

ODE Preliminary Exam

May 2023

Part A Answer 3 of the following 4 questions. Clearly indicate which 3 you choose in the boxes below. Any work on the remaining problems will not be factored into your grade.

Problems to grade:

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1. Consider the system

$$\begin{aligned}\dot{x} &= y^3 - x^3 \\ \dot{y} &= -x^3 - y^3.\end{aligned}$$

Prove that the origin is globally asymptotically stable.

2. Consider the system

$$\begin{aligned}\dot{x}_1 &= -x_1 \\ \dot{x}_2 &= -x_2 + x_1^2 \\ \dot{x}_3 &= x_3 + x_1^2.\end{aligned}$$

- (a) Identify all equilibria and classify their stability.
(b) Find the stable and unstable manifolds for the origin.

3. Consider the system

$$\begin{aligned}\dot{x}_1 &= \mu x_1 - x_1 x_2 \\ \dot{x}_2 &= x_1^2 - 4x_2.\end{aligned}$$

Does this system have any periodic orbits? Support your answer.

4. Consider the system

$$\begin{aligned}\dot{x} &= \mu x - x^2 \\ \dot{y} &= -y.\end{aligned}$$

- (a) Identify all equilibria and classify their stability.
(b) Identify and describe the bifurcation that takes place as μ is varied.
(c) Apply Sotomayor's Theorem to verify the bifurcation.

Part B Answer the following question.

1. Consider the system in polar coordinates

$$\dot{r} = 2r^2 - r^3 + \mu r$$

$$\dot{\theta} = 1.$$

- (a) Identify all equilibria and limit cycles and classify their existence and stability as they depend on parameter μ .
- (b) Sketch sample phase portraits for different values of μ .
- (c) Identify the bifurcation(s) that occur as μ is varied.