ODE Preliminary Exam

May 2022

Part A: Answer 3 of the following 4 questions. Clearly indicate which 3 you choose in the box below. Any work on the remaining problem will not be factored into your grade.

Problems to grade:

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1. Consider the system

$$\dot{x} = y^3 - x^3$$
$$\dot{y} = -x^3 - y^3$$

prove that the origin is globally asymptotically stable.

2. Consider the system

$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_2 + x_1^2$$
$$\dot{x}_3 = x_3 + x_1^2$$

   (a) Identify all equilibria and classify their stability.

   (b) Find the stable and unstable manifolds for the origin.

3. Consider the system

$$\dot{x_1} = \mu x_1 - x_1 x_2$$
$$\dot{x_2} = x_1^2 - 4x_2$$

Does this system have any periodic orbits? Support your answer.

4. Consider the system

$$\dot{x} = \mu x - x^2$$
$$\dot{y} = -y$$

   (a) Identify all equilibria and classify their stability.

   (b) Identify and describe the bifurcation that takes place as μ is varied.

   (c) Apply Sotomayar's theorem to verify the bifurcation.

Part B: Answer the following question.

1. Consider the system in polar coordinates

$$\dot{r} = 2r^2 - r^3 + \mu r$$
$$\dot{\theta} = 1$$

   (a) Identify all equilibria and limit cycles and classify their existence and stability as they depend on parameter μ.

   (b) Sketch sample phase portraits for different values of μ.

   (c) Identify the bifurcation(s) that occur as μ is varied.