TOPOLOGY PRELIMINARY EXAM, May, 2001

WORK ALL PROBLEMS. STATE PRECISELY ANY MAJOR THEOREMS AND RESULTS USED.

1. Prove that the closure of a connected set is connected.
2. Prove that a topological space satisfies the fourth separation axiom (axiom T_4) if and only if any neighborhood of any closed set contains the closure of some neighborhood of the same set.
3. Prove that for a metric space separability is equivalent to being second countable.
4. Prove the *Lindelöf Theorem:* Every open cover of a second countable space contains a countable subcover.
5. Prove that the cube I^n is compact.
6. Given manifold $X = (-4,4)$ find its class C^r and state orientability if the atlas of X consists of three charts $\left(U_1 = (-4,0),\ \ \varphi_1(x) = 1 - x\right)$, $\left(U_2 = (-2,2),\ \ \varphi_2(x) = x^3\right)$, and $\left(U_3 = (0,4),\ \ \varphi_3(x) = -x - 1\right)$. If X happens to be orientable then construct an orientation ω.
7. Compute X-polynomials for the link  .
8. Compute: (i) the fundamental group $\pi_1(X, x_0)$ of the space X (see Fig. 1), (ii) the fundamental group $\pi_1(X \times Y)$ where X and Y are depicted in Figs 1 and 2, and (iii) the fundamental group $\pi_1(X \vee Y)$ where X and Y are depicted in Figs 1 and 2.

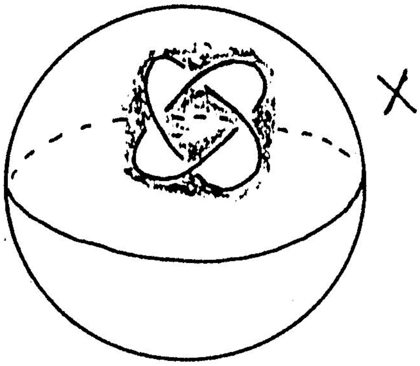

Fig. 1.

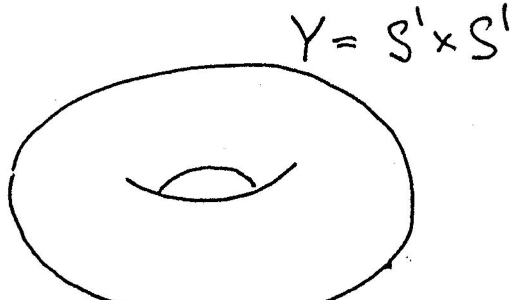

Fig. 2.

9. The generators g_1 and g_2 of the fundamental group of a topological space X homeomorphic to a disk with two holes are depicted in Fig. 3. Compute the homotopy class of the loop a depicted in Fig.4.

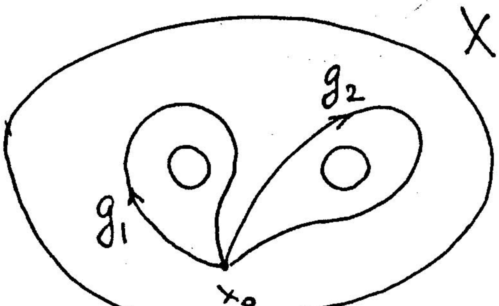

Fig. 3.

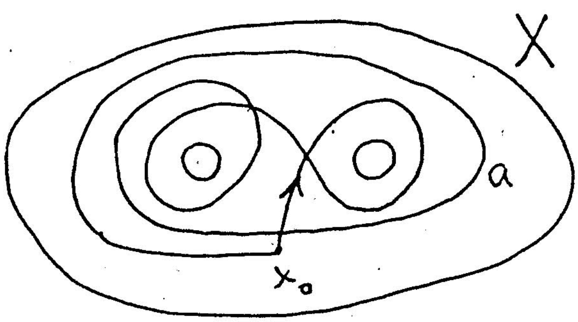

Fig. 4.

10. Compute the fundamental group of the space X which is homeomorphic to a projective plane with three holes.