

# PRELIMINARY EXAMINATION TOPICS FOR REAL VARIABLES

## 1. **Basic Analysis and Topology**

Metric spaces, compactness, connectedness, continuity, series and sequences, Arzela-Ascoli theorem, Stone-Weierstrass theorem

## 2. **Measure and Integration Theory**

Outer measure and the Caratheodory extension theorem, measures, Borel and Lebesgue measure, Egoroff's theorem, Lusin's theorem, integration with respect to a measure, Riemann-Stieltjes integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem, complex and signed measure, Hahn decomposition, Jordan decomposition, Radon-Nikodym theorem, Lebesgue decomposition, Fubine-Tonelli theorem.

## 3. **Differentiation Theory**

Functions of bounded variation, absolutely continuous functions, differentiation of monotone functions, Differentiation of Borel measures, fundamental theorem of calculus for Lebesgue integrals, Lebesgue differentiation theorem

## 4. **Functional Analysis**

Banach spaces, Hilbert spaces, bounded linear operators and functional duals spaces, Hahn-Banach theorem, closed graph theorem, open mapping theorem, uniform boundedness principle, Baire category theorem, weak topologies

## 5. **$L^p$ Spaces**

Inequalities of Jensen, Minkowski, and Holder, completeness, dual of  $L^p$

## 6. **Other Topics**

Basic Fourier analysis, Riesz representation theorems for various spaces of continuous functions