PRELIM TOPICS FOR PDE EXAM

(1) Linear and Quasilinear equations of First Order
  (a) Single first order equations
  (b) Complete integrals, envelopes
  (c) Vector fields and integral curves and surfaces
  (d) Quasilinear equations in $\mathbb{R}^2$: shocks, conservation laws, etc.
  (e) Characteristics (Transport equation)
  (f) Hamilton-Jacobi equation, weak solution
  (g) Hopf-Lax formula
  (h) Conservation Laws,
    (i) Burgers' equation,
    (ii) Lax-Oleinik formula,
    (iii) Integral solution
    (iv) Rankine-Hugoinot condition
  (i) Similarity solutions
  (j) Dispersive equations (Airy's equation, Schrodinger's equation, Korteweg-De Vries equation)
  (k) Smoothness of solutions, Cauchy-Kovalevski theorem
(2) Elliptic Equations
  (a) Laplace and Poisson's equations
  (b) Harmonic functions
  (c) Boundary value problems
  (d) Green's identities and uniqueness
  (e) Fundamental solutions and Green's functions
  (f) Maximum principle and its consequences
  (g) Mean value theorem
  (h) Separation of variables and eigenfunction expansions
(3) Parabolic Equations
  (a) Some results from real and functional analysis
    (i) Lebesgue dominated convergence theorem, monotone convergence theorem
    (ii) L^p spaces, H^s spaces, $W^{m,p}$ spaces, $C^{k,\alpha}$, etc.
    (iii) Weak derivatives
    (iv) Hölder's inequality
    (v) Young's inequality (Cauchy's inequality)
    (vi) Convolutions, Young's inequality for convolutions
    (vii) Jensen's inequality
    (viii) Rademacher's theorem
    (ix) Mollifiers and Approximate Identity

(x) Schwartz class $\mathcal{S}$, space of smooth functions with compact support
(xi) Fourier transform and Laplace transform (to solve equations)
(xii) Riemann-Lebesgue Lemma
(xiii) Plancherel Formula
(xiv) Sobolev inequalities, Poincaré inequalities
(b) Heat equation in $\mathbb{R}^n$
(c) Heat equation in bounded domains
(d) Maximum principle and uniqueness theorems
(e) Distributions
(f) Fundamental solutions
(g) Separation of variables and eigenfunction expansions

(4) Hyperbolic Equations
(a) One-dimensional wave equation
(b) D'Alembert's formula
(c) Energy methods
(d) Domain of dependance, range of influence
(e) Forward and backward characteristic cones
(f) Cauchy problem in $\mathbb{R}^n$
(g) Spherical means and the solution in $\mathbb{R}^3$
(h) The method of descent and the solution in $\mathbb{R}^2$
(i) Duhamel's principle
(j) Wave equation in bounded domains
(k) Conservation of energy
(l) Examples of boundary value problems
(m) Separation of variables and eigenfunction expansions