

Prelim Topics For ODE/PDE Exam

I. Ordinary Differential Equations

1. Examples of Explicit Solution Techniques
2. Initial Value Problems
 - (a) Existence and Uniqueness of Solutions
 - (b) Continuation and Maximal Intervals of Existence
 - (c) Dependence on Data
 - (d) Applications of Gronwall type inequalities
3. Linear Differential Equations
 - (a) N th Order Linear Equations
 - (b) Linear Systems with Constant Coefficients
 - (c) Matrix Algebra (including Matrix Exponentials and Canonical Forms)
 - (d) General Solutions and Vector Space Concepts
 - (e) Wronskian
 - (f) Abel Formula
 - (g) Fundamental Set, Fundamental Matrix, Fundamental Solution
 - (h) Variation of Parameters Formula for Nonhomogeneous Equations
4. Stability Theory
 - (a) Stability of Linear Systems with Constant Coefficients
 - (b) Routh-Hurwitz Criteria
 - (c) Stability of Linear Systems with Variable Coefficients
 - (d) Autonomous Systems and the Phase Plane
 - (e) Stability of Nonlinear Systems
 - (f) Stability by Linearization
 - (g) Lyapunov Functions and Lyapunov Theorems
 - (h) Center Manifold Theory
5. Global Theory
 - (a) Dynamical Systems and Global Existence Theorems
 - (b) Limit Set and Attractors
 - (c) Periodic Orbits and Limit Cycles
 - (d) Poincaré Map
 - (e) Poincaré-Bendixson Theorem
 - (f) Bendixson's Criteria
 - (g) Index Theory

II. Partial Differential Equations

1. Linear and Quasilinear equations of First Order
 - (a) Single First Order Equations
 - (b) Vector Fields and Integral Curves and Surfaces
 - (c) Quasilinear Equations in \mathbb{R}^2 : shocks, conservation laws, etc.
 - (d) Characteristics for Second Order Equations and Higher Order Equations
 - (e) The Cauchy Problem for Higher Order Equations and Well Posedness
 - (f) Holmgren Uniqueness Theory
 - (g) Smoothness of Solutions: Cauchy-Kovalevski Theorem, Classical Lewy example equation with no C^1 solution
2. Canonical Forms
 - (a) Reduction to Canonical Forms
 - (b) Classification of 2nd Order Equations in Two Variables: Hyperbolic, Elliptic, and Parabolic
3. Hyperbolic Equations
 - (a) The One Dimensional Wave Equation
 - (b) D'Alembert's Formula
 - (c) Energy Methods
 - (d) Domain of Dependence, Range of Influence
 - (e) Forward and Backward Characteristic Cones
 - (f) The Cauchy Problem in \mathbb{R}^n , $n = 2, 3$
 - (g) Spherical Means and the Solution in \mathbb{R}^3
 - (h) The method of Descent and the Solution in \mathbb{R}^2
 - (i) Duhamel's Principle
 - (j) Wave Equation in Bounded Domains
 - (k) Conservation of Energy
 - (l) Examples of Boundary Value Problems
 - (m) Separation of Variables and Eigenfunction Expansions
4. Elliptic Equations
 - (a) Laplace and Poisson's Equations
 - (b) Harmonic Functions
 - (c) Boundary Value Problems
 - (d) Green's Identities and Uniqueness
 - (e) Fundamental Solutions and Green's Functions
 - (f) The Maximum Principle and its Consequences
 - (g) The Mean Value Theorem
 - (h) Separation of Variables and Eigenfunction Expansions
5. Parabolic Equations
 - (a) Some Results from Functional Analysis

- i. The classical L^p function spaces
- ii. Hölder's inequality
- iii. Young's Inequality
- iv. Convolutions
 - v. Mollifiers and Approximate Identity
 - vi. Schwartz class \mathcal{D} of smooth compactly supported functions
 - vii. \mathcal{S} of rapidly decreasing functions
- viii. Fourier transform
 - ix. Riemann-Lebesgue Lemma
 - x. Plancherel Formula
- (b) The Heat Equation in \mathbb{R}^n
- (c) The Heat Equation in Bounded Domains
- (d) Maximum Principle and Uniqueness Theorems
- (e) Distributions
- (f) Fundamental Solutions
- (g) Separation of Variables and Eigenfunction Expansions