TOPOLOGY PRELIMINARY EXAM
AUGUST 2013

Endeavor to provide details for any arguments made during the course of a proof that are as complete as possible. If any major theorem is used in any argument, give a precise statement of that theorem. Do all 8 problems.

(1) Prove that every compact subspace of a Hausdorff topological space is closed.

(2) Compute, carefully and with all necessary details, the homology of a point.

(3) Prove that a topological space X is regular if and only if for every point $x \in X$ and every open neighborhood U of X, there is an open neighborhood V of x such that $\overline{V} \subset U$.

(4) Suppose that a group G acts properly discontinuously on a connected, locally path connected space X. Prove that the quotient map

$$\rho : X \to X/G$$

satisfies the axioms for a covering space.

(5) Give a statement and proof of Urysohn's Lemma.

(6) Suppose that

$$f, g : X \to Y$$

are two homotopic continuous maps from a space X to a space Y. Prove that they induce the same map in homology.

(7) Compute the fundamental group of the Klein bottle.

(8) Suppose that

$$\rho : (\widetilde{X}, \tilde{x}_0) \to (X, x_0)$$

is a covering space. Show that every path $\gamma : [0,1] \to X$ with initial point x_0 has a unique lift $\tilde{\gamma} : [0,1] \to \widetilde{X}$ with initial point $\tilde{x}_0$.