PRELIMINARY EXAMINATION TOPICS LIST FOR NUMERICAL ANALYSIS

1. Numerical Linear Algebra

- (a) Methods for solving linear equations
 - i. Direct methods
 - A. Gaussian elimination (LU and Cholesky factorizations)
 - B. Orthogonal decomposition
 - ii. Iterative methods
 - A. Jacobi, Gauss-Seidel, and SOR methods
 - B. Conjugate gradient method and Krylov subspace methods
- (b) Methods for solving for eigenvalues and eigenvectors
 - i. Power method
 - ii. Inverse power method
 - iii. QR method
- (c) Singular value decomposition
- 2. Computer arithmetic
 - (a) Floating point arithmetic
 - (b) Rounding errors
- 3. Approximation Theory
 - (a) Polynomial approximation
 - i. Weierstrass approximation theorem
 - ii. Lagrange interpolation
 - iii. Hermite interpolation
 - iv. Least squares (orthogonal polynomials)
 - v. Best approximation in the uniform norm (minimax)
 - (b) Piecewise polynomials
 - i. Continuous piecewise linear interpolation
 - ii. Cubic spline interpolation
 - (c) Trigonometric approximation
 - (d) Rational approximation (Pade approximations)
- 4. Numerical differentiation and integration
 - (a) Numerical differentiation

- (b) Numerical integration (quadrature)
 - i. Newton-Cotes methods
 - ii. Gaussian quadrature
 - iii. Romberg integration (Richardson extrapolation)
- 5. Initial-Value Differential Equation Systems
 - (a) Taylor series methods
 - (b) Runge-Kutta methods
 - (c) Multistep methods
 - (d) Predictor-Corrector methods
 - (e) Extrapolation methods
 - (f) Stiff systems
- 6. Numerical Solution of Nonlinear Equations
 - (a) Equations in one variable
 - i. Bisection method
 - ii. Fixed point method
 - iii. Newton's method
 - iv. Secant method, Muller's method
 - v. Acceleration of convergence (Aitken's method, Steffensen's method)
 - (b) Systems of nonlinear equations
 - i. Contraction mapping theorem
 - ii. Newton's method
 - iii. Quasi-Newton methods
 - iv. Minimization and the steepest descent method
- 7. Linear Two-point Boundary Value Problems
 - (a) Finite-difference methods
 - (b) Linear shooting method
 - (c) Galerkin method (finite element method)