus Theorem
 - e. Schwarz's Lemma
 - f. Liouville's Theorem
 - g. Fundamental Theorem of Algebra
- 7) Singularities
 - a. Laurent Expansions
 - b. Residue Theorem
 - c. Argument Principle
 - d. Rouché's Theorem
 - e. Jensen's Theorem
- 8) Spaces of Analytic Functions
 - a. Normality and Montel's Theorem
 - b. Hurwitz's Theorem
 - c. Riemann Mapping Theorem
 - d. Weierstrass Factorization

9) Spaces of Meromorphic Functions

- a. Runge's Theorem
- b. Mittag-Leffler Theorem

10) Special Functions

- a. Gamma Function
- b. Beta Function
- c. Zeta Function

11) Harmonic Functions

- a. Dirichlet Problem
- b. Mean-Value Theorem
- c. Maximum Principle
- d. Poisson Formula
- e. Harnack's Theorem
- f. Green's Function

12) Analytic Continuation

- a. Schwarz Reflection Principle
- b. Schwarz-Christoffel Formula