

Name: \_\_\_\_\_

**Part A: Answer 3 of the following 4 questions. Clearly indicate which 3 you choose.**

1. Consider the linear system
- $\dot{x} = Ax$
- where

$$A = \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- (a) Determine the fundamental matrix  
 (b) Use the method of variation of parameters to solve the linear nonhomogeneous

system  $\dot{x} = Ax + b(t)$  where  $x(0) = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$  and  $b(t) = \begin{bmatrix} \sin(2t) \\ \cos(2t) \\ 1 \end{bmatrix}$

2. Consider the system

$$\begin{aligned} \dot{x} &= 4y^3 - xy^2 \\ \dot{y} &= -3x + x^2y \end{aligned}$$

- (a) Find all the equilibrium points  
 (b) Apply LaSalle's Invariance Principle to determine the stability of the origin.

3. Consider the equation

$$\dot{x} = x - \mu x(1 - x)$$

where parameter  $\mu > 0$ .

- (a) Identify all equilibria as functions of  $\mu$  and classify their stability  
 (b) Identify any bifurcations that take place as  $\mu$  is varied. Sketch a bifurcation diagram

4. Consider the system

$$\begin{aligned} \dot{x} &= \mu x - y - x^3 \\ \dot{y} &= x + \mu y - y^3 \end{aligned}$$

- (a) Classify the stability of the equilibrium at the origin as a function of  $\mu$ .  
 (b) Does this system exhibit a Hopf bifurcation as  $\mu$  is varied? Provide evidence.  
*Hint:*  $\frac{1}{2} \leq \cos^4(\theta) + \sin^4(\theta) \leq 1$

**Part B: Answer the following question.**

1. Consider the system

$$\begin{aligned} \dot{x} &= y - x^2 \\ \dot{y} &= \alpha x + \beta - y \end{aligned}$$

- (a) Identify all equilibria and use linearization to investigate their stability as they depend on parameters  $\alpha$  and  $\beta$ .

- (b) Identify and describe any bifurcations that takes place as  $\alpha$  and  $\beta$  are varied. Provide sample phase-planes.
- (c) Identify regions of  $\alpha - \beta$  parameter space that yield qualitatively different behaviors (i.e. Quantify the number of equilibria in different regions of  $\alpha - \beta$  parameter space).