TOPOLOGY DOCTORAL PRELIMINARY EXAMINATION

May 2008

WORK ALL PROBLEMS. GIVE AS COMPLETE ARGUMENTS FOR PROOFS AND DESCRIPTIONS OF EXAMPLES AS POSSIBLE. IF ANY MAJOR THEOREM IS USED IN ANY ARGUMENT, GIVE A PRECISE STATEMENT OF THE THEOREM.

1. Let $f : X \to Y$ be a bijective continuous function. Show that if X is compact and Y is Hausdorff, then f is a homeomorphism.

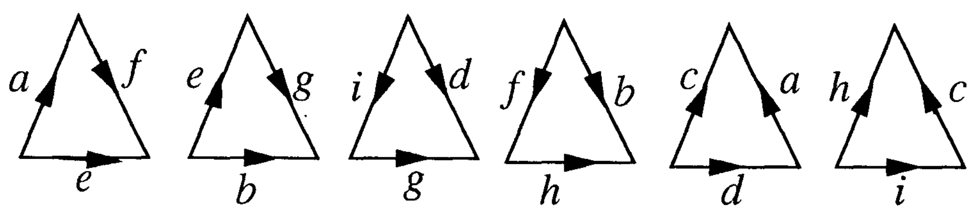
2. Is it true that a locally path connected space is path connected? Is it true that a path connected space is locally path connected? Justify your answers.

3. (a) Show that a subspace of a Hausdorff space is Hausdorff.
   (b) Show that a subspace of a regular space is regular.

4. State and prove the Lebesgue number theorem.

5. Write the Klein bottle as a semisimplicial (i.e. Δ)-complex, then compute its homology with coefficients in $\mathbb{Z}$.

6. Identify, according to the classification theorem of surfaces, the surface obtained by gluing the following triangles along edges as specified:



7. Find, with justification, a presentation of the fundamental group of a genus 3 orientable surface.

8. (a) Let $p : E \to B$ be a covering map, let $p(e_0) = b_0$. Show that any path $f : [0, 1] \to B$ beginning at b_0 has a unique lifting to a path $\tilde{f}$ in E beginning at e_0.
   (b) Let $p : E \to B$ be a covering map and let $p(e_0) = b_0$. Let the map $F : [0, 1] \times [0, 1] \to B$ be continuous with $F(0, 0) = b_0$. Show that there is a unique lifting of F to a continuous map $\tilde{F} : [0, 1] \times [0, 1] \to E$ such that $\tilde{F}(0, 0) = e_0$.